## Calculus Notes Ch. 2.6 - Related Rates

**Related Rates**: Problems involving finding the rate of change for a variable with respect to time

This is also an application of implicit differentiation: Finding derivatives of variables with respect to time t.

**Related Rates Steps:**

1. Write what you are given
2. Write what you are trying to find
3. Write an algebraic or geometric equation relating the variables (needs to be in terms of the rates that you are either given or are trying to find)
4. Differentiate equation with respect to time $t$
5. Substitute and solve

*Important Note: Remember that when the item is getting bigger, the rate is positive
If the item is getting smaller, the rate is negative – regardless of direction

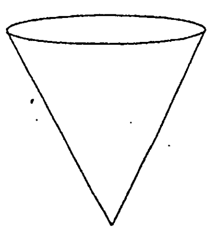
**Example 1:** The sides of a square are increasing at a rate of 5 cm/min. How fast is the area increasing when the sides measure 15 cm in length?

**Example 2:** A 25 foot ladder is leaning against a vertical wall. The bottom of the ladder is being pulled horizontally away from the wall at the constant rate of 3 ft / sec. a) How fast is the top of the ladder moving and in what direction when the bottom of the ladder is 15 ft from the wall? b) at what rate is the area changing when the bottom of the ladder is 15 ft from the wall?

**Example 3:** A spherical balloon is being inflated at a rate of 10 cubic centimeters per second. Find the rate of change of the surface area of the balloon at the moment when the surface area is $64\pi$.

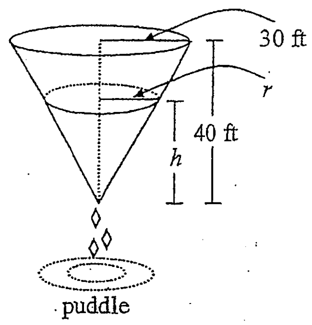
**Example 4:** Joe is standing 6 miles straight east of Moe. If Joe walks straight north at 3 mph while Moe walks straight south at 1 mph, at what rate is the distance between them changing after 2 hours?

**Example 5:** A conical tank (vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.



## A.P. Calculus AB   Worksheet 2-6 – Related Rates WS #1

1. A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 $ft^3$ / min. When the height ($h$) of the water in the tank is 30 ft, at what rate is the height of the water changing? (Volume of a cone $= \frac{1}{3}\pi r^2 h$)




2. The volume of a cube is decreasing at a rate of 10 $m^3$ / hour. How fast is the total surface area decreasing when the surface area is 54 $m^2$?

3. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.
**(a)** At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?
**(b)** At what rate is the tip of the shadow moving away from the person when the person is 25 ft from the pole?

4. A tank of water in the shape of a cone is leaking water at a constant rate of 2 $ft^3$/hr. The base radius of the tank is 5 ft and the height of the tank is 14 ft.

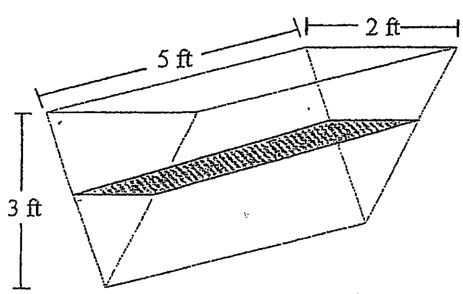
**(a)** At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?

**(b)** At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 ft?

5. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)

6. The trough shown in the figure below is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at a rate of 2 cubic feet per minute. At any time, *t*, let *h* be the depth and *V* be the volume of water in the trough.

a) Find the volume of water in the trough when it is full.
b) What is the rate of change in *h* at the instant when the trough is ¼ full by volume?
c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is ¼ full by volume?



3. Helium is pumped into a spherical balloon at the constant rate of 25 ft³/min. At what rate is the surface area of the balloon changing when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

**AP Calculus AB** **Related Rates** **Review**

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

3. Helium is pumped into a spherical balloon at the constant rate of $25\ \text{ft}^3/\text{min}$. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,

a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?
b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?

Solution Key

**Calculus Notes Ch. 2.6 - Related Rates**

**Related Rates**: Problems involving finding the rate of change for a variable with respect to time

This is also an application of implicit differentiation: Finding derivatives of variables with respect to time t.

