Related Rates Review Worksheet #2

1a) An 8-foot ladder is leaning against a wall. The top of the ladder is sliding down the wall at the rate of 2 feet per second. How fast is the bottom of the ladder moving along the ground at the point in time when the bottom of the ladder is 4 feet from the wall?

1b) How fast is the area of triangle changing when the bottom of ladder is 4 feet from the wall?

2) Two cars, one going due East at a rate of 90 km/hr and the other going due South at a rate of 60 km/hr, are traveling toward the intersection of the two roads. At what rate are the cars approaching each other at the instant when the first car is 0.2 km and the second car is 0.15 km from the intersection?

3) $V = \frac{4}{3}\pi r^3$ $S = 4\,\pi r^2$

Gas is escaping a spherical balloon at the rate of 4 cm^3 per minute. How fast is the surface area shrinking when the radius is 24 cm? For a sphere, $V = 4/3\pi r^3$ and $S = 4\pi r^2$ where V is volume, S is surface area and r is the radius of the balloon.

4) $V = \frac{\pi}{3} r^2 h$

A conical cup is 4 cm across and 6 cm deep. Water leaks out of the bottom at the rate of 2 cm^3/sec. How fast is the water level dropping when the height of the water is 3 cm?

Key

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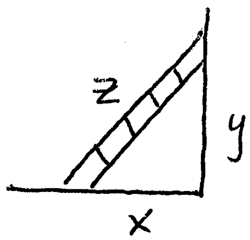


$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=4$ $\quad \frac{dx}{dt}=$ ___ ?

$y=\underline{4\sqrt{3}}$ $\quad \frac{dy}{dt}=-2$

$z=8$ $\quad \frac{dz}{dt}=0$

$4^2+y^2=8^2$ | $y=4\sqrt{3}$

$y^2=48$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(4)\left(\frac{dx}{dt}\right)+2(4\sqrt{3})(-2)=2(8)(0)$

$8\left(\frac{dx}{dt}\right)-16\sqrt{3}=0$

$8\frac{dx}{dt}=16\sqrt{3}$

$\boxed{\frac{dx}{dt}=\frac{16\sqrt{3}}{8}=2\sqrt{3}\approx 3.464 \text{ ft/sec}}$

1b) How fast is the area of triangle changing when the bottom of ladder is 4 feet from the wall?

$A=\overbrace{\tfrac{1}{2}x}^{f}\cdot\overbrace{y}^{g}$

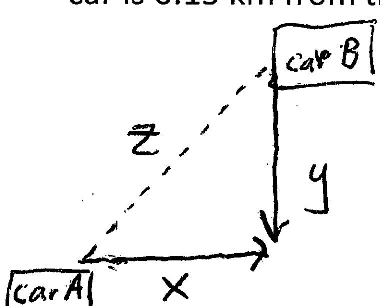
$\frac{dA}{dt}=\overbrace{\tfrac{1}{2}\left(\frac{dx}{dt}\right)}^{f'}\cdot\overbrace{y}^{g}+\overbrace{\tfrac{1}{2}x}^{f}\cdot\overbrace{\left(\frac{dy}{dt}\right)}^{g'}$

$\frac{dA}{dt}=\frac{1}{2}(2\sqrt{3})(4\sqrt{3})+\frac{1}{2}(4)(-2)$

$\frac{dA}{dt}=12-4$

$\boxed{\frac{dA}{dt}=8 \text{ ft}^2/\text{sec}}$

2) Two cars, one going due East at a rate of 90 km/hr and the other going due South at a rate of 60 km/hr, are traveling toward the intersection of the two roads. At what rate are the cars approaching each other at the instant when the first car is 0.2 km and the second car is 0.15 km from the intersection?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=0.2$ $\quad \frac{dx}{dt}=-90$

$y=0.15$ $\quad \frac{dy}{dt}=-60$

$z=\underline{0.25}$ $\quad \frac{dz}{dt}=$ ___ ?

$0.2^2+0.15^2=z^2$

$z=0.25$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(0.2)(-90)+2(0.15)(-60)=2(0.25)\left(\frac{dz}{dt}\right)$

$-36-18=0.5\left(\frac{dz}{dt}\right)$

$-54=0.5\left(\frac{dz}{dt}\right)$

$\frac{-54}{0.5}=\frac{dz}{dt}$

$\boxed{\frac{dz}{dt}=-108 \text{ km/hr}}$

3) $V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$

Gas is escaping a spherical balloon at the rate of 4 cm^3 per minute. How fast is the surface area shrinking when the radius is 24 cm? For a sphere, $V = 4/3\pi r^3$ and $S = 4\pi r^2$ where V is volume, S is surface area and r is the radius of the balloon.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$S = 4\pi r^2$

$\frac{dS}{dt} = 4\pi \cdot 2r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = -4\ cm^3/min \qquad \frac{dS}{dt} = \_\_\_ \qquad r = 24$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$-4 = 4\pi(24)^2\left(\frac{dr}{dt}\right)$

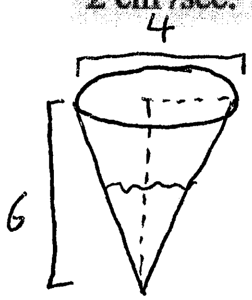
$-4 = 2304\pi\left(\frac{dr}{dt}\right)$

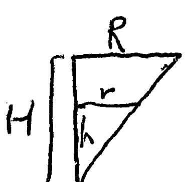
$\frac{-4}{2304\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{-1}{576\pi}\ cm/min}$

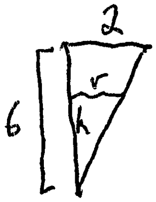
4) $V = \frac{\pi}{3} r^2 h$

A conical cup is 4 cm across and 6 cm deep. Water leaks out of the bottom at the rate of 2 cm^3/sec. How fast is the water level dropping when the height of the water is 3 cm?





$\frac{r}{R} = \frac{h}{H}$



$\frac{r}{2} = \frac{h}{6}$

$6r = 2h$

$r = \frac{2}{6}h \rightarrow \frac{h}{3}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{9} \cdot h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = -2$

$\frac{dh}{dt} = \_\_\_ ?$

$\boxed{h = 3\ cm}$ * save this until after derivative

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2\left(\frac{dh}{dt}\right)$

$-2 = \frac{\pi}{9}(3)^2\left(\frac{dh}{dt}\right)$

$-2 = \frac{\pi(9)}{9}\left(\frac{dh}{dt}\right)$

$-2 = \pi\left(\frac{dh}{dt}\right)$

$-\frac{2}{\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-2}{\pi}\ cm/sec}$