Calculus AB Related Rates WS #2

1) 1990 problem #4

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.
(Note: the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?
b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?
c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

2. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)

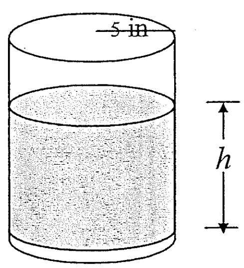
3. A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?

b. What is the rate at which the area of the triangle is changing?

4. A man 5.5 ft tall walks away at a rate of 2 ft/sec away from a lamppost that is 12 feet above ground. When the man is 8 ft away from the base of the light,
   a. At what rate is the length of his shadow changing? (ans: 1.69 ft/s)
   b. At what rate is the tip of his shadow moving? (ans: 3.69 ft/s)

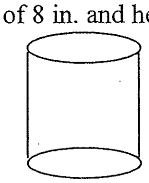
5. A spherical balloon is inflated so that its volume is increasing at the rate of 40 cubic feet per minute. How fast is the surface area of the balloon increasing when the radius is 5 feet? (Note: $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

6. 2003 AB problem #5



A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time, t, measured in seconds. The volume, V, of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$.) Find $\frac{dh}{dt}$ as a function of h. (This means your answer will contain the variable h)

7. Water is leaking out of a full cylindrical container at a rate of 2 in^3/hr. The container has a diameter of 8 in. and height of 12 in. At what rate is the height of the container changing when the container is half full?



$V = \pi r^2 h$

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a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?
b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?
c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

a) $\frac{dr}{dt} = 0.04\ cm/s$

$r = 10cm$

$\frac{dV}{dt} =$ ______

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi(10)^2(0.04)$

$= 16\pi\ cm^3/s$

b) $V = 36\pi$

$V = \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi r^3 = 36\pi$

$r^3 = 27$

$r = 3$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3)(0.04)$

$\boxed{= 0.24\pi\ cm^2/s}$

c) set $\frac{dV}{dt} = \frac{dr}{dt}$

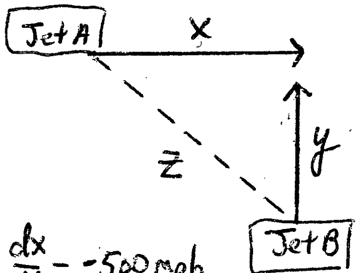
$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$1 = 4\pi r^2$

$\sqrt{\frac{1}{4\pi}} = r$, $\boxed{r = \frac{1}{2\sqrt{\pi}}\ cm}$

2. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)



$\frac{dx}{dt} = -500\ mph$

$\frac{dy}{dt} = -600\ mph$

$x = 100$

$z = 300$

$y = 282.843$ miles

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

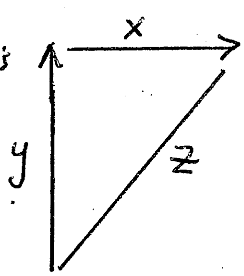
$2(100)(-500) + 2(282.843)(-600) = 2(300)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt} = -732.353\ mph}$

3. A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?

$\frac{dy}{dt} = -17 m/s$

$\frac{dz}{dt} = 5 m/s$



$x = 28.723$

$y = 20$

$z = 35$

$x^2 + 20^2 = 35^2$

$x = 28.723$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(28.723)\left(\frac{dx}{dt}\right) + 2(20)(-17) = 2(35)(5)$

$\boxed{\frac{dx}{dt} = 17.929 m/s}$

b. What is the rate at which the area of the triangle is changing?

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)y + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}(17.929)(20) + \frac{1}{2}(28.723)(-17)$

$\boxed{\frac{dA}{dt} = \cancel{179.821}\ m/s^2}$

$-64.84\ m/s^2$

4. A man 5.5 ft tall walks away at a rate of 2 ft/sec away from a lamppost that is 12 feet above ground. When the man is 8 ft away from the base of the light,
   a. At what rate is the length of his shadow changing? (ans: 1.69 ft/s)
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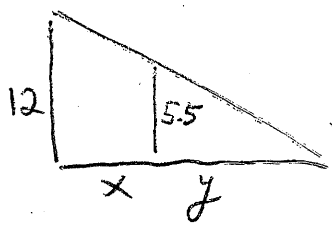


$\frac{dx}{dt} = 2 \text{ ft/s}$

$\frac{5.5}{12} = \frac{y}{x+y}$

$5.5x + 5.5y = 12y$

$5.5x = 6.5y$

$5.5\frac{dx}{dt} = 6.5\frac{dy}{dt}$

$5.5(2) = 6.5\frac{dy}{dt}$

a) $\frac{dy}{dt} = 1.692 \text{ ft/s}$

b) $\frac{dx}{dt} + \frac{dy}{dt} = 3.692 \text{ ft/s}$

5. A spherical balloon is inflated so that its volume is increasing at the rate of 40 cubic feet per minute. How fast is the surface area of the balloon increasing when the radius is 5 feet? (Note: $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

$\frac{dV}{dt} = 40 \text{ ft}^3/\text{min.}$

$\frac{dS}{dt} =$ _____

$r = 5 \text{ ft.}$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

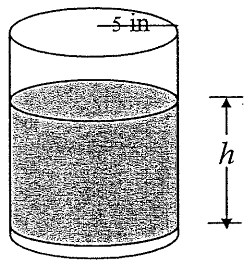
$40 = 4\pi(5)^2\left(\frac{dr}{dt}\right)$

$\frac{2}{5\pi} = \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi(5)\left(\frac{2}{5\pi}\right)$

$\boxed{\frac{dS}{dt} = 16 \text{ ft}^2/\text{min}}$

6. 2003 AB problem #5



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$\frac{dV}{dt} = -5\pi\sqrt{h} \text{ in}^3/\text{sec}$

$r = 5 \text{ in.}$

$V = \pi r^2 h$

$V = \pi(5)^2 h$

$V = 25\pi h$

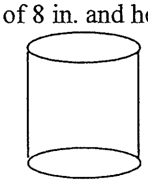
$\frac{dV}{dt} = 25\pi\left(\frac{dh}{dt}\right)$

$-5\pi\sqrt{h} = 25\pi\left(\frac{dh}{dt}\right)$

$\frac{-5\pi\sqrt{h}}{25\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-\sqrt{h}}{5} \text{ in/sec.}}$

7. Water is leaking out of a full cylindrical container at a rate of 2 in^3/hr. The container has a diameter of 8 in. and height of 12 in. At what rate is the height of the container changing when the container is half full?



$V = \pi r^2 h$

$\frac{dV}{dt} = -2 \text{ in}^3/\text{hr.}$

$r = 4 \text{ in.}$

$h = 12 \text{ in.}$

$\frac{dh}{dt} =$ _____

$V = \pi(4)^2 h$

$V = 16\pi h$

$\frac{dV}{dt} = 16\pi\left(\frac{dh}{dt}\right)$

$-2 = 16\pi\left(\frac{dh}{dt}\right)$

$\frac{dh}{dt} = \frac{-2}{16\pi} = \boxed{\frac{-1}{8\pi} \text{ in/hr.}}$