AP Calculus AB Related Rates Review WS #1

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

3. Helium is pumped into a spherical balloon at the constant rate of 25 ft^3/min. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

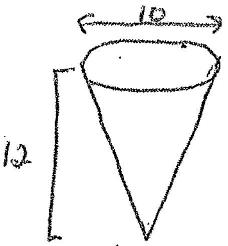
4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,

a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?

AP Calculus AB **Related Rates** WS #1 Solutions Key

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.



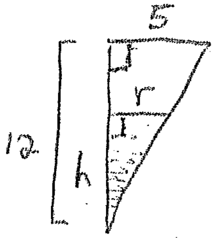
$\frac{dV}{dt} = -12 \text{ ft}^3/\text{min}$

$r = 4$ ft

* since $\frac{r}{5} = \frac{h}{12}$,

$\frac{4}{5} = \frac{h}{12}$, $5h = 48$

so $h = \frac{48}{5}$ ft.



$\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$r = \frac{5h}{12}$

Find $\frac{dh}{dt} =$ ____

$V = \frac{\pi}{3} r^2 h$

*Rewrite volume equation in terms of variable h.

$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h = \frac{\pi}{3}\left(\frac{25h^2}{144}\right)h = \frac{25\pi}{432}h^3$

$V = \frac{25\pi}{432}h^3$

$\frac{dV}{dt} = \frac{25\pi}{432} \cdot 3h^2 \frac{dh}{dt}$

$-12 = \frac{25\pi}{432} \cdot 3\left(\frac{48}{5}\right)^2 \left(\frac{dh}{dt}\right)$

$-12 = \frac{\cancel{25}\pi \cdot 3 \cdot 48^2}{432 \cdot \cancel{25}} \frac{dh}{dt}$

$-12 = 16\pi \frac{dh}{dt}$

$\frac{-12}{16\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-3}{4\pi} \text{ ft/min}}$

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

$V = \frac{\pi}{3} r^2 h$

$12r = 5h$

$\frac{12r}{5} = h$

Find $\frac{dr}{dt} =$ ____

$r = 4$

$V = \frac{\pi}{3} r^2 \left(\frac{12r}{5}\right)$

$V = \frac{12\pi}{15} r^3$

$V = \frac{4\pi}{5} r^3$

$\frac{dV}{dt} = \frac{4\pi}{5} \cdot 3r^2 \left(\frac{dr}{dt}\right)$

$-12 = \frac{4\pi}{5} \cdot 3(4)^2 \frac{dr}{dt}$

$-12 = \frac{4\pi \cdot 3 \cdot 16}{5} \cdot \frac{dr}{dt}$

$\frac{-12 \cdot 5}{4\pi \cdot 48} = \frac{dr}{dt}$

$\frac{-5}{16\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{-5}{16\pi} \text{ ft/min}}$

OR

Since we know from part a,

$\frac{dh}{dt} = \frac{-3}{4\pi}$

and $\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$\downarrow$

$12\left(\frac{dr}{dt}\right) = 5\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{-3}{4\pi}\right) = \frac{-15}{48\pi} = \boxed{\frac{-5}{16\pi} \text{ ft/min}}$

3. Helium is pumped into a spherical balloon at the constant rate of 25 ft^3/min. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

$\frac{dV}{dt} = 25 \text{ ft}^3/\text{min.}$

Find $\frac{dS}{dt} =$ ______

diameter $= 16$, so

$r = 8$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$25 = 4\pi(8)^2\left(\frac{dr}{dt}\right)$

$\frac{25}{4\pi \cdot 64} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{25}{256\pi} \text{ ft/min.}$

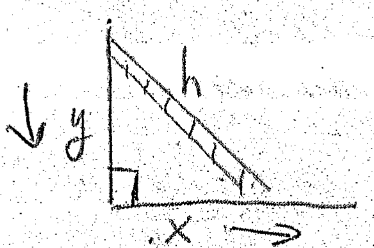
$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi(8)\left(\frac{25}{256\pi}\right)$

$\boxed{\frac{dS}{dt} = \frac{25}{4} \text{ ft}^2/\text{min.}}$

4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,
   a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?
   b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?



