Related Rates Test Review

1. Helium is pumped into a spherical balloon at the constant rate of 25 cubic feet/minute. At what rate is the surface area of the balloon increasing at the moment when its radius is 8 feet?

2. A spherical snowball with an outer layer of ice melts so that the volume of the snowball decreases at a rate of $2\ \frac{cm^3}{min}$. How fast is the radius changing when diameter of the snowball is 10 cm?

3. Two cars start moving from the same point. One travels south at 60 m/hr and the other travels west at 25 m/hr. At what rate is the distance between the cars increasing two hours later?

b) Find ROC of Area

4. A street light is mounted on a 15 foot pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? Write down the exact answer. b) Find ROC of Area

5. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?
b) Find ROC of Area

6) Find $\lim_{x \to 1} \frac{x^3-1}{4x^3-x-3}$

7) Use linear approximation to approximate $\frac{1}{\sqrt[3]{8.04}}$

Related Rates Test Review $V=\frac{4}{3}\pi r^3$ $\quad S=4\pi r^2$ Key

1. Helium is pumped into a spherical balloon at the constant rate of 25 cubic feet/minute. At what rate is the surface area of the balloon increasing at the moment when its radius is 8 feet?

$\frac{dV}{dt}=25\ ft^3/min.$

$\frac{ds}{dt}=$ ___

$r=8\ ft.$

$\frac{dV}{dt}=4\pi r^2\left(\frac{dr}{dt}\right)$

$25=4\pi(8)^2\left(\frac{dr}{dt}\right)$

$\frac{25}{256\pi}=\frac{dr}{dt}$

$\frac{ds}{dt}=8\pi r\left(\frac{dr}{dt}\right)$

$\frac{ds}{dt}=8\pi(8)\left(\frac{25}{256\pi}\right)$

$\boxed{\frac{ds}{dt}=\frac{25}{4}\ ft^2/min}$

2. A spherical snowball with an outer layer of ice melts so that the volume of the snowball decreases at a rate of $2\ \frac{cm^3}{min}$. How fast is the radius changing when diameter of the snowball is 10 cm?

$r=5$

$\frac{dV}{dt}=-2\ cm^3/min$

$V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt}=4\pi r^2\left(\frac{dr}{dt}\right)$

$-2=4\pi(5)^2\left(\frac{dr}{dt}\right)$

$\frac{-2}{25(4)\pi}=\frac{dr}{dt}$

$\boxed{\frac{dr}{dt}=\frac{-1}{50\pi}\ ft/min}$

36) Find Rate of Change in Area

$A=\frac{1}{2}xy$

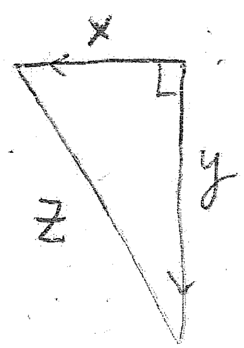
$\frac{dA}{dt}=\frac{1}{2}\frac{dx}{dt}y+\frac{1}{2}x\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt}=\frac{1}{2}(25)(120)+\frac{1}{2}(50)(60)$

$=1500+1500$

$=\boxed{3000\ m^2/hr.}$

3. Two cars start moving from the same point. One travels south at 60 m/hr and the other travels west at 25 m/hr. At what rate is the distance between the cars increasing two hours later?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(50)(25)+2(120)(60)=2(130)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=65\ mph}$

$\frac{dx}{dt}=25\ mph$ $\quad x=25(2)=50$

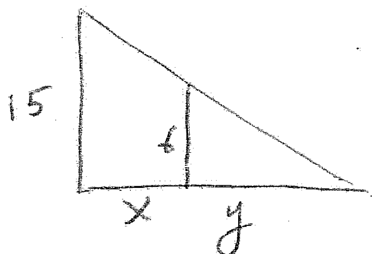
$\frac{dy}{dt}=60\ mph$ $\quad y=60(2)=120$

$x^2+y^2=z^2$

$50^2+120^2=z^2$

$z=130$

4. A street light is mounted on a 15 foot pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? Write down the exact answer.



$\frac{dx}{dt} = 5 \text{ ft/s}$

$x = 40 \text{ ft}$

$\frac{6}{15} = \frac{y}{x+y}$

$6x + 6y = 15y$

$6x = 9y$

$6\left(\frac{dx}{dt}\right) = 9\left(\frac{dy}{dt}\right)$

$6(5) = 9\left(\frac{dy}{dt}\right)$

$\frac{30}{9}\,3 = \frac{10}{3} \text{ ft/s}$

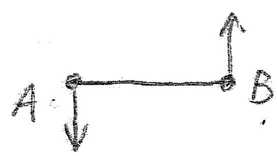
$\text{tip} = \frac{10}{3} + 5 = \boxed{\frac{25}{3} \text{ ft/s}}$

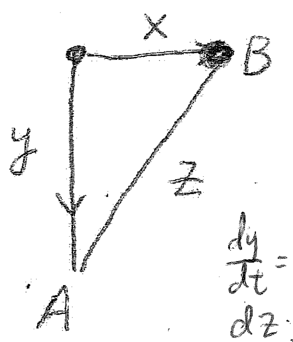
$\boxed{4B}$ How fast is area changing? (large triangle)

$A = \frac{1}{2}(15)(x+y)$

$\frac{dA}{dt} = \frac{15}{2}\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = \frac{15}{2}\left(\frac{25}{3}\right) = \boxed{62.5 \text{ ft}^2/\text{s}}$

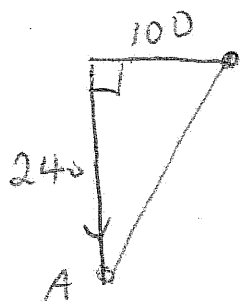
5. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?





$\frac{dy}{dt} = 60 \text{ km/hr}$

$\frac{dz}{dt} =$



$y = 60(4) = 240 \text{km}$

$100^2 + 240^2 = z^2$

$z = 260$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$\cancel{2(100)(0)} + 2(240)(60) = 2(260)\left(\frac{dz}{dt}\right)$

$\frac{dz}{dt} = 55.385 \text{ km/hr}$

$\boxed{5B}$ $A = \frac{1}{2}(100)(y)$

$\frac{dA}{dt} = 50\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt} = 50(60)$

$\frac{dA}{dt} = 3000 \text{km}^2/\text{hr.}$

6) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} = \frac{0}{0} \rightarrow \lim_{x\to 1} \frac{3x^2}{12x^2-1}$ $\boxed{\frac{3}{11}}$

7) $\frac{1}{\sqrt[3]{8.04}}$ $y = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$

$\left(8, \frac{1}{2}\right)$ $y' = -\frac{1}{3}x^{-4/3}$ $y' = \frac{-1}{3x^{4/3}}$

$y'(8) = \frac{-1}{3(8)^{4/3}} = \frac{-1}{48}$

$m = -1/48$

$y - \frac{1}{2} = \frac{-1}{48}(x-8)$

$y = \frac{-1}{48}(x-8) + \frac{1}{2}$

$y = \frac{-1}{48}(8.04-8) + \frac{1}{2} = \boxed{0.4992}$