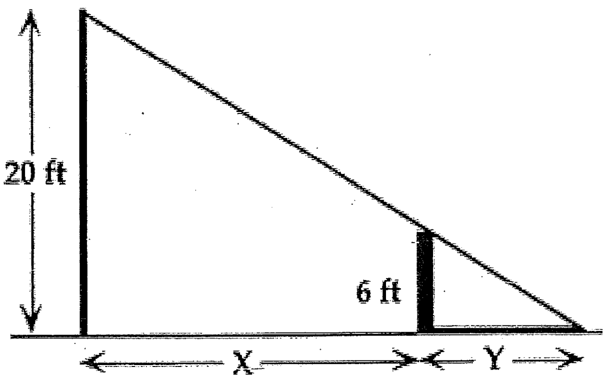
Related Rates Notes 2 - Similar Triangles and Shadow Problems

Example 1:

A man who is 6 feet tall is walking away from a lamp post at a rate of 5 feet per minute.

The lamp post is 20 feet tall. The person casts a shadow on the ground in front of them.

a) How fast is the shadow growing when the person is 30 feet from the lamp post?
b) How fast is the tip of the shadow moving when the person is 30 ft from the lamp post?



Notes:

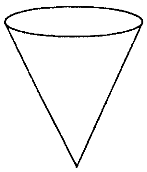
1) $\frac{dx}{dt} = rate\ of\ person\ walking$

2) $\frac{dy}{dt} = rate\ of\ change\ of\ shadow\ length$

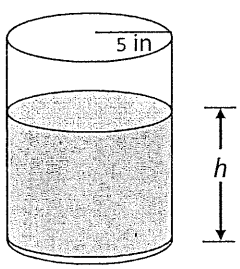
3) $\frac{dx}{dt} + \frac{dy}{dt} = rate\ of\ change\ of\ tip\ of\ shadow$

2. A street light is mounted at the top of a 15 ft pole. A man 5 ft tall walks towards the pole at a rate of 5ft per second. A) How fast is the tip of his shadow moving when he is 40 ft from the pole? B) How fast is the length of the shadow changing when he is 40 ft from the pole?

3. A conical tank (vertex down) is 40 feet across the top and 40 feet deep. If water is leaking out of the tank at a rate of 80 cubic feet per minute, find the rate of change of the radius of the water when the water is 8 feet deep. ($V = \frac{1}{3}\pi r^2 h$)



4. 2003 AB problem #5



A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time, t, measured in seconds. The volume, V, of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$.) Find $\frac{dh}{dt}$ as a function of h. (This means your answer will contain the variable h)

Key

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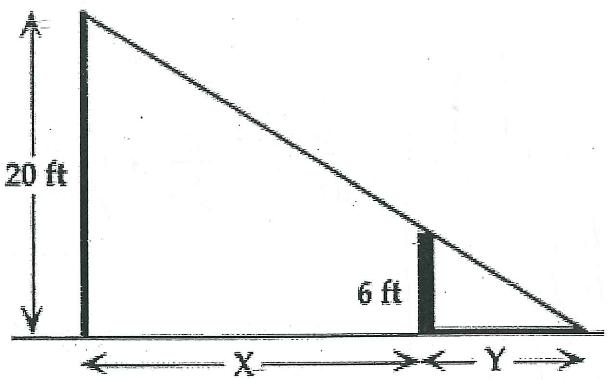
a) How fast is the shadow growing when the person is 30 feet from the lamp post? → $\frac{dy}{dt}$

b) How fast is the tip of the shadow moving when the person is 30 ft from the lamp post? → $\frac{dx}{dt}+\frac{dy}{dt}$



Notes:

