Spring Break Extra Credit MC (Non-Calculators) **Part 1 (30 Questions, 60 mins)**

Key

(Show work on test. Make corrections in different color ink (or use highlighter)

1) **Find the value of** $\lim_{x \to 2^-} \frac{2x}{x-2}$

(A) $-\infty$
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) ∞

$\boxed{A}$

*plug in Real number first (ignore one-sided limit)

$\lim_{x \to 2^-} \frac{2x}{x-2} \to \frac{2(2)}{2-2} = \frac{4}{0} \to$ Limit DNE (VA at $x=2$) $+\infty$, $-\infty$

1.9 $\frac{2(1.9)}{1.9-2} = \frac{+}{-} = \boxed{-\infty}$

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2) **Evaluate** $\lim_{x \to -1} \frac{x^2 - 5x - 6}{x^2 - 1}$.

(A) 1
(B) 3
(C) $\frac{7}{2}$
(D) 12
(E) indeterminate

$\boxed{C}$

$\frac{(-1)^2 - 5(-1) - 6}{(-1)^2 - 1} = \frac{0}{0}$ $\lim_{x \to -1} \frac{(x-6)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$\lim_{x \to -1} \frac{x-6}{x-1} = \frac{-7}{-2} = \boxed{\frac{7}{2}}$

---

3) $\lim_{x \to 2} \frac{2-x}{x^2-4} =$

(A) $-\frac{1}{2}$
(B) $-\frac{1}{4}$
(C) $\frac{1}{4}$
(D) $\frac{1}{2}$
(E) does not exist

$\boxed{B}$

$\frac{2-2}{2^2-4} = \frac{0}{0}$

$\lim_{x \to 2} \frac{2-x}{(x-2)(x+2)} = \lim_{x \to 2} \frac{-1\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$\lim_{x \to 2} \frac{-1}{x+2} = \boxed{\frac{-1}{4}}$

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4) $f(x) = 2x^3 - 6x^2 + 6x - 1$ **has a point of inflection located at**

(A) $(0, -1)$
(B) $(1, 1)$
(C) $(2, 3)$
(D) $(1, 0)$
(E) $(-1, 1)$

$\boxed{B}$

*steps:

a) find $f''(x)$
b) set $f''(x)=0$, find critical pts.
c) create sign line for $f''(x)$
d) check if critical point has a sign change

$f''(x)$: ∪ | a | ∪ | b | ∩

not a POI → a; b ← Is a POI

$f'(x) = 6x^2 - 12x + 6$
$f''(x) = 12x - 12$
$0 = 12(x-1)$
$x = 1$

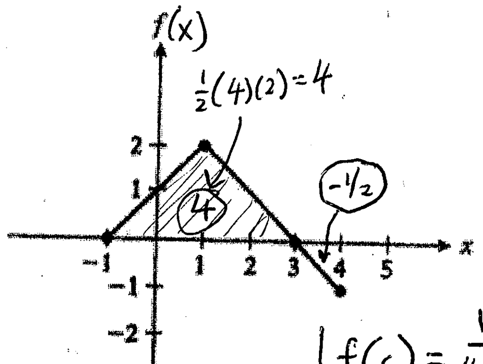
$f''(x)$: ∩ − | 1 | ∪ +

POI at $x=1$ b/c $f''(x)$ change signs

$f(1) = 2(1)^3 - 6(1)^2 + 6(1) - 1$
$f(1) = 1$

5) **Find the average value of $f(x)$ on the interval $[-1, 4]$ in the figure shown.**

$\boxed{B}$



(A) $-\frac{1}{5}$

$\boxed{\text{(B)}}$ $\frac{7}{10}$

(C) $\frac{9}{10}$

(D) $\frac{7}{2}$

(E) $\frac{35}{2}$

* Avg value theorem:

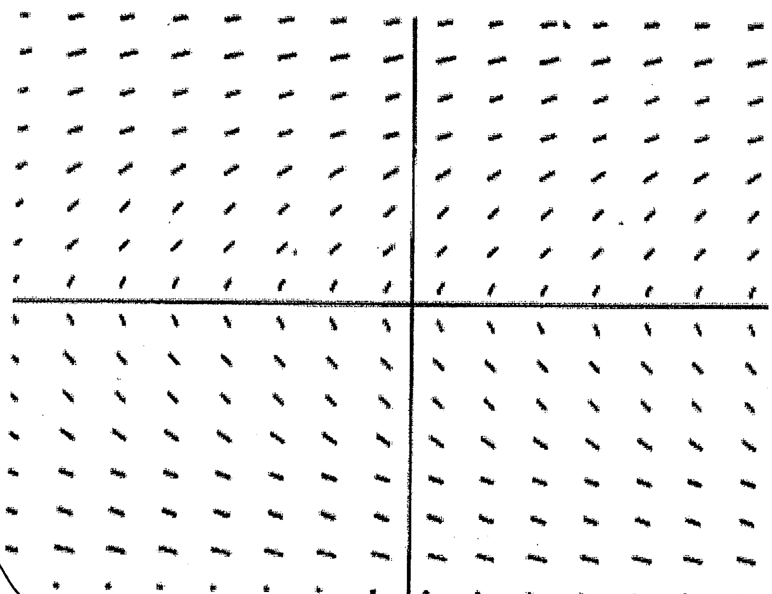
$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f(c) = \frac{1}{4-(-1)}\int_{-1}^{4} f(x)\,dx$$

$$f(c) = \frac{1}{5}\left[4 - \frac{1}{2}\right] = \frac{1}{5}\left(\frac{7}{2}\right) = \boxed{\frac{7}{10}}$$

6) **Which equation has the slope field shown below?**

$\boxed{A}$



If true, slope at (-1,1) should be negative

$\boxed{\text{(A)}}$ $\frac{dy}{dx} = \frac{5}{y}$ ~~(B)~~ $\frac{dy}{dx} = \frac{5}{x}$ ~~(C)~~ $\frac{dy}{dx} = \frac{x}{y}$ If true, (-1,1) should show positive slope

~~(D)~~ $\frac{dy}{dx} = 5y$ ~~(E)~~ $\frac{dy}{dx} = x + y$

* If true, slope when y=5 should be steeper than at y=1

If true, (-1,1) should show slope=0

7) An object moving in a straight line has velocity given by the equation $v(t) = 4t + e^{t-2}$. At time $t = 2$ the object's position, $y(t)$, is given by $y(2) = 3$. The function, $y(t)$, describing the object's position for any time $t > 0$ is

(A) $y(t) = 9t - 15$

(B) $y(t) = 2t^2 + e^{t-2} + 9$

(C) $y(t) = 9t + 9$

(D) $y(t) = 2t^2 + e^{t-2} - 6$

(E) $y(t) = 9t - 6$

D

$y(t) = \int v(t)\,dt$ $\quad y(2) = 3$

$y(t) = \int 4t + e^{t-2}\,dt$ $\quad \int e^u du = e^u + c$

$y(t) = \frac{4t^2}{2} + e^{t-2} + c$

$y(t) = 2t^2 + e^{t-2} + c \leftarrow y(2) = 3$

$3 = 2(2)^2 + e^{2-2} + c$

$3 = 8 + e^0 + c$

$3 = 8 + 1 + c$

$-6 = c$

$\boxed{y(t) = 2t^2 + e^{t-2} - 6}$

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8) Consider a continuous function f with the properties that f is concave up on the interval $[-1, 3]$ and concave down on the interval $[3, 5]$. Which of the following statements is true?

✓ (A) $f''(2) > 0$ and $f''(4) < 0$.

✗ (B) $f''(2) < 0$ and $f''(4) > 0$.

✗ (C) $f''(3) > 0$ and $x = 3$ is a point of inflection of f.

(D) Both (A) and (C)

(E) Both (B) and (C)

A

$f''(x)$ $\quad \cup \; + \quad \cap \; -$

$-1 \quad 3 \quad 5$

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9) A spherical balloon is being filled with water so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm? The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

(A) $\frac{1}{100\pi}$ cm/s

(B) $\frac{1}{25\pi}$ cm/s

(C) $\frac{1}{50\pi}$ cm/s

(D) $\frac{1}{75\pi}$ cm/s

(E) There is not enough information to determine the answer.