**Related Rates Steps:**

1. Write what you are given   *use units of measurement to help match appropriate variable to values
2. Write what you are trying to find
3. Write an algebraic or geometric equation relating the variables (needs to be in terms of the rates that you are either given or are trying to find)
4. Differentiate equation with respect to time *t*
5. Substitute and solve

*Important Note: Remember that when the item is getting bigger, the rate is positive
If the item is getting smaller, the rate is negative – regardless of direction

**Example 1:** The sides of a square are increasing at a rate of 5 cm/min. How fast is the area increasing when the sides measure 15 cm in length?

represents change in Area w/ respect to time

represents change in side length with respect to time

$A = x^2$

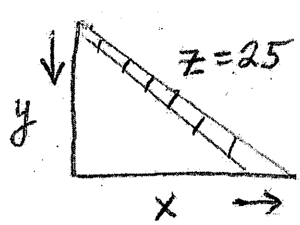
$\frac{dA}{dt} = 2x\left(\frac{dx}{dt}\right)$

Given: $\frac{dx}{dt} = 5\ cm/min$   Find $\frac{dA}{dt} = \_\_\_$

$x = 15cm$

$\frac{dA}{dt} = 2(15)(5) = \boxed{150 cm^2/min}$

**Example 2:** A 25 foot ladder is leaning against a vertical wall. The bottom of the ladder is being pulled horizontally away from the wall at the constant rate of 3 ft / sec. a) How fast is the top of the ladder moving and in what direction when the bottom of the ladder is 15 ft from the wall? b) at what rate is the area changing when the bottom of the ladder is 15 ft from the wall?




$x = 15$

$y = 20$

$z = 25$

$15^2 + y^2 = 25^2$

$y = 20$

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = \_\_\_$

$\frac{dz}{dt} = 0$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(15)(3) + 2(20)\frac{dy}{dt} = 2(25)(0)$

$40\left(\frac{dy}{dt}\right) = -90$   $\boxed{\frac{dy}{dt} = -9/4\ ft/s}$

$A = \frac{1}{2}xy$   $f'g + fg'$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)(y) + \frac{1}{2}(x)\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}(3)(20) + \frac{1}{2}(15)(-9/4)$

$= 30 - \frac{135}{8} = \boxed{\frac{105}{8} ft^2/s}$

$\approx 13.125 ft^2/s$

change in volume $(cm^3/s)$

**Example 3:** A spherical balloon is being inflated at a rate of 10 cubic centimeters per second. Find the rate of change of the surface area of the balloon at the moment when the surface area is $64\pi$.

$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$\frac{dV}{dt} = 10\,cm^3/s$

$\frac{dS}{dt} = $ ____

$S = 64\pi$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$64\pi = 4\pi r^2$

$16 = r^2$

$r = 4$

$10 = 4\pi(4)^2\left(\frac{dr}{dt}\right)$

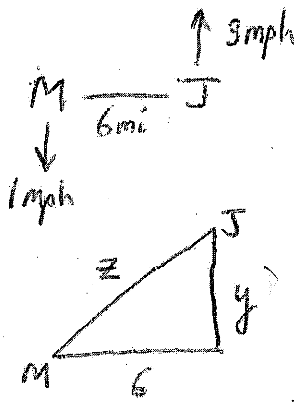
$\frac{10}{64\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{32\pi}$

$\frac{dS}{dt} = 8\pi(4)\left(\frac{5}{32\pi}\right)$

$\boxed{\frac{dS}{dt} = 5\,cm^2/sec}$

**Example 4:** Joe is standing 6 miles straight east of Moe. If Joe walks straight north at 3 mph while Moe walks straight south at 1 mph, at what rate is the distance between them changing after 2 hours?




$\frac{dy}{dt} = 4$ mph

$y = (4)(2) = 8$ mi

$z = 10$ mi

$6^2 + 8^2 = z^2$

$z = 10$

Find $\frac{dz}{dt} = $ ____

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

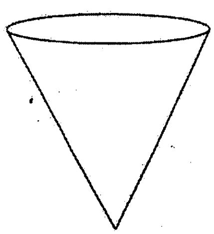
$\cancel{2(6)(0)} + 2(8)(4) = 2(10)\left(\frac{dz}{dt}\right)$

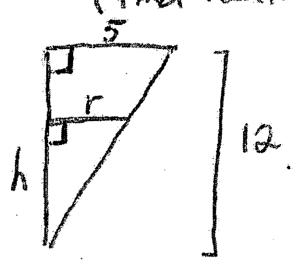
$64 = 20\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{64}{20} = \frac{16}{5} = \boxed{3.2\text{ mph}}$

**Example 5:** A conical tank (vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

*Use similiar triangles to rewrite equation using less variables. (find relationship between height and radius, use substitution)






$V = \frac{\pi}{3}r^2 h$

$\frac{dV}{dt} = 10\,ft^3/min$

$\frac{dh}{dt} = $ ____

$h = 8$ ft.