$x^2 + y^2 = h^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 15$ $\frac{dx}{dt} = 5 \text{ ft/s}$

$y = 8$ $\frac{dy}{dt} =$ ______

$h = 17$ $\frac{dh}{dt} = 0$

a) $2(15)(5) + 2(8)\left(\frac{dy}{dt}\right) = \cancel{2(17)}(0)$

$150 + 16\left(\frac{dy}{dt}\right) = 0$ $\frac{dy}{dt} = \frac{-150}{16} = \boxed{\frac{-75}{8} \text{ ft/s}}$

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)y + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$= \frac{1}{2}(5)(8) + \frac{1}{2}(15)\left(\frac{-75}{8}\right)$

$= 20 - \frac{1125}{16}$

$\boxed{\frac{dA}{dt} = \frac{-805}{16} \text{ ft}^2/\text{s}}$

AB Calculus **Related Rates** WS #2

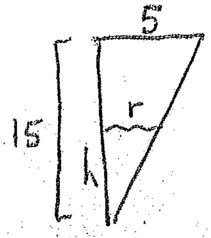
1. A conical paper cup is 15 cm tall with a radius of 5 cm. Water is being poured into the cup at a rate of 32π cm³/sec. At the moment when the depth of water is rising at 72 cm/sec, find the radius of the water level at that time. ($V = \frac{1}{3}\pi r^2 h$)

2. The area of a circle increases at a rate of $1\text{cm}^2/\text{s}$. How fast is the diameter changing when the circumference is 2cm? $A = \pi r^2$ $C = 2\pi r$ diameter = 2r

3. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

AB Calculus **Related Rates** WS #2 Key

1. A conical paper cup is 15 cm tall with a radius of 5 cm. Water is being poured into the cup at a rate of 32π cm³/sec. At the moment when the depth of water is rising at 72 cm/sec, find the radius of the water level at that time. ($V = \frac{1}{3}\pi r^2 h$)



$\frac{r}{5} = \frac{h}{15}$

$5h = 15r$

$h = 3r$

$\frac{h}{3} = r$

$\frac{dV}{dt} = 32\pi \text{ cm}^3/\text{sec}$

$\frac{dh}{dt} = 72 \text{ cm/sec}$

$h = \_\_\_$

$r = \_\_\_$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{9} \cdot h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$

$32\pi = \frac{\pi}{9} h^2 (72)$

$32\pi = 8\pi h^2$

$4 = h^2$

$\underline{2 = h}$

Since $h = 3r$,

$2 = 3r$

$\frac{2}{3} = r$

$\boxed{r = \frac{2}{3} \text{ cm}}$

2. The area of a circle increases at a rate of $1 \text{cm}^2/\text{s}$. How fast is the diameter changing when the circumference is 2cm? $A = \pi r^2$ $C = 2\pi r$ diameter $= 2r$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

$\frac{dA}{dt} = 1 \text{ cm}^2/\text{s}$

$C = 2 \text{ cm}$

$C = 2\pi r$

$2 = 2\pi r$

$\frac{2}{2\pi} = r$

$\frac{1}{\pi} = r$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$1 = 2\pi\left(\frac{1}{\pi}\right)\left(\frac{dr}{dt}\right)$

$1 = 2\left(\frac{dr}{dt}\right)$

$\frac{1}{2} = \frac{dr}{dt}$

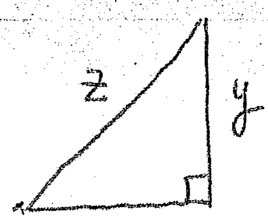
$d = 2r$

$\frac{dd}{dt} = 2\left(\frac{dr}{dt}\right)$

$\frac{dd}{dt} = 2\left(\frac{1}{2}\right)$

$\boxed{\frac{dd}{dt} = 1 \text{ cm/s}}$

3. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?



$x = 700$ (constant)

$700^2 + 2400^2 = z^2$

$2500 = z$

$\frac{dy}{dt} = 900 \text{ ft/s}$

$\frac{dz}{dt} = \_\_\_$

$y = 2400$

$z = \underline{2500}$

$700^2 + y^2 = z^2$

$0 + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(2400)(900) = 2(2500)\left(\frac{dz}{dt}\right)$

$\frac{2(2400)(900)}{2(2500)} = \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 864 \text{ ft/s}}$