B

$V = \frac{4}{3}\pi r^3$ $\quad$ * Related Rates

$\frac{dV}{dt} = +100 cm^3/sec$

$\frac{dr}{dt} = \_\_\_$ $\quad d = 50 \; (r = 25)$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

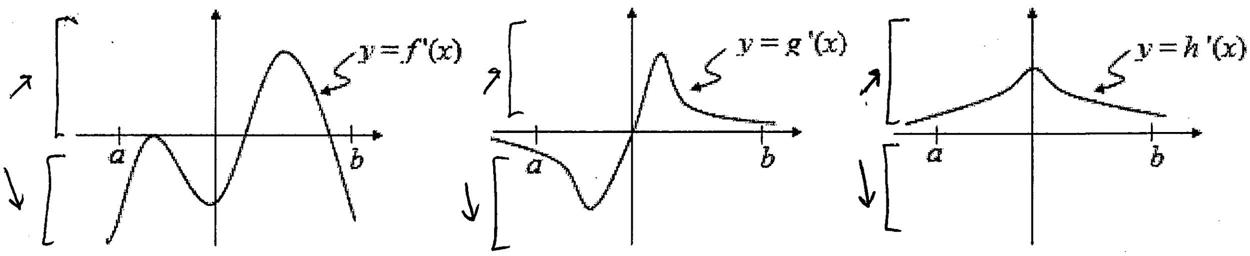
$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$100 = 4\pi(25)^2 \cdot \frac{dr}{dt}$

$\frac{\cancel{100}}{\cancel{4}\pi \cdot \cancel{25} \cdot 25} =$

$\boxed{\frac{1}{25\pi} cm/s = \frac{dr}{dt}}$

10)



The graphs of the derivatives of functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

$\boxed{A}$

$\boxed{a)}$ f only   b) g only   c) h only   d) f and g only   e) f, g, and h

$f'(x)$: ↘ − | ↘ − | ↗ + | ↘ − $g'(x)$: ↘ − | ↗ + $h'(x)$: ↗

Rel. max

11)

__________ Find the derivative of $f(x) = \dfrac{3x^2}{x-7}$

a) $\dfrac{42x-3x^2}{(x-7)^2}$ b) $\dfrac{9x^2-42x}{(x-7)^2}$ c) $6x$ $\boxed{d)}$ $\dfrac{3x(x-14)}{(x-7)^2}$ e) none of these

$\boxed{D}$

* quotient rule $\dfrac{f'g - fg'}{g^2}$

$$\frac{6x(x-7)-3x^2(1)}{(x-7)^2}$$

$$f'(x) = \frac{6x^2-42x-3x^2}{(x-7)^2}$$

$$f'(x) = \frac{3x^2-42x}{(x-7)^2} = \boxed{\frac{3x(x-14)}{(x-7)^2}}$$

12)

__________ Find $\dfrac{dy}{dx}$ for the following: $y^2-3xy+7x=2$

a) $\dfrac{dy}{dx}=\dfrac{3y-5}{2y-3x}$ b) $\dfrac{dy}{dx}=\dfrac{3y-7}{2y-3}$ c) $\dfrac{dy}{dx}=\dfrac{3y-5}{2y-3}$ $\boxed{d)}$ $\dfrac{dy}{dx}=\dfrac{3y-7}{2y-3x}$

e) none of these

$\boxed{D}$

*implicit differentiation
*product rule

$$y^2-3xy+7x=2$$

$$2y\left(\frac{dy}{dx}\right) + \overbrace{(-3)}^{f'}\cdot\overbrace{y}^{g} + \overbrace{(-3x)}^{f}\cdot\overbrace{1\frac{dy}{dx}}^{g'} + 7 = 0$$

$$2y\left(\frac{dy}{dx}\right) - 3y - 3x\left(\frac{dy}{dx}\right) + 7 = 0$$

$$\frac{dy}{dx}(2y-3x) = 3y-7$$

$$\boxed{\frac{dy}{dx} = \frac{3y-7}{2y-3x}}$$

13) An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

a) $y = -6x - 6$ $\boxed{\text{b)}}$ $y = -3x + 1$ c) $y = 2x + 10$ d) $y = 3x - 1$ e) $y = 4x + 1$