$\frac{r}{5} = \frac{h}{12}$

$5h = 12r$

$\frac{5}{12}h = r$

*Rewrite volume in terms of "h"

$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h = \frac{\pi}{3}\cdot\frac{25h^2}{144}\cdot h = \frac{25\pi}{432}h^3$

$V = \frac{25\pi}{432}h^3$

$\frac{dV}{dt} = \frac{25\pi}{432}\cdot 3h^2\left(\frac{dh}{dt}\right)$

$10 = \frac{25\pi}{432}\cdot 3(8)^2\frac{dh}{dt}$

$10 = \frac{4800\pi}{432}\cdot\frac{dh}{dt}$

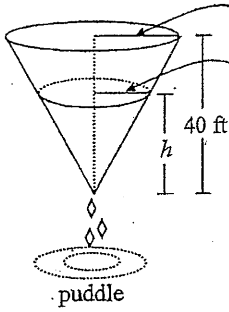
$10\cdot\frac{432}{4800\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{9}{10\pi}\,ft/min}$

Solution Key 2014

**A.P. Calculus AB Worksheet 2-6 – Related Rates WS #1**

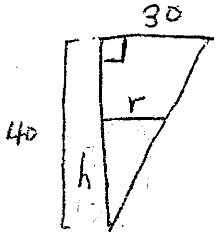
1. A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft$^3$ / min. When the height ($h$) of the water in the tank is 30 ft, at what rate is the height of the water changing? (Volume of a cone $= \frac{1}{3}\pi r^2 h$)




$\frac{dV}{dt} = -2\ ft^3/min$

$\boxed{h = 30}$ save until the end to use

$\frac{dh}{dt} = \_\_\_\_$




$\frac{r}{30} = \frac{h}{40}$

$40r = 30h$

$r = \frac{3}{4}h$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{3h}{4}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{9h^2}{16} h$

$V = \frac{3\pi}{16} h^3$

$\frac{dV}{dt} = \frac{3\pi}{16} \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{9\pi}{16} h^2 \frac{dh}{dt}$

$-2 = \frac{9\pi}{16}(30)^2 \frac{dh}{dt}$

$\frac{-2 \cdot 16}{9\pi \cdot 30^2} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-8}{2025\pi}\ ft/min.}$

$\frac{ds}{dt}$

2. The volume of a cube is decreasing at a rate of 10 m$^3$ / hour. How fast is the total surface area ~~decreasing when the surface area~~ is 54 m$^2$?

$V = x^3$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$

$\frac{dV}{dt} = -10\ m^3/hr.$

$S = 6x^2$

$\boxed{\frac{dS}{dt} = 12x \frac{dx}{dt}}$

$S = 54 m^2$

$54 = 6x^2 \quad x = 3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$-10 = 3(3)^2\left(\frac{dx}{dt}\right)$

$\boxed{\frac{-10}{27}} = \frac{dx}{dt}$

$\frac{dS}{dt} = 12(3)\left(\frac{-10}{27}\right)$

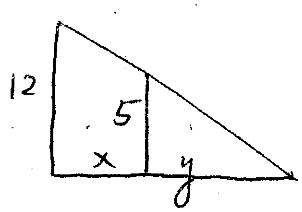
$= \boxed{-\frac{40}{3}\ m^2/hr.}$

3. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.

**(a)** At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?

**(b)** At what rate is the ~~tip of the shadow~~ moving away from the person when the ~~person is 25 ft from the~~ pole?

ROC for shadow length




$\frac{5}{12} = \frac{y}{x+y}$

$5x + 5y = 12y$

$5x = 7y$

$5\frac{dx}{dt} = 7\frac{dy}{dt}$

$\frac{dx}{dt} = 2\ ft/s$

$x = 25$

$5(2) = 7\frac{dy}{dt}$

$10 = 7\frac{dy}{dt}$

$\frac{10}{7} = \frac{dy}{dt}$

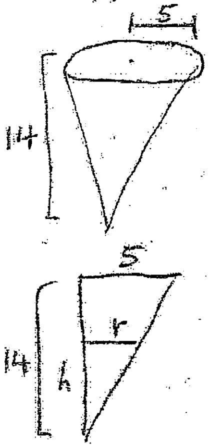
a) $\frac{dx}{dt} + \frac{dy}{dt} = 10/7 + 2$

$= \frac{24}{7}\ ft/s$

b) $\frac{dy}{dt} = \frac{10}{7}\ ft/s$

4. A tank of water in the shape of a cone is leaking water at a constant rate of 2 $ft^3$/hr. The base radius of the tank is 5 ft and the height of the tank is 14 ft.
(a) At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?

(b) At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 ft?




