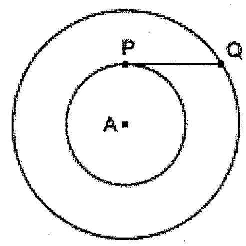
3. Tom found the value of $3^{21} = 10{,}4A0{,}353{,}20B$. He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?

(A) 0 (B) 2 (C) 3 (D) 6 (E) 8

4.

Let A be the centers of two concentric circles, as shown. $\overline{PQ}$ is a line segment tangent to the smaller circle at point P and intersecting the larger circle at point Q. If PQ = 10, what is the area of the region lying between the two circles?

(A) 10π (B) 25π (C) 50π (D) 64π (E) 100π



5. Let x = m + n where m and n are positive integers satisfying $2^6 + m^n = 2^7$. The sum of all of the possible values of x is:

(A) 18 (B) 25 (C) 75 (D) 82 (E) 90

8. If q and r are the zeros of the quadratic polynomial $x^2 + 15x + 31$, find the quadratic polynomial whose zeros are $q + 1$ and $r + 1$.

(A) $x^2 + 17x + 31$ (B) $x^2 + 15x + 33$ (C) $x^2 + 13x + 17$

(D) $x^2 + 19x + 37$ (E) None of these

11. In triangle ABC, AB = 3, BC = 5, and AC = 7. The angle bisectors of the two smallest angles of the triangle meet at point P. Compute the ratio of the measure of $\angle APC$ to the measure of $\angle B$?

(A) $\frac{11}{10}$ (B) $\frac{10}{9}$ (C) $\frac{8}{7}$ (D) $\frac{7}{6}$ (E) $\frac{5}{4}$

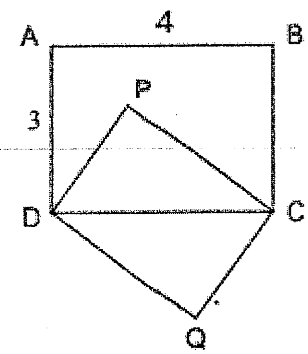
17. On Dr. Garner's multiple choice tests, each question has 4 choices, exactly one of which is the correct answer. On one such test, a student knows the correct answer to exactly 70% of the questions. For the other 30% of the questions he selects one of the choices at random. If he gets the correct answer to a particular question, what is the probability that he knew the answer rather than guessed it?

(A) $\frac{3}{40}$ (B) $\frac{7}{20}$ (C) $\frac{17}{40}$ (D) $\frac{31}{40}$ (E) $\frac{28}{31}$

20. The number 2011 can be written as $a^2 - b^2$ where a and b are integers. Compute the value of $a^2 + b^2$.

(A) 2018041 (B) 2022061 (C) 2024072 (D) 2026085 (E) 2033051

22. Rectangle ABCD has sides of length 3 and 4. Rectangle PCQD is similar to rectangle ABCD, with P inside rectangle ABCD. Compute the distance from P to AB.



(A) 1 (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) $\frac{21}{17}$ (E) $\frac{27}{25}$

18. In a geometric sequence of real numbers, the sum of the first two terms is 7 and the sum of the first six terms is 91. What is the sum of the first four terms?

(A) 13 (B) 14 (C) 26 (D) 28 (E) 32

20. Suppose $f(x) = ax + b$, $g(x) = bx + a$ (a, b integers). If $f(1) = 8$ and $f(g(50)) - g(f(50)) = 28$, find the product of a and b.

(A) 5 (B) 12 (C) 48 (D) 182 (E) 210

21. One of the roots of the equation $x^3 - 14x^2 + 26x + c = 0$ is the quotient of the other two roots. Compute the product of all possible values of c.

(A) 456 (B) 529 (C) 576 (D) 625 (E) 676

22. A vertical line divides the triangle with vertices (0, 0), (1, 1), (9, 1) into two regions of equal area. The equation of the line is $x = a$. Compute the value of a.

(A) 2 (B) 2.5 (C) 3 (D) 3.5 (E) 4

Math Team Sept 14 2017 **Practice from Kennesaw Competition 2011-12**

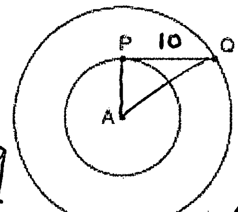
*consider multiples of 3: digit pattern is 3, 9, 7, 1, ...
R1 – $3^1 = 3$
R2 – $3^2 = 9$
R3 – $3^3 = 27$
R0 – $3^4 = 81$
$3^5 = 243$
$4 \overline{)21}$ 5 R 1
so $B = 3$

3. Tom found the value of 3^{21} = 10,4A0,353,20B. He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?

(A) 0 (B) 2 (C) 3 **(D) 6** (E) 8

* Since 3^{21} is a factor of 9, all digits must add to a multiple of 9: 104A353203 → adds to be 21, so $A = 6$

4. Let A be the centers of two concentric circles, as shown. $\overline{PQ}$ is a line segment tangent to the smaller circle at point P and intersecting the larger circle at point Q. If PQ = 10, what is the area of the region lying between the two circles?



