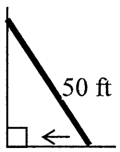
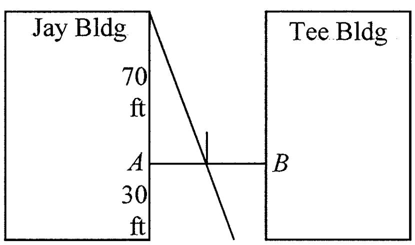
A.P. Calculus AB   Related Rates Test Review

1. A ladder, 50 feet long, is being pushed against the wall at a rate of 5 ft / sec. When the bottom of the ladder is 30 ft from the wall:
    a) What is the top of the ladder "doing" (velocity)?
    b) At what rate is the area of the triangle enclosed by the ladder, wall, and floor changing?



2. 1991 problem # 6: A tightrope is stretched 30 feet above the ground between the Jay and Tee buildings which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.

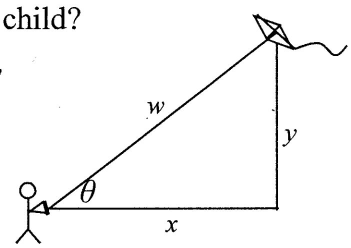


a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units.)

3. A kite is moving away from a child in a manner so that the angle θ stays constant, as in the picture below. The rate of change of the horizontal distance between the kite and the child is 6 ft / sec. How fast is the string, w, being let out when the kite is 30 ft high and at a horizontal distance of 40 ft from the child?



4. When a circular plate of metal is heated in an oven, its diameter increases at the rate of 0.02 cm/sec. (Solve without using radius)
    a) At what rate is the plate's area increasing when the diameter is 100 cm?
    b) At what rate is the plate's circumference increasing when the diameter is 100 cm?

5. Water is leaking out of a cylindrical container at a rate of 5 cm^3/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when container has height of 3 cm?

6. The length, x, of a rectangle is decreasing at the rate of 2cm/sec, while the width, y, is increasing at the rate of 2cm/sec. When x =12cm and y= 5cm,
   Find the rates of change of:
   a) The area
   b) The perimeter
   c) the length of the diagonal of the rectangle

7. A circle of radius 3 inches is inside a square with 12 inch sides. How fast is the area between the circle and square changing if the radius is increasing at 4 inches per minute and the sides are increasing at 2 inches per minute?

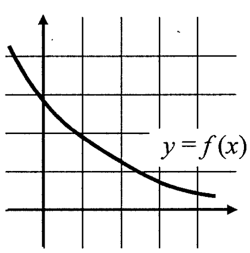
8. A rectangle is inscribed in a circle of radius 5 inches. If the length of the rectangle is decreasing at a rate of 2 inches per second, how fast is the area of the rectangle changing at the instant when the length is 6 inches? HINT: A diagonal of a rectangle is a diameter of the circle.

9. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?

10. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec
    a) How fast is the boat approaching the dock when 10 ft of rope are out?
    b) At what rate is area of triangle changing at that moment?

11. Find the local linear approximation $f(x) = \sqrt{3x^2 - 5x}$ at a = -5 and use it to approximate $f(-4.9)$.

12. Suppose the only information we have about a function f is that f '(1) = 5 and the graph of f(x) is as shown below.
    a. Use a linear approximation to estimate f(0.9) and f(1.1).
    b. Are your estimates in part a too large or too small? Explain why.



13. Find the limit:

a) $\lim_{x \to 3} \frac{4x^2 - 5x}{1 - 3x^2}$

b) $\lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$

Related Rates Test Review Answer Key

(1) 

$\frac{dx}{dt} = -5$, $x = 30$ a) $\frac{dy}{dt} = ?$ $\quad x^2 + y^2 = 50^2$ $\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\quad \frac{dy}{dt} = \frac{x\frac{dx}{dt}}{y} = \frac{(30)(-5)}{40} = \boxed{\frac{-15}{4}\ \frac{ft}{sec}}$

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}\cdot y + \frac{1}{2}x\frac{dy}{dt} = \frac{1}{2}(-5)(40) + \frac{1}{2}(30)\left(-\frac{15}{4}\right) = \boxed{\frac{-625}{4}\ \frac{ft^2}{sec}}$

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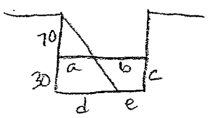
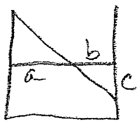
(2) 

$\frac{da}{dt} = 2$ a) $\frac{dd}{dt} = ?$ $\quad \frac{a}{70} = \frac{d}{100} \to 100a = 70d$ $\quad 100\frac{da}{dt} = 70\frac{dd}{dt}$ $\quad \frac{dd}{dt} = \frac{100(2)}{70} = \boxed{\frac{20}{7}\ \frac{ft}{sec}}$

b) $\frac{a}{70} = \frac{50}{100}$ $\quad a = \boxed{35\ ft}$

c) 

$b = 10$, $a = 40$ $\quad \frac{a}{70} = \frac{b}{c}$ $\quad \frac{40}{70} = \frac{10}{c} \to c = \frac{35}{2}$

$\frac{a}{70} = \frac{50-a}{c}$ $\quad ac = 3500 - 70a$

$\frac{da}{dt}\cdot c + a\frac{dc}{dt} = -70\frac{da}{dt}$

$(2)\left(\frac{35}{2}\right) + 40\frac{dc}{dt} = (-70)(2)$

$\frac{dc}{dt} = \frac{-140-35}{40} = \frac{-35}{8}\ \frac{ft}{sec}$

moving at a rate of $\boxed{\frac{35}{8}\ \frac{ft}{sec}}$ up the wall

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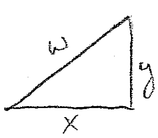
(3) 