$\frac{r}{5}=\frac{h}{14}$

$\frac{dV}{dt}=-2\,ft^3/hr$

$\frac{dh}{dt}=$ ____

$h=6$ ft.

$r=\frac{5h}{14}$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{\pi}{3}\left(\frac{5h}{14}\right)^2 h$

$V=\frac{\pi}{3}\cdot\frac{25}{196}h^3$

$V=\frac{25\pi}{588}h^3$

$\frac{dV}{dt}=\frac{25\pi}{588}\cdot 3h^2\frac{dh}{dt}$

$-2=\frac{75\pi}{588}(6)^2\frac{dh}{dt}$

$\frac{-2\cdot 588}{75\pi\cdot 36}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-98}{225\pi}\text{ ft/hr.}}$

Find $\frac{dr}{dt}$

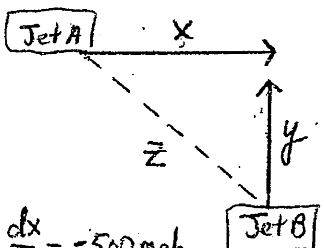
$\frac{r}{5}=\frac{h}{14}$ $\quad 14r=5h$

$14\left(\frac{dr}{dt}\right)=5\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt}=\frac{5\cdot -98}{225\pi}\cdot\frac{1}{14}$

$\boxed{\frac{dr}{dt}=\frac{-7}{45\pi}\text{ ft/hr.}}$

5. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)




$\frac{dx}{dt}=-500$ mph

$\frac{dy}{dt}=-600$ mph

$x=100$

$z=300$

$y=282.843$ miles

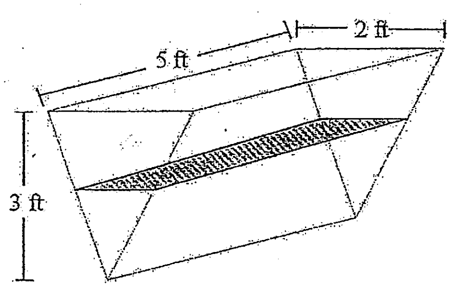
$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(100)(-500)+2(282.843)(-600)=2(300)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=-732.353\text{ mph}}$

6. The trough shown in the figure below is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at a rate of 2 cubic feet per minute. At any time, $t$, let $h$ be the depth and $V$ be the volume of water in the trough.
a) Find the volume of water in the trough when it is full.
b) What is the rate of change in $h$ at the instant when the trough is ¼ full by volume?
c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is ¼ full by volume?

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AP Review: Related Rates

#6

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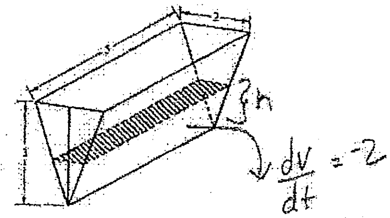



The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time $t$, let $h$ be the depth and $V$ be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

means volume of trough.

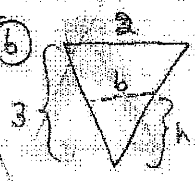
(b) What is the rate of change in $h$ at the instant when the trough is $\frac{1}{4}$ full by volume?

$\frac{dh}{dt}$

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

(a) $V = (\text{Area of } \Delta)(\text{height}) = \frac{1}{2} \cdot 2 \cdot 3 \cdot 5 = 15 \text{ ft}^3$

(b)




Similar triangles

$\frac{3}{2} = \frac{h}{b}$

$3b = 2h$

$b = \frac{2h}{3}$

$V = \frac{1}{2} bh \cdot 5 = \frac{5}{2} bh$

$V = \frac{5}{\cancel{2}}\left(\frac{\cancel{2}h}{3}\right)h = \frac{5}{3}h^2$

$\frac{dV}{dt} = \frac{5}{3} \cdot 2h \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{10h}{3} \cdot \frac{dh}{dt}$

When trough is $\frac{1}{4}$ full:

$V = \frac{15}{4}$

$V = \frac{15}{4} = \frac{5}{3}h^2$

$h = 3/2$

$-2 = \frac{10}{3}\left(\frac{3}{2}\right) \cdot \frac{dh}{dt}$

$-\frac{2}{5} = \frac{dh}{dt}$

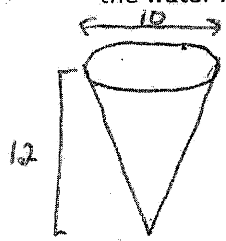
Surface rectangle

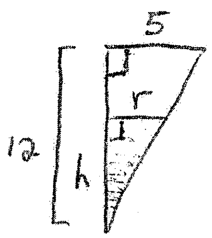
(c) $A = 5b = 5\left(\frac{2}{3}h\right) = \frac{10}{3}h$

$\frac{dA}{dt} = \frac{10}{3} \cdot \frac{dh}{dt} = \frac{10}{3} \cdot \frac{-2}{5} = \frac{-4}{3}$

**AP Calculus AB** **Related Rates Quiz Review** Solutions Key

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.




