AB Calculus **Related Rates Morning Quiz Review**

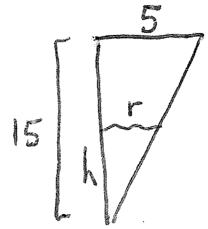
1. A conical paper cup is 15 cm tall with a radius of 5 cm. Water is being poured into the cup at a rate of 32π cm³/sec. At the moment when the depth of water is rising at 72 cm/sec, find the radius of the water level at that time. $(V = \frac{1}{3}\pi r^2 h)$

2. The area of a circle increases at a rate of 1cm^2/s. How fast is the diameter changing when the circumference is 2cm? $A = \pi r^2$ $C = 2\pi r$ diameter = 2r

3. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

AB Calculus **Related Rates Morning Quiz Review** Key

1. A conical paper cup is 15 cm tall with a radius of 5 cm. Water is being poured into the cup at a rate of 32π cm³/sec. At the moment when the depth of water is rising at 72 cm/sec, find the radius of the water level at that time. $(V = \frac{1}{3}\pi r^2 h)$



$\frac{r}{5} = \frac{h}{15}$

$5h = 15r$

$h = 3r$

$\frac{h}{3} = r$

$\frac{dV}{dt} = 32\pi \text{ cm}^3/\text{sec}$

$\frac{dh}{dt} = 72 \text{ cm/sec}$

$h = \_\_\_$

$r = \_\_\_$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{9} \cdot h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$

$32\pi = \frac{\pi}{9} h^2 (72)$

$32\pi = 8\pi h^2$

$4 = h^2$

$\underline{2 = h}$

Since $h = 3r$,

$2 = 3r$

$\frac{2}{3} = r$

$\boxed{r = \frac{2}{3} \text{ cm}}$

2. The area of a circle increases at a rate of 1cm²/s. How fast is the diameter changing when the circumference is 2cm? $A = \pi r^2$ $C = 2\pi r$ diameter = 2r

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

$\frac{dA}{dt} = 1 \text{ cm}^2/\text{s}$

$C = 2 \text{ cm}$

$C = 2\pi r$

$2 = 2\pi r$

$\frac{2}{2\pi} = r$

$\frac{1}{\pi} = r$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$1 = 2\pi\left(\frac{1}{\pi}\right)\left(\frac{dr}{dt}\right)$

$1 = 2\left(\frac{dr}{dt}\right)$

$\frac{1}{2} = \frac{dr}{dt}$

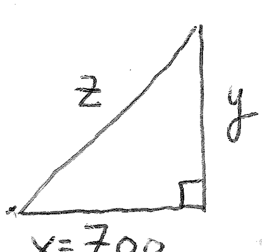
$d = 2r$

$\frac{dd}{dt} = 2\left(\frac{dr}{dt}\right)$

$\frac{dd}{dt} = 2\left(\frac{1}{2}\right)$

$\boxed{\frac{dd}{dt} = 1 \text{ cm/s}}$

3. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?



$x = 700$ (constant)

$700^2 + 2400^2 = z^2$

$2500 = z$

$\frac{dy}{dt} = 900 \text{ ft/s}$

$\frac{dz}{dt} = \_\_\_$

$y = 2400$

$z = \underline{2500}$

$700^2 + y^2 = z^2$

$0 + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(2400)(900) = 2(2500)\left(\frac{dz}{dt}\right)$

$\frac{2(2400)(900)}{2(2500)} = \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 864 \text{ ft/s}}$