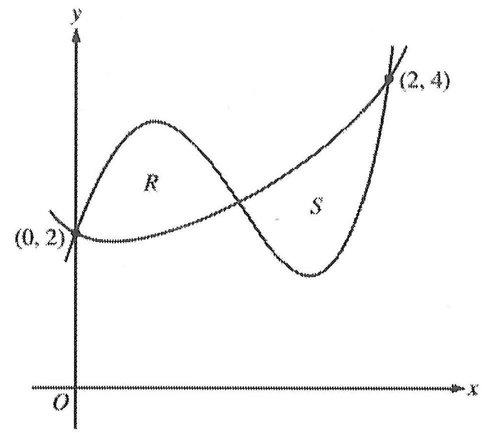
1)

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.



(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\text{Area} = \int_0^A [g(x) - f(x)]\,dx + \int_A^2 [f(x) - g(x)]\,dx$$
$$= 0.997427 + 1.006919 = 2.004$$

$4: \begin{cases} 1: \text{limits} \\ 2: \text{integrands} \\ 1: \text{answer} \end{cases}$

(b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2\,dx = 1.283$

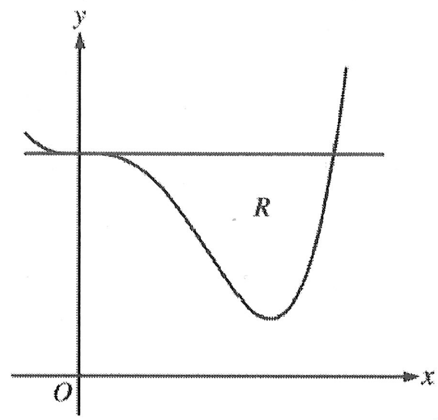
$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$
$h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

$2: \begin{cases} 1: \text{considers } h' \\ 1: \text{answer} \end{cases}$

2)

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\text{Volume} = \pi \int_0^{2.3} \left[(4+2)^2 - (f(x)+2)^2 \right] dx$$
$$= 98.868 \text{ (or } 98.867)$$

$4: \begin{cases} 2: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $$\text{Volume} = \int_0^{2.3} \frac{1}{2}(4 - f(x))^2\, dx$$
$$= 3.574 \text{ (or } 3.573)$$

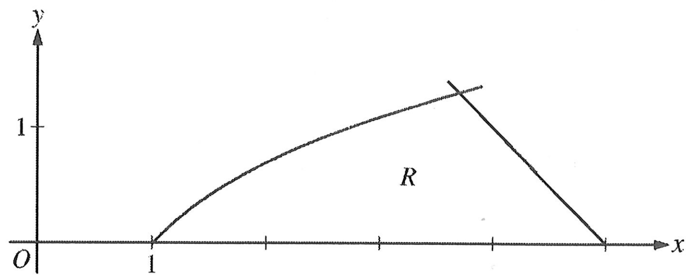
$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $$\int_0^k (4 - f(x))\, dx = \int_k^{2.3} (4 - f(x))\, dx$$

$2: \begin{cases} 1: \text{area of one region} \\ 1: \text{equation} \end{cases}$

3)

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \Rightarrow x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) Area $= \int_0^B \left(5 - y - e^y\right) dy$
$= 2.986$ (or 2.985)

OR

Area $= \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$
$= 2.986$ (or 2.985)

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

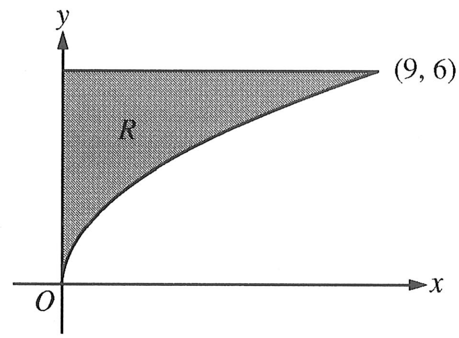
(b) Volume $= \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$

$3 : \begin{cases} 2 : \text{integrands} \\ 1 : \text{expression for total volume} \end{cases}$

(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

4)



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2 \right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$.

Volume $= \int_0^6 \frac{3}{16}y^4 \, dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$