$\frac{dV}{dt} = -12\,ft^3/min$

$r = 4\,ft$

$*$ since $\frac{r}{5} = \frac{h}{12}$,

$\frac{4}{5} = \frac{h}{12}$, $5h = 48$

so $h = \frac{48}{5}\,ft.$

Find $\frac{dh}{dt} =$ ____

$\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$r = \frac{5h}{12}$

$V = \frac{\pi}{3} r^2 h$

$*$Rewrite volume equation in terms of variable $h$.

$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h = \frac{\pi}{3}\left(\frac{25h^2}{144}\right)h = \frac{25\pi}{432}h^3$

$V = \frac{25\pi}{432}h^3$

$\frac{dV}{dt} = \frac{25\pi}{432}\cdot 3h^2 \frac{dh}{dt}$

$-12 = \frac{25\pi}{432}\cdot 3\left(\frac{48}{5}\right)^2\left(\frac{dh}{dt}\right)$

$-12 = \frac{\cancel{25}\pi\cdot 3\cdot 48^2}{432\cdot \cancel{25}}\frac{dh}{dt}$

$-12 = 16\pi \frac{dh}{dt}$

$\frac{-12}{16\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-3}{4\pi}\,ft/min}$

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

$V = \frac{\pi}{3} r^2 h$

$12r = 5h$

$\frac{12r}{5} = h$

Find $\frac{dr}{dt} =$ ____

$r = 4$

$V = \frac{\pi}{3} r^2\left(\frac{12r}{5}\right)$

$V = \frac{12\pi}{15} r^3$

$V = \frac{4\pi}{5} r^3$

$\frac{dV}{dt} = \frac{4\pi}{5}\cdot 3r^2\left(\frac{dr}{dt}\right)$

$-12 = \frac{4\pi}{5}\cdot 3(4)^2 \frac{dr}{dt}$

$-12 = \frac{4\pi\cdot 3\cdot 16}{5}\cdot\frac{dr}{dt}$

$\frac{-12\cdot 5}{4\pi\cdot 48} = \frac{dr}{dt}$

$\frac{-5}{16\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{-5}{16\pi}\,ft/min}$

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OR

Since we know from part a, $\frac{dh}{dt} = \frac{-3}{4\pi}$

and $\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$\downarrow$

$12\left(\frac{dr}{dt}\right) = 5\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{-3}{4\pi}\right) = \frac{-15}{48\pi} = \boxed{\frac{-5}{16\pi}\,ft/min}$

3. Helium is pumped into a spherical balloon at the constant rate of 25 ft$^3$/min. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

$\frac{dV}{dt} = 25\ ft^3/min$

Find $\frac{dS}{dt} =$ ______

diameter $= 16$, so $r = 8$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$25 = 4\pi(8)^2\left(\frac{dr}{dt}\right)$

$\frac{25}{4\pi \cdot 64} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{25}{256\pi}\ ft/min.$

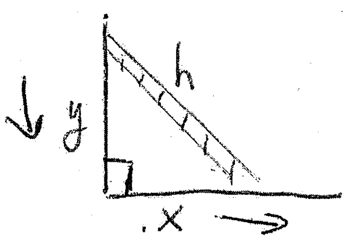
$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi(8)\left(\frac{25}{256\pi}\right)$

$\boxed{\frac{dS}{dt} = \frac{25}{4}\ ft^2/min.}$

4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,
a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?
b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?




$x^2 + y^2 = h^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 15$ $\frac{dx}{dt} = 5\ ft/s$

$y = 8$ $\frac{dy}{dt} =$ ______

$h = 17$ $\frac{dh}{dt} = 0$

a) $2(15)(5) + 2(8)\left(\frac{dy}{dt}\right) = \cancel{2(17)(0)}$

$150 + 16\left(\frac{dy}{dt}\right) = 0$ $\frac{dy}{dt} = \frac{-150}{16} = \boxed{\frac{-75}{8}\ ft/s}$

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)y + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$= \frac{1}{2}(5)(8) + \frac{1}{2}(15)\left(\frac{-75}{8}\right)$

$= 20 - \frac{1125}{16}$

$\boxed{\frac{dA}{dt} = \frac{-805}{16}\ ft^2/s}$