$\boxed{B}$

* find $f''(x)=0$, determine POI
* find point
* find slope from $f'(x)$
* tangent line equation

$y - y_1 = m(x - x_1)$

$y' = 3x^2 + 6x$

$y'' = 6x + 6$

$0 = 6(x+1)$

$x = -1$ (POI)

$\cap$ $\cup$

− +

−1

$y(-1) = (-1)^3 + 3(-1)^2 + 2 = -1 + 3 + 2 = 4$

$y'(-1) = 3(-1)^2 + 6(-1) = 3 - 6 = -3$

point: $(-1, 4)$

slope: $m = -3$

$y - 4 = -3(x+1)$

$y - 4 = -3x - 3$

$\boxed{y = -3x + 1}$

14) **If $y = \cos^2 x - \sin^2 x$, then $y' =$**

(A) −1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

$\boxed{\text{(E)}}$ $-4(\cos x)(\sin x)$

$\boxed{E}$

* chain rule:

$y = [\cos x]^2 - [\sin x]^2$

$y' = 2[\cos x](-\sin x) - 2[\sin x](\cos x)$

$y' = -2\sin x\cos x - 2\sin x\cos x$

$\boxed{y' = -4\sin x\cos x}$

15) $\int \frac{x-2}{x-1}\,dx =$

(A) $-\ln|x-1| + C$

(B) $x + \ln|x-1| + C$

$\boxed{\text{(C)}}$ $x - \ln|x-1| + C$

(D) $x + \sqrt{x-1} + C$

(E) $x - \sqrt{x-1} + C$

$\boxed{C}$

* Long Division OR * Synthetic Division

$$\begin{array}{r} 1 - \frac{1}{x-1} \\ x-1 \overline{)\, x - 2} \\ \ominus x \oplus 1 \\ \hline -1 \end{array}$$

$$\begin{array}{c|cc} 1 & 1 & -2 \\ & \downarrow & 1 \\ \hline & 1 & -1 \end{array}$$

$\int 1 - \frac{1}{x-1}\,dx$

$\int 1 - \frac{1}{x-1}\,dx$

$\boxed{x - \ln|x-1| + C}$

16) If $\int_2^4 f(x)\,dx = 6$, then $\int_2^4 (f(x)+3)\,dx =$

$\boxed{D}$

(A) 3

(B) 6

(C) 9

(D) 12

(E) 15

$\int_2^4 [f(x)+3]\,dx$

$= \int_2^4 f(x)\,dx + \int_2^4 3\,dx$

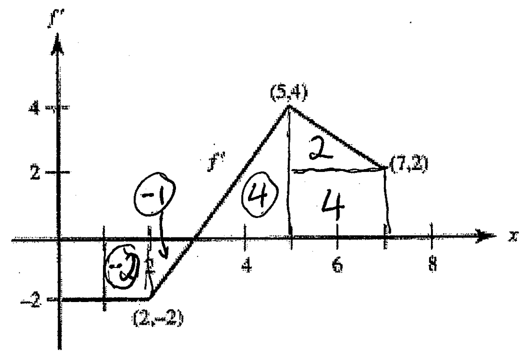
$\int_2^4 f(x)\,dx + \int_2^4 3\,dx$

$3x\Big]_2^4 = 3(4)-3(2) = 6$

$6 + 6$

$= \boxed{12}$

17) The graph of f' is shown below. If $f(7) = 3$ then $f(1) =$



$\int_1^7 f'(x)\,dx = -3 + 10 = 7$

$\boxed{B}$

(A) −10 (B) −4 (C) −3 (D) 10 (E) 16

* final position = initial position + displacement

$x(b) = x(a) + \int_a^b v(t)\,dt$

$f(1) = f(7) + \int_7^1 f'(x)\,dx$

$f(1) = f(7) - \int_1^7 f'(x)\,dx$

$f(1) = 3 - (7)$

$\boxed{f(1) = -4}$

18) Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5\,dx$ is equal to which of the following?

