Calculus AB Related Rates Test Review

1. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?

2. Use local linear approximation to estimate $\frac{-1}{15.7\sqrt[4]{15.7}}$

3. Find the limit:

a) $\lim_{x\to 3}\frac{4x^2-5x}{1-3x^2}$

b) $\lim_{x\to 1}\frac{5x^4-4x^2-1}{10-x-9x^3}$

4. Water is leaking out of a cylindrical container at a rate of 5 cm^3/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when container has height of 3 cm?

5. A sphere of diameter 4 in. is inside a cylinder with radius of 12 in and constant height of 8 in. How fast is the volume between the sphere and cylinder changing if the diameter of the sphere is increasing at a rate of 2 in/min and the radius of the cylinder is decreasing at a rate of 4 in/min?

6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec

a) How fast is the boat approaching the dock when 10 ft of rope are out?

b) At what rate is area of triangle changing at that moment?

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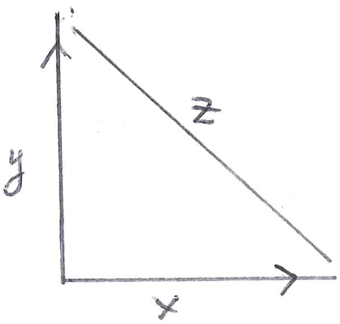


$\frac{dy}{dt} = 1$ ft/s $\frac{dx}{dt} = 17$ ft/s

$y = 65$

Find $\frac{dz}{dt}$ 3 seconds later

$y = 65 + 3 = 68$ ft

$x = 17(3) = 51$ ft.

$z^2 = x^2 + y^2$

$z^2 = 51^2 + 68^2$

$z = 85$ ft.

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(51)(17) + 2(68)(1) = 2(85)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt} = 11 \text{ ft/s}}$

2. Use local linear approximation to estimate $\dfrac{-1}{15.7\sqrt[4]{15.7}}$

$y = \frac{-1}{x\sqrt[4]{x}}$

$y(16) = \frac{-1}{16\sqrt[4]{16}} = \frac{-1}{16(2)} = \frac{-1}{32}$

point: $(16, -\frac{1}{32})$

$y(x) = \frac{-1}{x \cdot x^{1/4}} = \frac{-1}{x^{5/4}} = -x^{-5/4}$

$y' = \frac{5}{4}x^{-9/4} = \frac{5}{4x^{9/4}}$

$y'(16) = \frac{5}{4(16)^{9/4}} = \frac{5}{4(2)^9} = \frac{5}{2^{11}}$

slope: $m = \frac{5}{2^{11}}$

$y + \frac{1}{32} = \frac{5}{2^{11}}(x-16)$ $y = \frac{5}{2^{11}}(x-16) - \frac{1}{32}$

$\frac{-1}{15.7\sqrt[4]{15.7}} \approx \frac{5}{2^{11}}(15.7-16) - \frac{1}{32} \approx \boxed{-0.0319}$

3. Find the limit:

a) $\lim_{x \to \infty} \frac{4x^2 - 5x}{1 - 3x^2} = \frac{\infty}{-\infty}$

$f'(x) = 8x - 5$

$g'(x) = -6x$

$\lim_{x \to \infty} \frac{8x-5}{-6x} \rightarrow \frac{8}{-6}$

$= \boxed{\frac{-4}{3}}$

b) $\lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3} = \frac{0}{0}$

$\lim_{x \to 1} \frac{20x^3 - 8x}{-1 - 27x^2} = \frac{20-8}{-1-27} = \frac{12}{-28} = \boxed{\frac{-3}{7}}$

4. Water is leaking out of a cylindrical container at a rate of 5 cm^3/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when container has height of 3 cm? $V = \pi r^2 h$

$\frac{dV}{dt} = -5 cm^3/hr.$

$r = 6$ cm

$h = 16$ cm

$\frac{dh}{dt} =$ ____

$h = 3$ cm

$r = 6$ cm

$V = \pi r^2 h$

$V = \pi (6)^2 h$

$V = 36\pi h$

$\frac{dV}{dt} = 36\pi\left(\frac{dh}{dt}\right)$

$-5 = 36\pi\left(\frac{dh}{dt}\right)$

$\frac{-5}{36\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-5}{36\pi} \text{ cm/hr.}}$

5. A sphere of diameter 4 in. is inside a cylinder with radius of 12 in and constant height of 8 in. How fast is the volume between the sphere and cylinder changing if the diameter of the sphere is increasing at a rate of 2 in/min and the radius of the cylinder is decreasing at a rate of 4 in/min?

$V_{Between} = V_{cylinder} - V_{sphere}$

$\frac{dV}{dt_B} = \frac{dV}{dt_c} - \frac{dV}{dt_s}$

Sphere $V = \frac{4}{3}\pi r^3$

$r = 2$

$\frac{dr}{dt} = 1$ in/min

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt_s} = 4\pi(2)^2(1)$

$= 16\pi$ in^3/min

Cylinder: $V = \pi r^2 h$

$h = 8$ (constant)

$r = 12$ in.

$\frac{dr}{dt} = -4$ in/min

$V = \pi r^2(8)$

$V = 8\pi r^2$

$\frac{dV}{dt} = 16\pi r \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt_c} = 16\pi(12)(-4)$

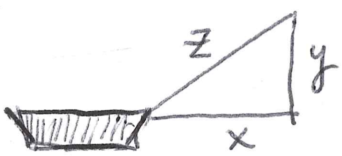
$\frac{dV}{dt_c} = -768\pi$ in^3/min

$\frac{dV}{dt_B} = \frac{dV}{dt_c} - \frac{dV}{dt_s}$

$= -768\pi - 16\pi$

$= \boxed{-784\pi \text{ in}^3/\text{min}}$

6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec



$\frac{dz}{dt} = -2$ ft/s

$y = 6$

$z = 10$

$x = 8$

$\frac{dz}{dt} = -2$ ft/s

$\frac{dy}{dt} = 0$

a) How fast is the boat approaching the dock when 10 ft of rope are out?

Find $\frac{dx}{dt}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(8)\left(\frac{dx}{dt}\right) + 2(10)(0) = 2(10)(-2)$

$16\frac{dx}{dt} = -40$

$\boxed{\frac{dx}{dt} = -2.5 \text{ ft/s}}$

b) At what rate is area of triangle changing at that moment?

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$

$= \frac{1}{2}(-2.5)(6) + \frac{1}{2}(8)(0)$

$\boxed{\frac{dA}{dt} = -7.5 \text{ ft}^2/\text{s}}$