$\frac{dx}{dt} = 6$ $\quad \frac{dw}{dt} = ?$ $\quad y = 30$, $x = 40$, $w = 50$

$\frac{x}{w} = \frac{40}{50}$ $\quad 50x = 40w$ $\quad 50\frac{dx}{dt} = 40\frac{dw}{dt}$ $\quad \frac{dw}{dt} = \frac{50(6)}{40} = \boxed{\frac{15}{2}\ \frac{ft}{sec}}$

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(4) $\frac{dd}{dt} = -.02$ a) $A = \pi\left(\frac{1}{2}d\right)^2 = \frac{1}{4}\pi d^2 \to \frac{dA}{dt} = \frac{1}{2}\pi d\frac{dd}{dt} = \frac{1}{2}\pi(100)(-.02) = \boxed{-\pi\ \frac{cm^2}{sec}}$

b) $C = \pi d$ $\quad \frac{dC}{dt} = \pi\frac{dd}{dt} = \boxed{-.02\pi\ \frac{cm}{sec}}$

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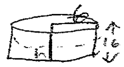
(5) $\frac{dV}{dt} = -5$, $r = 6$, 

$h = 3$, $\frac{dh}{dt} = ?$

$V = \pi r^2 h = 36\pi h$ $\quad \frac{dV}{dt} = 36\pi\frac{dh}{dt} \to -5 = 36\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{\frac{-5}{36\pi}\ \frac{cm}{hr}}$

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(6) $\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 2$, $x = 12$, $y = 5$

a) $A = xy \to \frac{dA}{dt} = \frac{dx}{dt}\cdot y + x\frac{dy}{dt} = -2(5) + 2(12) = \boxed{14\ \frac{cm^2}{sec}}$

b) $P = 2x + 2y \to \frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt} = 2(-2) + 2(2) = \boxed{0\ \frac{cm}{sec}}$

c) $x^2 + y^2 = z^2 \to 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt} \to \frac{dz}{dt} = \frac{2(12)(-2) + 2(5)(2)}{2(13)} = \boxed{\frac{-14}{13}\ \frac{cm}{sec}}$

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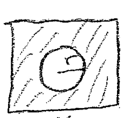
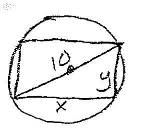
(7) 

$r = 3$, $x = 12$, $\frac{dr}{dt} = 4$, $\frac{dx}{dt} = 2$

$A = x^2 - \pi r^2$

$\frac{dA}{dt} = 2x\frac{dx}{dt} - 2\pi r\frac{dr}{dt} = 2(12)(2) - 2\pi(3)(4)$

$= \boxed{48 - 24\pi\ \frac{in^2}{min}}$

(8) 

$\frac{dx}{dt} = -2$ $\quad x = 6 \quad y = 8$

$A = xy = x\sqrt{100-x^2}$ $\quad x^2+y^2=100 \quad y = \sqrt{100-x^2}$

$\frac{dA}{dt} = \frac{dx}{dt}\sqrt{100-x^2} + x\left(\frac{1}{2}\right)(100-x^2)^{-\frac{1}{2}}\left(-2x\frac{dx}{dt}\right)$

$= (-2)\sqrt{100-36} + 6\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{100-36}}\right)(-2)(6)(-2)$

$= \boxed{-7 \frac{in^2}{sec}}$

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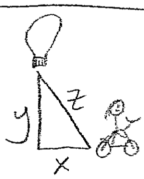
(9) 

$\frac{dy}{dt} = 1, \ \frac{dx}{dt} = 17, \ \frac{dz}{dt} = ?, \ y = 65+3 = 68, \ x = 51, \ z = 85$

$x^2 + y^2 = z^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{51(17)+68(1)}{85} = \boxed{11 \frac{ft}{sec}}$

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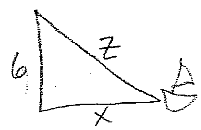
(10) 

$\frac{dz}{dt} = -2$ $\quad z = 10 \quad x = 8$

a) $\frac{dx}{dt} = ?$ $\quad x^2 + 6^2 = z^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$

$\frac{dx}{dt} = \frac{10(-2)}{8} = \boxed{\frac{-5}{2} \frac{ft}{sec}}$

b) $A = \frac{1}{2}(6)(x) = 3x \quad \frac{dA}{dt} = 3\frac{dx}{dt} = 3\left(-\frac{5}{2}\right) = \boxed{\frac{-15}{2} \frac{ft^2}{sec}}$

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(11) $f(-5) = 10$ point: $(-5, 10)$

$f(x) = \sqrt{3x^2-5x}$

$f'(x) = \frac{1}{2}(3x^2-5x)^{-\frac{1}{2}}(6x-5) = \frac{6x-5}{2\sqrt{3x^2-5x}}$

$f'(-5) = \frac{-35}{20} = -\frac{7}{4}$

$y - 10 = -\frac{7}{4}(x+5)$

$y - 10 = -\frac{7}{4}(-4.9+5)$

$y = \frac{407}{40}$

$f(-4.9) \approx \frac{407}{40}$

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(12) $f'(1) = 5$

$f(1) = 2$

a) $y - 2 = 5(x-1)$

$y = 5x - 3$

$f(0.9) \approx 5(0.9) - 3 = 1.5$

$f(1.1) \approx 5(1.1) - 3 = 2.5$

b) estimates are too small because $f(x)$ is concave up

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(13) $\lim_{x \to 3} \frac{4x^2-5x}{1-3x^2} = \boxed{\frac{21}{-26}}$

$\lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$

$= \lim_{x \to 1} \frac{20x^3 - 8x}{-1 - 27x^2} = \frac{12}{-28} = \boxed{\frac{-3}{7}}$