*u-sub

$\boxed{C}$

(A) $2\int_{-2}^{13} u^5\,du$

(B) $\int_{-2}^{13} u^5\,du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5\,du$

(D) $\int_{-1}^{4} u^5\,du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5\,du$

$u = x^2 - 3$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

*convert bounds

If $x=-1$, $u=x^2-3$, $u=(-1)^2-3=-2$

If $x=4$, $u=x^2-3$, $u=4^2-3=13$

$\int_{-2}^{13} \cancel{x} \cdot u^5 \cdot \frac{du}{2\cancel{x}} = \boxed{\frac{1}{2}\int_{-2}^{13} u^5\,du}$

19)

$\overbrace{\ \ }^{1}\overbrace{\ \ }^{2}\overbrace{\ \ }^{3}\overbrace{\ \ }^{5}$

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

$\boxed{D}$

The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50

* trapezoid Area

$A = \frac{w}{2}[h_1 + h_2]$

$$\int_2^{13} f(x)\,dx \approx \frac{1}{2}[6+-2] + \frac{2}{2}[-2+-1] + \frac{3}{2}[-1+3] + \frac{5}{2}[3+9]$$

$$= \frac{1}{2}(4) + 1(-3) + \frac{3}{2}(2) + \frac{5}{2}(12) = 2-3+3+30 = \boxed{32}$$

20)

The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

$\boxed{E}$

* Steps

1) find $y''(x)$, find critical points

2) create $y''(x)$ sign line

$y(x) = 3x^4 - 16x^3 + 24x^2 + 48$

$y'(x) = 12x^3 - 48x^2 + 48x$

$y''(x) = 36x^2 - 96x + 48$

$0 = 12(3x^2 - 8x + 4)$

$0 = 12(3x-2)(x-2)$

$x = 2/3,\ x = 2$

∪ ∩ ∪

\+ − +

0 2/3 1 2 3

$f(x)$ concave down in interval $(\frac{2}{3}, 2)$ b/c $y''(x) < 0$

21)

$\overbrace{\ \ }^{2}\overbrace{\ \ }^{2}\overbrace{\ \ }^{2}$

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

$\boxed{E}$

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

* $v(b) = v(a) + \int_a^b a(t)\,dt$

$v(6) = v(0) + \int_0^6 a(t)\,dt$

11

$\int_0^6 a(t)\,dt \approx 2(5) + 2(2) + 2(8) = 30$

$v(6) = 11 + 30 = \boxed{41 \text{ ft/s}}$

22)

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

B

(A) -4 $\boxed{\text{(B)}}$ -2 (C) 0 (D) 2 (E) 4

*piecewise function is continuous (share same y-value, set equations equal) and differentiable (shares same slope, set derivatives equal)

at $x=2$
$cx+d = x^2 - cx$
$2c + d = 2^2 - 2c$
$d = 4 - 4c$

at $x=2$
$c = 2x - c$
$2c = 4$
$\boxed{c=2}$

$d = 4 - 4c$
$d = 4 - 4(2)$
$\boxed{d = -4}$
$c + d = 2 - 4 = \boxed{-2}$

23)

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f ?

✓ I. $\lim_{x \to 3} f(x)$ exists.

✓ II. f is continuous at $x = 3$.

X III. f is differentiable at $x = 3$.

$\lim_{x\to 3^-} x + 2 = 5$ $\lim_{x \to 3^+} 4x - 7 = 5$ $\lim_{x\to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

Limit exists ✓

$$f'(x) = \begin{cases} 1 & , x \le 3 \\ 4 & , x \ge 3 \end{cases}$$

since the slopes do not match at $x=3$, not a smooth curve at $x=3$, not differentiable

D

(A) None

(B) I only

(C) II only

$\boxed{\text{(D)}}$ I and II only

(E) I, II, and III

24)

Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

E

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

$\boxed{\text{(E)}}$ $y = -\sqrt{\frac{2x^3}{3} - 14}$

*separate variables

$y\,dy = x^2 dx$

$\int y\,dy = \int x^2 dx$

$\frac{y^2}{2} = \frac{x^3}{3} + c$ (3,-2)