(A) 10π (B) 25π (C) 50π (D) 64π **(E) 100π**

$(AP)^2 + (PQ)^2 = (AQ)^2$
$(AP)^2 + 100 = (AQ)^2$
$(AQ)^2 - (AP)^2 = 100$

Area(small circle) $= \pi(AP)^2$

Area(large circle) $= \pi(AQ)^2$

Area(large − small) $= \pi[(AQ)^2 - (AP)^2] = \pi[100] = 100\pi$

5. Let x = m + n where m and n are positive integers satisfying $2^6 + m^n = 2^7$. The sum of all of the possible values of x is:

(A) 18 (B) 25 (C) 75 (D) 82 **(E) 90**

$2^6 + m^n = 2(2^6)$
$m^n = 2^6$

$m^n = (2)^6 = 2^{3\cdot2} = 4^3 = 8^2 = (2^6)^1$
(a) $m = 64, n = 1 \rightarrow x = 65$
(b) $m = 2, n = 6 \rightarrow x = 8$
(c) $m = 4, n = 3 \rightarrow x = 7$
(d) $m = 8, n = 2 \rightarrow x = 10$

$7 + 10 + 65 + 8 = 90$

8. If q and r are the zeros of the quadratic polynomial $x^2 + 15x + 31$, find the quadratic polynomial whose zeros are $q + 1$ and $r + 1$.

(A) $x^2 + 17x + 31$ (B) $x^2 + 15x + 33$ **(C) $x^2 + 13x + 17$**
(D) $x^2 + 19x + 37$ (E) None of these

$x^2 + (-q-r)x + qr$

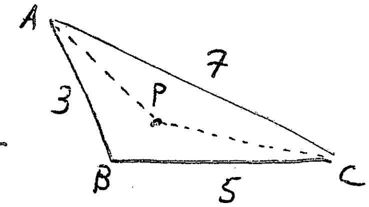
$q + r = -15$ $qr = 31$

$q + 1 + r + 1 = -15 + 2$
$q + r + 2 = -13$

$(q+1)(r+1) = qr + r + q + 1 = 31 + (-15) + 1 = 17$

$x^2 + 13x + 17$

11. In triangle ABC, AB = 3, BC = 5, and AC = 7. The angle bisectors of the two smallest angles of the triangle meet at point P. Compute the ratio of the measure of $\angle APC$ to the measure of $\angle B$?



(A) $\frac{11}{10}$ (B) $\frac{10}{9}$ (C) $\frac{8}{7}$ (D) $\frac{7}{6}$ **(E) $\frac{5}{4}$**

$7^2 = 3^2 + 5^2 - 2(3)(5)\cos B$

$49 = 9 + 25 - 30\cos B$

$\cos B = -\frac{1}{2}$ $B = \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3} = 120°$

$m\angle BAC + m\angle BCA = 60°$

$m\angle PAC + m\angle PCA = 30°$

$m\angle P = 150°$

$\frac{\angle APC}{\angle B} = \frac{150}{120} = \boxed{\frac{5}{4}}$

17. On Dr. Garner's multiple choice tests, each question has 4 choices, exactly one of which is the correct answer. On one such test, a student knows the correct answer to exactly 70% of the questions. For the other 30% of the questions he selects one of the choices at random. If he gets the correct answer to a particular question, what is the probability that he knew the answer rather than guessed it?

(A) $\frac{3}{40}$ (B) $\frac{7}{20}$ (C) $\frac{17}{40}$ (D) $\frac{31}{40}$ **(E) $\frac{28}{31}$**

P(student knowing answer) $= \frac{7}{10}$

P(not know answer) $= \frac{3}{10}$

P(guess correctly) $= \frac{1}{4}$

P(not know answer and guess correctly) $= \frac{3}{10} \cdot \frac{1}{4} = \frac{3}{40}$

P(getting correct answer) $= \frac{7}{10} + \frac{3}{40} = \frac{28}{40} + \frac{3}{40} = \frac{31}{40}$

$\frac{\text{P(know answer)}}{\text{P(getting correct answer)}} = \frac{7/10}{31/40} = \boxed{\frac{28}{31}}$

20. The number 2011 can be written as $a^2 - b^2$ where a and b are integers. Compute the value of $a^2 + b^2$. *2011 is a prime number

(A) 2018041 **(B) 2022061** (C) 2024072 (D) 2026085 (E) 2033051

$(a-b)(a+b) = 2011$

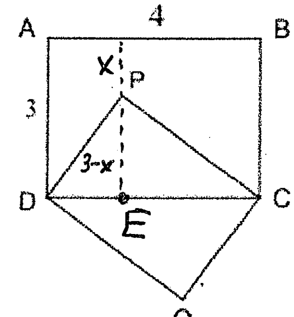
$a - b = 1$ since 2011 is prime

so since $1006 - 1005 = 1$,

$(1006-1005)(1006+1005) = 2011$

$1006^2 + 1005^2 = \boxed{2022061}$

22. Rectangle ABCD has sides of length 3 and 4. Rectangle PCQD is similar to rectangle ABCD, with P inside rectangle ABCD. Compute the distance from P to AB.



