Milton Math Team Ciphering Practice #1

A regular polygon has an interior angle of measure 140° and sides of length 10. What is its perimeter?

Interior Angle $= \dfrac{180(n-2)}{n}$

$140 = \dfrac{180(n-2)}{n}$

$140n = 180n - 360$

$-40n = -360$

$n = 9$

Perimeter:

$9 \times 10 = \boxed{90}$

Milton Math Team Ciphering Practice #1

A regular polygon has an interior angle of measure 140° and sides of length 10. What is its perimeter?

Milton Math Team Ciphering Practice #2

Simplify $\dfrac{\log 333 - \log 111 - \log 15 - \log 5}{\log 5}$

$$\frac{\log\left(\frac{333}{111 \cdot 15 \cdot 5}\right)}{\log 5} = \frac{\log \frac{1}{25}}{\log 5} = \frac{\log 5^{-2}}{\log 5}$$

$$= \frac{-2 \cdot \cancel{\log 5}}{\cancel{\log 5}} = \boxed{-2}$$

Milton Math Team Ciphering Practice #2

Simplify $\dfrac{\log 333 - \log 111 - \log 15 - \log 5}{\log 5}$

Milton Math Team Ciphering Practice #3

Find the sum of the roots of $8^{2x+1} = 6(8^x) - 1$.

$$8^{2x} \cdot 8 - 6(8^x) + 1 = 0$$

$$8[8^x]^2 - 6[8^x] + 1 = 0$$

let $y = 8^x$

$$8y^2 - 6y + 1 = 0$$

$$(4y - 1)(2y - 1) = 0$$

$$y = 1/4 \quad , \quad y = 1/2$$

$8^x = 1/4$	$8^x = 1/2$
$2^{3x} = 2^{-2}$	$2^{3x} = 2^{-1}$
$3x = -2$	$3x = -1$
$x = -2/3$	$x = -1/3$

Sum of Roots =

$$-\frac{2}{3} + -1/3 = \boxed{-1}$$

Milton Math Team Ciphering Practice #3

Find the sum of the roots of $8^{2x+1} = 6(8^x) - 1$.

Milton Math Team Ciphering Practice #4

The polynomial with rational coefficients $f(x) = x^4 + bx^3 + cx^2 + dx + e$ has roots $\sqrt{3}$ and $1 - 5i$. Find $f(1)$.

The other roots are $-\sqrt{3}$ and $1+5i$

$$f(x) = (x-\sqrt{3})(x+\sqrt{3})(x-(1-5i))(x-(1+5i))$$

$$f(x) = (x^2-3)\left[(x-1+5i)(x-1-5i)\right]$$

$$f(1) = (1-3)\left[(1-1+5i)(1-1-5i)\right]$$

$$= (-2)(5i)(-5i)$$

$$= (-2)(-25i^2) = \boxed{-50}$$

Milton Math Team Ciphering Practice #4

The polynomial with rational coefficients $f(x) = x^4 + bx^3 + cx^2 + dx + e$ has roots $\sqrt{3}$ and $1 - 5i$. Find $f(1)$.

Milton Math Team Ciphering Practice #5

Find all values of x for which $x^2 \sin x - \pi^2 \sin x - 2x^2 + 2\pi^2 > 0$.

$$x^2 \sin x - \pi^2 \sin x - 2x^2 + 2\pi^2 = 0$$

$$\sin x(x^2 - \pi^2) - 2(x^2 - \pi^2) = 0$$

$$(\sin x - 2)(x^2 - \pi^2) = 0$$

$\sin x - 2 = 0$	$x^2 - \pi^2 = 0$
$\sin x = 2$	$x^2 = \pi^2$
no solution	$x = \pm\sqrt{\pi^2} = \pm \pi$

$-$ $\quad$ $+$ $\quad$ $-$

$-\pi$ $\quad$ π

* Test intervals

* $\sin x - 2$ always negative

$$\boxed{-\pi < x < \pi}$$

Milton Math Team Ciphering Practice #5

Find all values of x for which $x^2 \sin x - \pi^2 \sin x - 2x^2 + 2\pi^2 > 0$.