$\frac{(-2)^2}{2} = \frac{3^3}{3} + c$

$2 = 9 + c$ $\underline{-7 = c}$

$\frac{y^2}{2} = \frac{x^3}{3} - 7$

$2\left(\frac{y^2}{2} = \frac{x^3}{3} - 7\right)$

$y^2 = \frac{2}{3}x^3 - 14$

$y = \pm\sqrt{\frac{2}{3}x^3 - 14}$

$\boxed{y = -\sqrt{\frac{2}{3}x^3 - 14}}$

choose this b/c ordered pair is (3,-2) *y-value is negative

25)

E

$$\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ [✓ (E) $2x\sin(x^6)$]

*SFTC

$$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f[p(x)]\cdot p'(x)$$

$$\frac{d}{dx}\int_0^{x^2} \sin(t^3)\,dt = \sin(x^2)^3 \cdot 2x$$

$$= \boxed{2x\sin(x^6)}$$

26)

E

If $y = (x^3+1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3+1)$ (C) $2(3x^2+1)$ (D) $3x^2(x^3+1)$ [✓ (E) $6x^2(x^3+1)$]

* chain rule

$$\frac{d}{dx}f[g(x)] = f'[g(x)]\cdot g'(x)$$

$y=(x^3+1)^2$

outside : $(\ )^2$

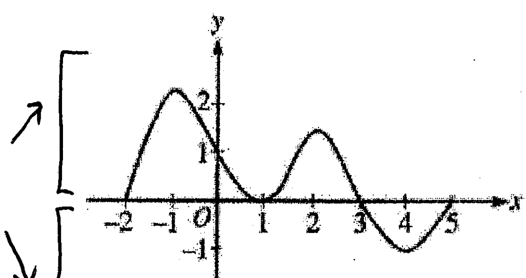
inside : x^3+1

$y' = 2(\ )^1 \cdot 3x^2$

$y' = 2(x^3+1)\cdot 3x^2$

$$\boxed{y' = 6x^2(x^3+1)}$$

27)



Graph of f'

B

The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

[✓ (B) $[-2, 3]$]

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

$f'(x)$ + + −

−2 1 3 5

$f(x)$ increasing in interval

$(-2, 1), (1, 3)$

or

$(-2, 3)$

28)

If $\frac{dy}{dx} = (1+\ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$

IBP (Integration by parts)

$\boxed{E}$

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1+\ln x$

(C) $\ln x$

(D) $e^{2x+x\ln x-2}$

(E) $e^{x\ln x}$

*separation of variables

$\frac{dy}{dx} = (1+\ln x)y$

$dy = (1+\ln x)\cdot y \cdot dx$

$\int \frac{dy}{y} = \int (1+\ln x)\,dx$

$\ln|y| = \int 1\,dx + \int \ln x\,dx$

$u = \ln x \quad \int dv = \int 1\,dx$

$du = \frac{1}{x}dx \quad v = x$

$= uv - \int v\,du$

$= x\ln x - \int x \cdot \frac{1}{x}dx$

$= x\ln x - x + C$

$\ln|y| = x + x\ln x - x + C$

$\ln|y| = x\ln x + C$

$e^{\ln|y|} = e^{x\ln x} \cdot e^{C}$

$|y| = Ce^{x\ln x}$ ← point (1,1)

$1 = Ce^{1\ln(1)}$

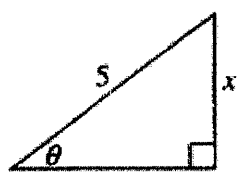
$1 = C$

$\boxed{y = e^{x\ln x}}$

29)

$\boxed{E}$

find $\frac{dx}{dt}$ when $x=3$



$\frac{d\theta}{dt} = 3$ rad/min

In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

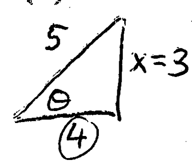
*Trig Related Rates

$\sin\theta = \frac{x}{5} = \frac{1}{5}x$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dx}{dt}\right)$

$\cos\theta(3) = \frac{1}{5}\left(\frac{dx}{dt}\right)$

$5\cdot 3\cos\theta = \frac{dx}{dt}$



$\cos\theta = \frac{4}{5}$

$\frac{dx}{dt} = 15\cos\theta$

$\frac{dx}{dt} = 15\left(\frac{4}{5}\right) = \boxed{12}$

30)