(A) 1 (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) $\frac{21}{17}$ **(E) $\frac{27}{25}$**

*set up proportions:

$\frac{AD}{PD} = \frac{AC}{DC}$

$\frac{3}{PD} = \frac{5}{4}$

$PD = \frac{12}{5}$

$\frac{AD}{PD} = \frac{AB}{PC}$

$\frac{3}{12/5} = \frac{4}{PC}$

$3PC = \frac{48}{5}$

$PC = \frac{16}{5}$

$\triangle DPE \sim \triangle DCP$

$\frac{DP}{DC} = \frac{PE}{PC}$

$DP \cdot PC = DC \cdot PE$

$(\frac{12}{5}) \cdot (\frac{16}{5}) = (4)(3-x)$

$\frac{192}{25} = (3-x) \cdot 4$

$\frac{192}{100} = 3 - x$

$x = -\frac{192}{100} + \frac{300}{100} = \frac{108}{100} = \boxed{\frac{27}{25}}$

18. In a geometric sequence of real numbers, the sum of the first two terms is 7 and the sum of the first six terms is 91. What is the sum of the first four terms?

(A) 13 (B) 14 (C) 26 **[(D) 28]** (E) 32

*geometric sequence: $a, ar, ar^2, ar^3, ar^4, ar^5$

$a + ar = 7$

$a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 91$

$1(a+ar) + r^2(a+ar) + r^4(a+ar)$

$(a+ar)[1+r^2+r^4] = 91$

$7[1+r^2+r^4] = 91$

$1+r^2+r^4 = 13$

$r^4+r^2-12=0$

$(r^2+4)(r^2-3)=0$

~~$r^2=-4$~~, $r^2=3$ ✓

1^{st} 4 terms: $1(a+ar)+r^2(a+ar)$

$=1(7)+3(7)$

$= 7+21 = \boxed{28}$

20. Suppose f(x) = ax + b, g(x) = bx + a (a, b integers). If f(1) = 8 and f(g(50)) − g(f(50)) = 28, find the product of a and b.

(A) 5 **[(B) 12]** (C) 48 (D) 182 (E) 210

$f(g(x)) = a(bx+a)+b = abx+a^2+b$

$g(f(x)) = b(ax+b)+a = abx+b^2+a$

$f(g(x))-g(f(x)) = abx+a^2+b-(abx+b^2+a)$

$28 = a^2+b-b^2-a$

$f(1)=a(1)+b=8 \quad b=8-a$

$a^2+b-b^2-a=28$

$a^2+(8-a)-(8-a)^2-a=28$

$a^2+8-a-(64-16a+a^2)-a=28$

$a^2+8-a-64+16a-a^2-a-28=0$

$14a-56-28=0$

$a=6,\ b=2$

$ab=6(2)=\boxed{12}$

21. One of the roots of the equation $x^3 - 14x^2 + 26x + c = 0$ is the quotient of the other two roots. Compute the product of all possible values of c.

(A) 456 (B) 529 (C) 576 (D) 625 **[(E) 676]**

roots: p, q, and $r \rightarrow p, q$, and p/q

$p+q+r=14 \qquad p+q+p/q=14 \rightarrow pq+q^2+p=14q \rightarrow q^2+p=14q-pq$

$pq+qr+pr=26 \qquad pq+q(p/q)+p(p/q)=26$

$pqr=-c \qquad pq^2+pq+p^2=26q \rightarrow p(q^2+q+p)=26q$

$p(q^2+p+q)=26q$

$p(14q-pq+q)=26q$

$p(15q-pq)=26q$

$15pq-p^2q=26q$

$15p-p^2=26$

$p^2-15p+26=0$

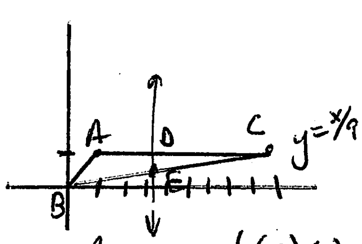
$(p-13)(p-2)$

$p=2, 13 \qquad c=-4, -169$

$(-4)(-169) = \boxed{676}$

22. A vertical line divides the triangle with vertices (0, 0), (1, 1), (9, 1) into two regions of equal area. The equation of the line is x = a. Compute the value of a.

(A) 2 (B) 2.5 **[(C) 3]** (D) 3.5 (E) 4



$Area_{ABC} = \frac{1}{2}(8)(1) = 4$

point $E(a, \frac{a}{9})$

$\Delta DEC = \frac{1}{2}(1-\frac{a}{9})(9-a) = 2$

$(9-a)^2 = 36$

$a = 15$ or 3

$a < 9$, so $\boxed{a=3}$