Milton Math Team Ciphering Practice #6

There are three distinct integers $x < y < z$, each greater than 2005, such that their mean, median, and range are equal. Find the smallest possible value of z.

median is y: $\frac{x+z}{2} = y$

range is y: $z - x = y$

$$x + z = 2y$$
$$-x + z = y$$
$$2z = 3y$$
$$z = \frac{3}{2}y$$

$$x + z = 2y$$
$$x + \frac{3}{2}y = 2y$$
$$x = \frac{1}{2}y$$
$$\underline{2x = y} \rightarrow z - x = y$$
$$z - x = 2x$$
$$\underline{z = 3x}$$

$$x < y < z$$
$$x < 2x < 3x$$

the smallest value for x is $\underline{2006}$

So the relationship must be

$$2006 < 2(2006) < 3(2006)$$
$$x < y < z$$

$$\boxed{z = 6018}$$

Milton Math Team Ciphering Practice #6

There are three distinct integers $x < y < z$, each greater than 2005, such that their mean, median, and range are equal. Find the smallest possible value of z.

Milton Math Team Ciphering Practice #7

If $f(x) = x^2 + 2x + 2$ and $g(f(x)) = 2x^2 + 4x + 1$, then find $g(7)$.

*set $f(x) = 7$

$x^2 + 2x + 2 = 7$

$x^2 + 2x - 5 = 0$

$\frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$

$= \frac{-2 \pm \sqrt{24}}{2}$

$= \frac{-2 \pm 2\sqrt{6}}{2}$

$= -1 \pm \sqrt{6}$

$g(-1+\sqrt{6}) = 2(-1+\sqrt{6})^2 + 4(-1+\sqrt{6}) + 1$

$= 2(1 - 2\sqrt{6} + 6) - 4 + 4\sqrt{6} + 1$

$= 2 \underline{- 4\sqrt{6}} + 12 - 4 + \underline{4\sqrt{6}} + 1$

$= \boxed{11}$

Milton Math Team Ciphering Practice #7

If $f(x) = x^2 + 2x + 2$ and $g(f(x)) = 2x^2 + 4x + 1$, then find $g(7)$.

Milton Math Team Ciphering Practice #8

What is the sum of all the distinct factors of 3599?

$$3599 = 3600 - 1$$

Difference of squares
$a^2 - b^2 = (a-b)(a+b)$

$$= 60^2 - 1^2$$

$$= (60-1)(60+1)$$

$$= (59)(61)$$

Distinct Factors are: 1, 59, 61, 3599

$\boxed{\text{sum is } 3720}$

Milton Math Team Ciphering Practice #8

What is the sum of all the distinct factors of 3599?

Milton Math Team Ciphering Practice #9

Convert the hexadecimal number $AD14_{16}$ to base 8.

*convert hexadecimal to binary first

$$\begin{array}{cccc} (10) & (13) & & \\ A & D & 1 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1010 & 1101 & 0001 & 0100 \end{array}$$

* convert from binary to base 8 (group 3 binary digits at a time)

1010 1101 0001 0100

$$\begin{array}{cccccc} 1 & 010 & 110 & 100 & 010 & 100 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 6 & 4 & 2 & 4 \end{array}$$

$\boxed{126424}$

Milton Math Team Ciphering Practice #9

Convert the hexadecimal number $AD14_{16}$ to base 8.

Milton Math Team Ciphering Practice #10

Let k be a real number such that $x^2 + x\sqrt{20} - k = 0$. If k is chosen from the interval $[-7, -2]$, then what is the probability that the roots of the equation are real?

$1x^2 + \sqrt{20}x - k = 0$

Discriminant: $b^2 - 4ac \geq 0$

$(\sqrt{20})^2 - 4(1)(-k) \geq 0$

$20 + 4k = 0$

$k = -5$

$- \;|\; +$

-5

$k \geq -5$

$$\text{probability} = \frac{\text{favorable outcome}}{\text{possible outcome}} \rightarrow \frac{[-5,-2]}{[-7,-2]} = \boxed{\frac{3}{5}}$$

Milton Math Team Ciphering Practice #10

Let k be a real number such that $x^2 + x\sqrt{20} - k = 0$. If k is chosen from the interval $[-7, -2]$, then what is the probability that the roots of the equation are real?