$\boxed{B}$

$\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

expand

$\int_1^e (x^2+1)(x^{-1})\,dx$

$\int x + \frac{1}{x}\,dx$

$\left.\frac{x^2}{2} + \ln|x|\right]_1^e$

$\frac{e^2}{2} + \ln|e| - \left(\frac{1^2}{2} + \ln|1|\right)$

$\frac{e^2}{2} + 1 - \frac{1}{2} + 0 = \frac{e^2}{2} + \frac{1}{2} = \boxed{\frac{e^2+1}{2}}$

Spring Break Extra Credit MC (Calculator Active) **Part 2(15 Questions, 45 mins)**

(Show work on test. Make corrections in different color ink (or use highlighter)

Calculator section

1)

Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

$\boxed{C}$

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

* parallel tangent lines when slopes equal
* set derivatives equal $(f'(x)=g'(x))$, solve for x

$f'(x)=3e^{2x}\cdot 2=6e^{2x}$
$g'(x)=18x^2$

$6e^{2x}=18x^2$

$6e^{2x}-18x^2=0$

$\boxed{x\approx -0.391}$

2)

The base of a solid is the region in the first quadrant enclosed by the graph of $y=2-x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

$\boxed{B}$

(A) $\pi\int_0^2(2-y)^2\,dy$

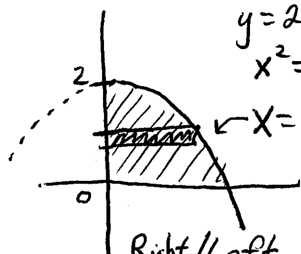
(B) $\int_0^2(2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}}\left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}}\left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}}\left(2-x^2\right)dx$

$y=2-x^2$
$x^2=2-y \rightarrow x=\pm\sqrt{2-y}$



$\leftarrow x=\sqrt{2-y}$

Right/Left

base $=\sqrt{2-y}-0$

Area (square) $=[\text{base}]^2$

Area $=\left[\sqrt{2-y}\right]^2$

Volume $=\int_{y_1}^{y_2}[\text{Area}]\,dy$

$V=\int_0^2\left[\sqrt{2-y}\right]^2 dy$

$\boxed{V=\int_0^2(2-y)\,dy}$

3)

Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

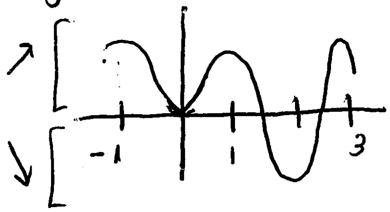
(D) $1.772 \le x \le 2.507$

(E) $2.802 \le x \le 3$

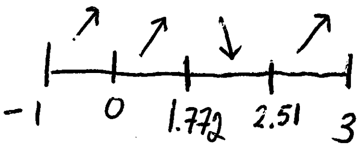
$\boxed{D}$

$g'(x) = \frac{d}{dx}\int_0^x \sin(t^2)\,dt$

$g'(x) = \sin(x^2)$



$g'(x)$



g(x) decreasing in interval (1.772, 2.51) b/c $g'(x) < 0$

4)

The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what intervals is f increasing?

(A) $1 \le x \le 1.445$ only

(B) $1 \le x \le 1.691$

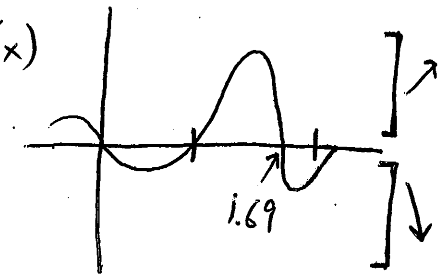
(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

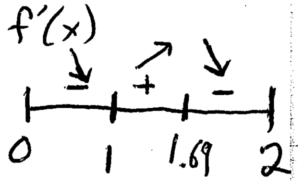
(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

