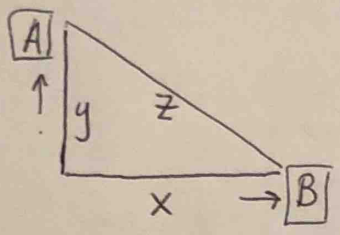
Key

Related Rates Morning Review WS #3

1. Two cyclists leave from the same position. Cyclist A travels due North at 10 mph. **One hour later,** the cyclist B leaves from the position and travels due East at 20 mph. At what rate is the distance between the two cyclists changing **2 hours after cyclist B leaves**?



$x = 20\text{mph} \times 2\text{hrs} = 40\text{mi}$

$y = 10\text{mph} \times 3\text{hrs} = 30\text{mi}$

$z = \underline{\quad 50 \quad}$
$30^2+40^2=z^2$

$\frac{dx}{dt} = 20\text{mph}$

$\frac{dy}{dt} = 10\text{mph}$

$\frac{dz}{dt} = \_\_\_\_\_ ?$

$x^2+y^2=z^2$

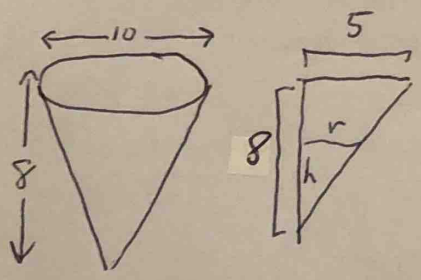
$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$ $\longrightarrow$ $2(40)(20)+2(30)(10)=2(50)\left(\frac{dz}{dt}\right)$

$1600 + 600 = 100\left(\frac{dz}{dt}\right)$

$2200 = 100\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt} = 22\text{mph}}$

2) Water is being pumped into a conical tank that is 8 feet tall and has a diameter of 10 feet. If the water is being pumped in at a constant rate of 3/5 cubic feet per hour, at what rate is the depth of the water in the tank changing when the tank is half full? $\left(V = \frac{\pi}{3}r^2h\right)$



$\frac{dV}{dt} = \frac{3}{5}\text{ ft}^3/\text{hr}$

$\boxed{h = 4\text{ ft}}$ save until after derivative to plug in.

$\frac{dh}{dt} = \_\_\_ ?$

Rewrite Volume equation in terms of h.

$\frac{r}{5} = \frac{h}{8}$

$8r = 5h$

$r = \boxed{\frac{5}{8}h}$

$V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{5h}{8}\right)^2h$

$V = \frac{\pi}{3}\cdot\frac{25h^2}{64}\cdot h$

$V = \frac{25\pi}{192}h^3$

$\frac{dV}{dt} = \frac{25\pi}{192}\cdot 3h^2\cdot\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{75\pi}{192}h^2\left(\frac{dh}{dt}\right)$

$\frac{3}{5} = \frac{75\pi}{192}\cdot(4)^2\left(\frac{dh}{dt}\right)$

$\frac{3}{5} = \frac{1200\pi}{192}\left(\frac{dh}{dt}\right)$

$\frac{3}{5} = \frac{25\pi}{4}\left(\frac{dh}{dt}\right)$

$\frac{3}{5}\cdot\frac{4}{25\pi} = \frac{dh}{dt}$

$\frac{12}{125\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{12}{125\pi}\text{ ft/hr}}$

3) **The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?**

$(A = \pi r^2 \quad C = 2\pi r)$

$\frac{dr}{dt} = 0.2\ m/s \quad \frac{dA}{dt} = \_\_?$ $\quad C = 20\pi$ meters

$A = \pi r^2$ | $C = 2\pi r$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$ | $\frac{dC}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

$C = 2\pi r$

$\frac{20\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$10 = r$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

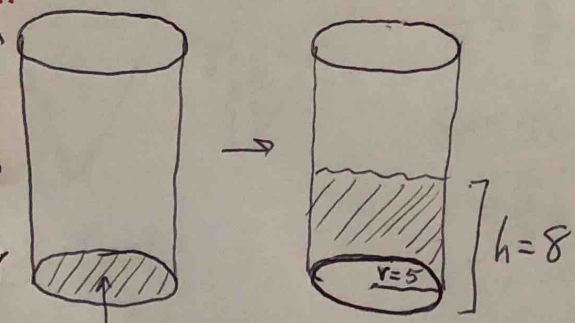
$\frac{dA}{dt} = 2\pi(10)(0.2)$

$\boxed{\frac{dA}{dt} = 4\pi\ m^2/sec}$

4) A cylindrical tank has a height of 16 feet with the area of the circular base being 25π ft^2.

Water flows at 8 cubic feet per minute into the tank. How fast is the water level rising when the tank is half full? $(Area\ of\ circle = \pi r^2)$ $(Volume\ of\ cylinder = \pi r^2 h)$

not used in problem!



$A = 25\pi$

$A = \pi r^2$

$25\pi = \pi r^2$

$25 = r^2$

$r = 5$

$\frac{dV}{dt} = 8\ ft^3/min \quad h = 8$ ← * not used!

$\frac{dh}{dt} = \_\_?$

* Radius r is constant:

$V = \pi r^2 h$

$V = \pi(5)^2 h$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi\left(\frac{dh}{dt}\right)$

$8 = 25\pi\left(\frac{dh}{dt}\right)$

$\frac{8}{25\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{8}{25\pi}\ ft/min}$