AP Calculus AB **Related Rates Quiz Review**

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

3. Helium is pumped into a spherical balloon at the constant rate of $25 \text{ ft}^3/\text{min}$. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

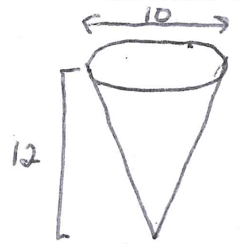
4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,

a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?

AP Calculus AB **Related Rates Quiz Review** Solutions Key

1. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find the rate at which the water level is dropping when radius of the water level is 4 ft.



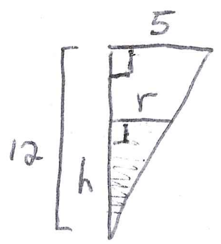
$\frac{dV}{dt} = -12\ ft^3/min$

$r = 4\ ft$

* since $\frac{r}{5} = \frac{h}{12}$,

$\frac{4}{5} = \frac{h}{12}$, $5h = 48$

so $h = \frac{48}{5}\ ft.$



$\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$r = \frac{5h}{12}$

Find $\frac{dh}{dt} =$ ____

$V = \frac{\pi}{3} r^2 h$

*Rewrite volume equation in terms of variable h.

$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h = \frac{\pi}{3}\left(\frac{25h^2}{144}\right)h = \frac{25\pi}{432}h^3$

$V = \frac{25\pi}{432}h^3$

$\frac{dV}{dt} = \frac{25\pi}{432} \cdot 3h^2 \frac{dh}{dt}$

$-12 = \frac{25\pi}{432} \cdot 3\left(\frac{48}{5}\right)^2\left(\frac{dh}{dt}\right)$

$-12 = \frac{25\pi \cdot 3 \cdot 48^2}{432 \cdot 25}\frac{dh}{dt}$

$-12 = 16\pi \frac{dh}{dt}$

$\frac{-12}{16\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-3}{4\pi}\ ft/min}$

2. A conical tank (vertex down) full of water is 10 feet across the top and 12 feet deep. If water is flowing out of the tank at a rate of 12 cubic feet per minute, find **the rate of change of the radius of water level** when radius of the water level is 4 ft.

$V = \frac{\pi}{3} r^2 h$

$12r = 5h$

$\frac{12r}{5} = h$

Find $\frac{dr}{dt} =$ ____

$r = 4$

$V = \frac{\pi}{3} r^2\left(\frac{12r}{5}\right)$

$V = \frac{12\pi}{15} r^3$

$V = \frac{4\pi}{5} r^3$

$\frac{dV}{dt} = \frac{4\pi}{5} \cdot 3r^2\left(\frac{dr}{dt}\right)$

$-12 = \frac{4\pi}{5} \cdot 3(4)^2 \frac{dr}{dt}$

$-12 = \frac{4\pi \cdot 3 \cdot 16}{5} \cdot \frac{dr}{dt}$

$\frac{-12 \cdot 5}{4\pi \cdot 48} = \frac{dr}{dt}$

$\frac{-5}{16\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{-5}{16\pi}\ ft/min}$

OR

Since we know from part a, $\frac{dh}{dt} = \frac{-3}{4\pi}$

and $\frac{r}{5} = \frac{h}{12}$

$12r = 5h$

$\downarrow$

$12\left(\frac{dr}{dt}\right) = 5\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{5}{12}\left(\frac{-3}{4\pi}\right) = \frac{-15}{48\pi} = \boxed{\frac{-5}{16\pi}\ ft/min}$

3. Helium is pumped into a spherical balloon at the constant rate of 25 ft^3/min. At what rate is the surface area of the balloon increasing the moment when the diameter is 16 ft? $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

$\frac{dV}{dt} = 25 \text{ ft}^3/\text{min}$

Find $\frac{dS}{dt} =$ ______

diameter $= 16$, so

$r = 8$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$25 = 4\pi (8)^2 \left(\frac{dr}{dt}\right)$

$\frac{25}{4\pi \cdot 64} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{25}{256\pi} \text{ ft/min}$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \left(\frac{dr}{dt}\right)$

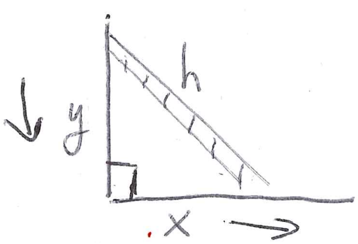
$\frac{dS}{dt} = 8\pi (8)\left(\frac{25}{256\pi}\right)$

$\boxed{\frac{dS}{dt} = \frac{25}{4} \text{ ft}^2/\text{min.}}$

4. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s,

a) how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

b) At what rate is the area of the triangle changing when the ladder is 8 ft above ground?



$x^2 + y^2 = h^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 15$ $\quad \frac{dx}{dt} = 5 \text{ ft/s}$

$y = 8$ $\quad \frac{dy}{dt} =$ ______

$h = 17$ $\quad \frac{dh}{dt} = 0$

a) $2(15)(5) + 2(8)\left(\frac{dy}{dt}\right) = 2(17)(0)$

$150 + 16\left(\frac{dy}{dt}\right) = 0$ $\quad \frac{dy}{dt} = \frac{-150}{16} = \boxed{\frac{-75}{8} \text{ ft/s}}$

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)y + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$= \frac{1}{2}(5)(8) + \frac{1}{2}(15)\left(\frac{-75}{8}\right)$

$= 20 - \frac{1125}{16}$

$\boxed{\frac{dA}{dt} = \frac{-805}{16} \text{ ft}^2/\text{s}}$