Related Rates Review Worksheet #1

1a)

A ladder 10 meters long is leaning against a vertical wall with its other end on the ground. The top end of the ladder is sliding down the wall. When the top end is 6 meters from the ground it is sliding down at 2 m/sec. How fast is the bottom moving away from the wall at this instant?

1b) How fast is the area of triangle changing when the top end is 6 meters from the ground?

2) Two cars leave from the same position at the same time. One travels due North at 30 mph, while the other travels due East at 40 mph. At what rate is the distance between the cars changing after 2 hours?

3) $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Air is escaping from a spherical balloon at the rate of 2 cm^3 per minute. How fast is the surface area shrinking when the radius is 1 cm? $V = 4/3\ \pi r^3$ and $S = 4\pi r^2$ where V is the volume and S is the surface area, r is the radius.

4) $V = \frac{\pi}{3}r^2h$

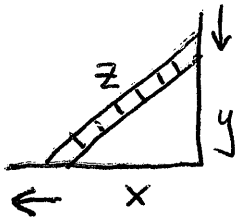
A funnel in the shape of an inverted cone is 30 cm deep and has a diameter across the top of 20 cm. Liquid is flowing out of the funnel at the rate of 12 cm^3/sec. At what rate is the height of the liquid decreasing at the instant when the liquid in the funnel is 20 cm deep?

Related Rates Review Worksheet #1

Key

1a)

A ladder 10 meters long is leaning against a vertical wall with its other end on the ground. The top end of the ladder is sliding down the wall. When the top end is 6 meters from the ground it is sliding down at 2 m/sec. How fast is the bottom moving away from the wall at this instant?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=8$ $\quad \frac{dx}{dt}=$ ___

$y=6$ $\quad \frac{dy}{dt}=-2$

$z=10$ $\quad \frac{dz}{dt}=0$

$2(8)\left(\frac{dx}{dt}\right)+2(6)(-2)=2(10)(0)$

$16\left(\frac{dx}{dt}\right)-24=0$

$16\frac{dx}{dt}=24$

$\frac{dx}{dt}=\frac{24}{16}$

$\frac{dx}{dt}=1.5$ m/s

$\boxed{\frac{dx}{dt}=\frac{3}{2}\text{ m/s}}$

1b) How fast is the area of triangle changing when the top end is 6 meters from the ground?

$A=\frac{1}{2}xy$

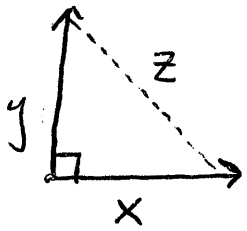
$\frac{dA}{dt}=\overbrace{\frac{1}{2}\left(\frac{dx}{dt}\right)}^{f'}\cdot\overbrace{y}^{g}+\overbrace{\frac{1}{2}x}^{f}\cdot\overbrace{\left(\frac{dy}{dt}\right)}^{g'}$

$\frac{dA}{dt}=\frac{1}{2}\left(\frac{3}{2}\right)(6)+\frac{1}{2}(8)(-2)$

$\frac{dA}{dt}=4.5-8=-3.5$ or $-\frac{7}{2}$

$\boxed{\frac{dA}{dt}=-3.5\text{ m}^2/\text{s}}$

2) Two cars leave from the same position at the same time. One travels due North at 30 mph, while the other travels due East at 40 mph. At what rate is the distance between the cars changing after 2 hours?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=2(40)=80$ $\quad \frac{dx}{dt}=40$

$y=2(30)=60$ $\quad \frac{dy}{dt}=30$

$z=100$ $\quad \frac{dz}{dt}=$ ___ ?

$2(80)(40)+2(60)(30)=2(100)\left(\frac{dz}{dt}\right)$

$50=\frac{dz}{dt}$

$\boxed{\frac{dz}{dt}=50\text{ mph}}$

3) $V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

Air is escaping from a spherical balloon at the rate of 2 cm^3 per minute. How fast is the surface area shrinking when the radius is 1 cm? V= 4/3 πr^3 and S = 4πr^2 where V is the volume and S is the surface area, r is the radius.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$-2 = 4\pi(1)^2\left(\frac{dr}{dt}\right)$

$\frac{-2}{4\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-1}{2\pi}$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi(1)\left(\frac{-1}{2\pi}\right)$

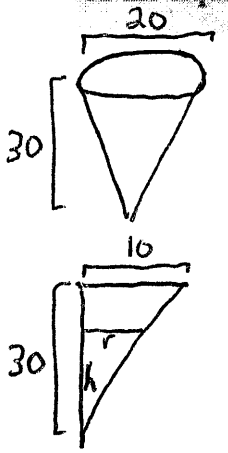
$\frac{dS}{dt} = -4\ cm^2/min$

$\frac{dS}{dt} = $? $\quad \frac{dV}{dt} = -2$

$r = 1$ cm

4) $V = \frac{\pi}{3}r^2h$

A funnel in the shape of an inverted cone is 30 cm deep and has a diameter across the top of 20 cm. Liquid is flowing out of the funnel at the rate of 12 cm^3/sec. At what rate is the height of the liquid decreasing at the instant when the liquid in the funnel is 20 cm deep?



$\frac{dV}{dt} = -12\ cm^3/sec$

$\frac{dh}{dt} = $ ___

$h = 20$ (save until end to use)

$\frac{r}{10} = \frac{h}{30}$

$30r = 10h$

$r = \frac{10h}{30}$ $\quad r = \frac{h}{3}$

$V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2h$

$V = \frac{\pi}{3}\cdot\frac{h^2}{9}h$

$V = \frac{\pi}{27}h^3$

$\frac{dV}{dt} = \frac{\pi}{27}\cdot 3h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\left(\frac{dh}{dt}\right)$

$-12 = \frac{\pi}{9}(20)^2\left(\frac{dh}{dt}\right)$

$-12 = \frac{400\pi}{9}\left(\frac{dh}{dt}\right)$

$-12\cdot\frac{9}{400\pi} = \frac{dh}{dt}$

$\frac{-27}{100\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-27}{100\pi}\ cm/sec$