

Problem 1-1

If we call the number x , then the problem statement says that $1 = 2020x$. Solving, $x = \boxed{\frac{1}{2020}}$.

Problem 1-2

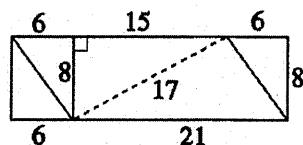
Since the primes n and $n+2$ differ by 2, n is odd, so $n+2$ and $n+4$ are both odd. If an odd integer is divided by 3, the only possible remainders are 0, 1, or 2. If n leaves a remainder of 0, n would be the one divisible by 3. If n leaves a remainder of 1 when divided by 3, then $n+2$ would be divisible by 3. If n leaves a remainder of 2 when divided by 3, then $n+4$ would be divisible by 3. The only prime divisible by 3 is 3. Since 1 isn't prime, the only such sequence of primes is 3, 5, 7—so there's only $\boxed{1}$ value of n .

Problem 1-3

Method I: Coordinatize, placing the rectangle's lower left vertex at $(0,0)$. The length of the dashed line segment with endpoints $(6,0)$ and $(21,8)$ is $\boxed{17}$.

Method II:

Solution without words:



Problem 1-4

In the employee-intern pairings, equal numbers of employees and interns are selected. Of those who remain, there are as many employees as interns, so the probability that the number of copper rings equals the number of brass rings is $\boxed{1}$.

Problem 1-5

Call the number N . The sum of N 's two largest divisors is odd, so one of N 's two largest divisors is even, hence 2 is also a divisor of N . Since 2 is clearly N 's smallest divisor > 1 , N 's largest proper divisor is $N/2$. Since N 's largest divisor is N itself, $N + N/2 = 111$, so $N = \boxed{74}$.

Problem 1-6

Since a and b have greatest common divisor 1, the highest power of each prime divisor of ab must be a factor of a or a factor of b , but not of both. The prime factors of $30!$ are the primes below 30, and the highest power of each such prime will be a factor of exactly one of a or b . The primes less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29, ten primes in all. The total number of possible integer pairs (a,b) is 2^{10} . In half of these pairs, $a < b$; in half, $a > b$. The answer to the question is $2^{10}/2 = 2^9$ or $\boxed{512}$.

[NOTE: By factoring into primes, we get $30! = 2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17^1 \times 19^1 \times 23^1 \times 29^1$.]