Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding Extrema on a Closed Interval In Exercises 1–8, find the absolute extrema of the function on the closed interval.

1. $f(x) = x^2 + 5x, \quad [-4, 0]$
2. $f(x) = x^3 + 6x^2, \quad [-6, 1]$
3. $f(x) = \sqrt{x} - 2, \quad [0, 4]$
4. $h(x) = 3\sqrt{x} - x, \quad [0, 9]$
5. $f(x) = \dfrac{4x}{x^2 + 9}, \quad [-4, 4]$
6. $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}, \quad [0, 2]$
7. $g(x) = 2x + 5\cos x, \quad [0, 2\pi]$
8. $f(x) = \sin 2x, \quad [0, 2\pi]$

Using Rolle's Theorem In Exercises 9–12, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

9. $f(x) = 2x^2 - 7, \quad [0, 4]$
10. $f(x) = (x - 2)(x + 3)^2, \quad [-3, 2]$
11. $f(x) = \dfrac{x^2}{1 - x^2}, \quad [-2, 2]$
12. $f(x) = \sin 2x, \quad [-\pi, \pi]$

Using the Mean Value Theorem In Exercises 13–18, determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

If the Mean Value Theorem cannot be applied, explain why not.

13. $f(x) = x^{2/3}, \quad [1, 8]$
14. $f(x) = \dfrac{1}{x}, \quad [1, 4]$
15. $f(x) = |5 - x|, \quad [2, 6]$
16. $f(x) = 2x - 3\sqrt{x}, \quad [-1, 1]$
17. $f(x) = x - \cos x, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
18. $f(x) = \sqrt{x} - 2x, \quad [0, 4]$

19. **Mean Value Theorem** Can the Mean Value Theorem be applied to the function

$$f(x) = \frac{1}{x^2}$$

on the interval $[-2, 1]$? Explain.

20. **Using the Mean Value Theorem**
    (a) For the function $f(x) = Ax^2 + Bx + C$, determine the value of c guaranteed by the Mean Value Theorem on the interval $[x_1, x_2]$.
    (b) Demonstrate the result of part (a) for $f(x) = 2x^2 - 3x + 1$ on the interval $[0, 4]$.

Intervals on Which f Is Increasing or Decreasing In Exercises 21–26, identify the open intervals on which the function is increasing or decreasing.

21. $f(x) = x^2 + 3x - 12$
22. $h(x) = (x + 2)^{1/3} + 8$
23. $f(x) = (x - 1)^2(x - 3)$
24. $g(x) = (x + 1)^3$
25. $h(x) = \sqrt{x}(x - 3), \quad x > 0$
26. $f(x) = \sin x + \cos x, \quad [0, 2\pi]$

Applying the First Derivative Test In Exercises 27–34, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

27. $f(x) = x^2 - 6x + 5$
28. $f(x) = 4x^3 - 5x$
29. $h(t) = \dfrac{1}{4}t^4 - 8t$
30. $g(x) = \dfrac{x^3 - 8x}{4}$
31. $f(x) = \dfrac{x + 4}{x^2}$
32. $f(x) = \dfrac{x^2 - 3x - 4}{x - 2}$
33. $f(x) = \cos x - \sin x, \quad (0, 2\pi)$
34. $g(x) = \dfrac{3}{2}\sin\left(\dfrac{\pi x}{2} - 1\right), \quad [0, 4]$

Finding Points of Inflection In Exercises 35–40, find the points of inflection and discuss the concavity of the graph of the function.

35. $f(x) = x^3 - 9x^2$
36. $f(x) = 6x^4 - x^2$
37. $g(x) = x\sqrt{x + 5}$
38. $f(x) = 3x - 5x^3$
39. $f(x) = x + \cos x, \quad [0, 2\pi]$
40. $f(x) = \tan\dfrac{x}{4}, \quad (0, 2\pi)$

Using the Second Derivative Test In Exercises 41–46, find all relative extrema. Use the Second Derivative Test where applicable.

41. $f(x) = (x + 9)^2$
42. $f(x) = 2x^3 + 11x^2 - 8x - 12$
43. $g(x) = 2x^2(1 - x^2)$
44. $h(t) = t - 4\sqrt{t + 1}$
45. $f(x) = 2x + \dfrac{18}{x}$
46. $h(x) = x - 2\cos x, \quad [0, 4\pi]$