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$\frac{dx}{dt} = 5$ ft/s

$x = 30$ ft

$\frac{6}{20} = \frac{y}{x+y}$

$6(x+y) = 20y$

$6x + 6y = 20y$

$6x = 14y$

$6\left(\frac{dx}{dt}\right) = 14\left(\frac{dy}{dt}\right)$

$6(5) = 14\left(\frac{dy}{dt}\right)$

$\frac{30}{14} = \frac{dy}{dt}$

$\frac{15}{7} = \frac{dy}{dt}$

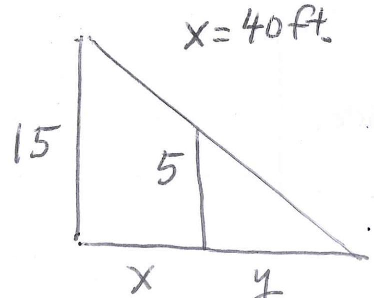
a) $\frac{dy}{dt} = \boxed{\frac{15}{7}\ ft/s}$

b) $\frac{dx}{dt} + \frac{dy}{dt} = 5 + \frac{15}{7}$

$= \boxed{\frac{50}{7} \text{ or } 7.14\ ft/s}$

2. A street light is mounted at the top of a 15 ft pole. A man 5 ft tall walks towards the pole at a rate of 5ft per second. A) How fast is the tip of his shadow moving when he is 40 ft from the pole? B) How fast is the length of the shadow changing when he is 40 ft from the pole?

$x = 40$ ft



$\frac{5}{15} = \frac{y}{x+y}$

$\frac{dx}{dt} = -5$ ft/s

$\frac{1}{3} = \frac{y}{x+y}$

$3y = x + y$

$2y = x$

$2\left(\frac{dy}{dt}\right) = \frac{dx}{dt}$

$2\left(\frac{dy}{dt}\right) = -5$

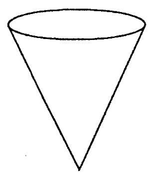
$\frac{dy}{dt} = \frac{-5}{2}$ ft/s

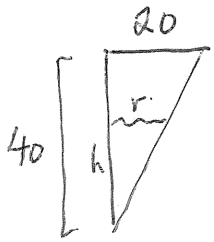
a) $\frac{dx}{dt} + \frac{dy}{dt} = -5 - \frac{5}{2}$

$= \boxed{-7.5\ ft/s}$

b) $\frac{dy}{dt} = \boxed{\frac{-5}{2}\ ft/s \text{ or } -2.5\ ft/s}$

3. A conical tank (vertex down) is 40 feet across the top and 40 feet deep. If water is leaking out of the tank at a rate of 80 cubic feet per minute, find the rate of change of the radius of the water when the water is 8 feet deep. ($V = \frac{1}{3}\pi r^2 h$)





$\frac{dV}{dt} = -80\ ft^3/min$

$h = 8\ ft$

$\frac{dr}{dt} =$ ____

$\frac{r}{20} = \frac{h}{40}$

$20h = 40r$

$h = \frac{40}{20}r$

$h = 2r$, $r = 4$

* Since $h = 2r$ and $h = 8$, $r = 4$

$V = \frac{\pi}{3}r^2 h = \frac{\pi}{3}r^2(2r)$

$V = \frac{2\pi}{3}r^3$

$\frac{dV}{dt} = \frac{2\pi}{3}\cdot 3r^2\left(\frac{dr}{dt}\right)$

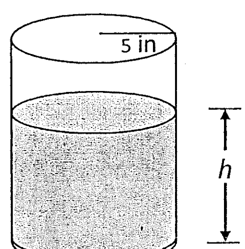
$\frac{dV}{dt} = 2\pi r^2\left(\frac{dr}{dt}\right)$

$-80 = 2\pi(4)^2\left(\frac{dr}{dt}\right)$

$\frac{-80}{32\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = -\frac{5}{2\pi}\ ft/min}$

4. 2003 AB problem #5



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$\frac{dV}{dt} = -5\pi\sqrt{h}$

$r = 5\ in.$

$V = \pi r^2 h$

$V = \pi(5)^2 h$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi\left(\frac{dh}{dt}\right)$

$-5\pi\sqrt{h} = 25\pi\frac{dh}{dt}$

$\frac{-5\pi\sqrt{h}}{25\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-\sqrt{h}}{5}\ in/sec.}$