1)

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim_{x \to 0^-} (1 - 2\sin x) = 1$ $\lim_{x \to 0^+} e^{-4x} = 1$ $f(0) = 1$ So, $\lim_{x \to 0} f(x) = f(0)$. Therefore f is continuous at $x = 0$.	2 : analysis
(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$ $-2\cos x \neq -3$ for all values of $x < 0$. $-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$. Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.	$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$
(c) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$ $= \int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx$ $= \Big[x + 2\cos x\Big]_{x=-1}^{x=0} + \Big[-\frac{1}{4}e^{-4x}\Big]_{x=0}^{x=1}$ $= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)$ Average value $= \frac{1}{2}\int_{-1}^{1} f(x)\,dx$ $= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$	$4 : \begin{cases} 1 : \int_{-1}^{0} (1 - 2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

2)

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$	2 : $f'(x)$
(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$ $f(-3) = \sqrt{25 - 9} = 4$ An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.	2 : $\begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$
(c) $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$ $\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4$ Therefore, $\lim_{x \to -3} g(x) = 4$. $g(-3) = f(-3) = 4$ So, $\lim_{x \to -3} g(x) = g(-3)$. Therefore, g is continuous at $x = -3$.	2 : $\begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$
(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$ $\int_0^5 x\sqrt{25 - x^2}\, dx = -\frac{1}{2}\int_{25}^0 \sqrt{u}\, du$ $= \left[-\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\right]_{u=25}^{u=0}$ $= -\frac{1}{3}(0 - 125) = \frac{125}{3}$	3 : $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$