1.

The sum of the digits of a certain two-digit number is 14.
If the digits are reversed, the number is increased by 18. Find the number.

Solutions #1

Let the number be $10a + b$.

$$10b + a = 10a + b + 18$$

$$b - a = 2$$

$$b + a = 14$$

$$2b = 16 \quad b = 8, \quad a = 6$$

The number is 68

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2.

$$\text{Let } x = \frac{\sqrt{7}+\sqrt{3}}{2} \quad \text{and } y = \frac{\sqrt{7}-\sqrt{3}}{2}$$

then the value of $x^2 - xy + y^2$ is...

**Solution #2**

$$x - y = 3 \quad \text{and} \quad xy = 1$$

$$x^2 - xy + y^2 = (x-y)^2 + xy = 3 + 1 = \underline{4}$$

Alternate Solution:

$$\left.\begin{aligned} x^2 &= \frac{7+3+2\sqrt{21}}{4} = \frac{10+2\sqrt{21}}{4} \\ y^2 &= \frac{7+3-2\sqrt{21}}{4} = \frac{10-2\sqrt{21}}{4} \end{aligned}\right\} = \frac{20}{4} = 5$$

$$xy = \frac{7-3}{4} = 1$$

So, $x^2 - xy + y^2$
$= x^2 + y^2 - xy = \underline{4}$

3. If $y - 2\sqrt{y} - 8 = 0$, Let $x = \sqrt{y}$

then what is the value of y?

Solutions for #3

**Answer: y = 16**

then $x^2 - 2x - 8 = 0$

which has roots $x = 4$ and $x = -2$.

But x must be positive, so $x = 4$

and so $y = x^2$

4.

If the sum of the first 100 positive odd integers is subtracted from the sum of the first 100 positive even integers, then the result is ...

$$\begin{array}{r} 2+4+6+\cdots+198+200 \\ -\ 1+3+5+\cdots+197+199 \\ \hline 1+1+1+\cdots+1+1=\underline{\underline{100}} \end{array}$$

5.

If $\sin 2\theta = 0.69$, and $\theta$ is an acute angle, then what is the value of $\sin\theta + \cos\theta$?

(Recall $\sin^2 x + \cos^2 x = 1$ and $\sin 2x = 2\sin x\cos x$)

Let $x = \sin\theta + \cos\theta$, then

$$x^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$$
$$= 1 + \sin 2\theta = 1.69$$

So, $x = \sqrt{1.69} = 1.3$ (Note: $x > 0$ )

6.

If x and y are non-zero and $x^2 + y^2 = 3xy$ then what is the value of $\frac{x}{y} + \frac{y}{x}$?

**Solution #6:**

Divide both sides by xy.

$$\frac{x^2}{xy} + \frac{y^2}{xy} = 3$$

$$\frac{x}{y} + \frac{y}{x} = \underline{3}$$

7)

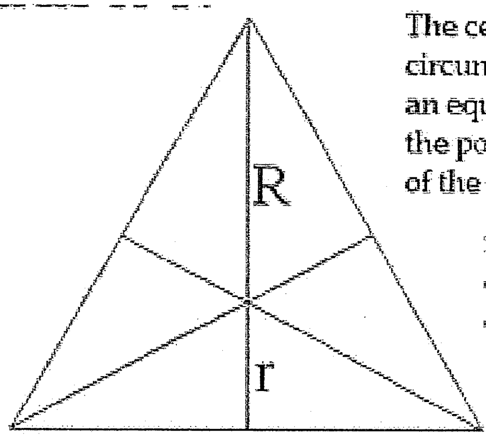
Compute $\sqrt{3+\sqrt{3+\sqrt{3+\cdots}}}$.

#7 Solution:

**Solution:** Let $x=\sqrt{3+\sqrt{3+\sqrt{3+\cdots}}}$. Then $x=\sqrt{3+x}$, so $x^2-x-3=0$. By the quadratic equation, $x=\frac{1\pm\sqrt{13}}{2}$; since $x$ must be positive, we have $x=\frac{1+\sqrt{13}}{2}$.

8.

Let T denote an equilateral triangle. If the inscribed circle of T has an area of 5, then what is the area of the circumscribed circle of T?




The center of both the inscribed and circumscribed circle is the same for an equilateral triangle. This point is the point of intersection of the medians of the triangle.

r is the radius of the inscribed circle and R is the radius of the circumscribed circle.

The medians meet at a point that is 2/3 of the way from each vertex of the triangle to the opposite side.

**So R = 2r**

$$\frac{\pi R^2}{\pi r^2} = \left(\frac{R}{r}\right)^2 = 4$$

$$\frac{\text{Area of circumscribed circle}}{\text{Area of inscribed circle}} \quad \frac{\pi R^2}{\pi r^2} \qquad \pi R^2 = 4\pi r^2$$

**So 4 x 5 = 20**