$\boxed{B}$

$f'(x)$



$f'(x)$



f(x) is increasing in interval (1, 1.69) b/c $f'(x) > 0$

5)

x	0	1	2	3
$f''(x)$	5	0	-7	4

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

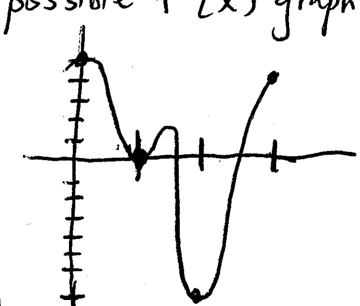
(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

$\boxed{E}$

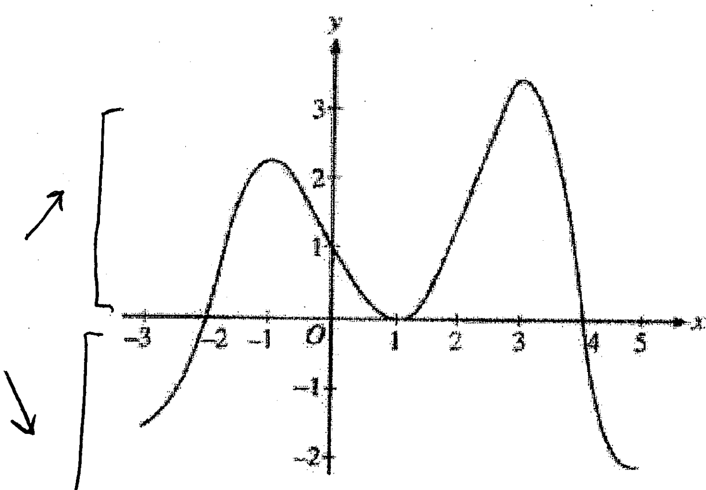
*possible $f''(x)$ graph:



* POI not necessarily at x=1, x-intercept at x=1 doesn't guarantee graph will cross x-axis at that point.

6)

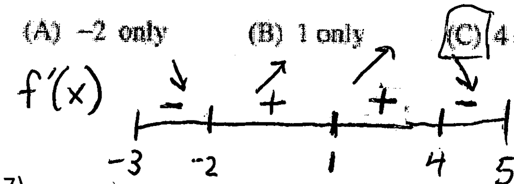
$\boxed{C}$



Graph of f'

The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

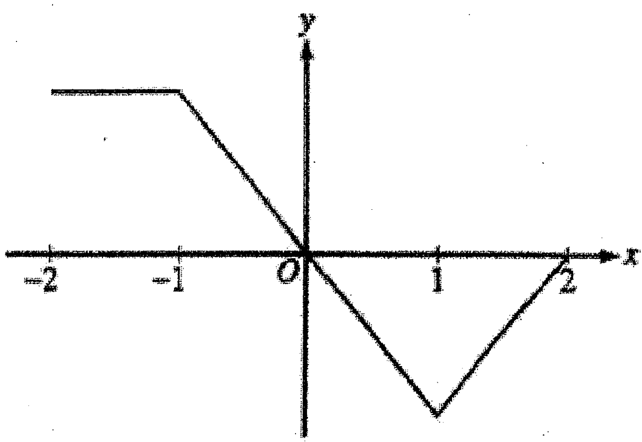
(A) -2 only (B) 1 only (C) 4 only (D) -1 and 3 only (E) -2, 1, and 4



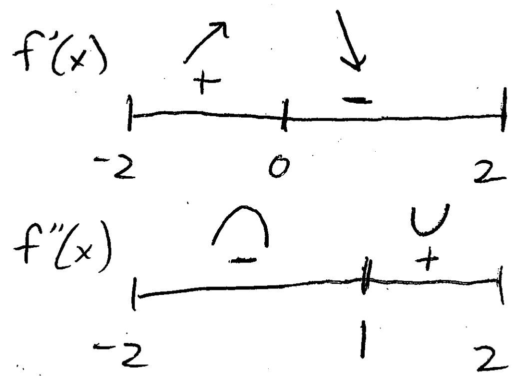
Rel. max at $x=4$ since f' changes from + to −

7)

$\boxed{B}$



Graph of f'



The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$. ✗

(B) f is increasing for $-2 \le x \le 0$. ✓

(C) f is increasing for $1 \le x \le 2$. ✗

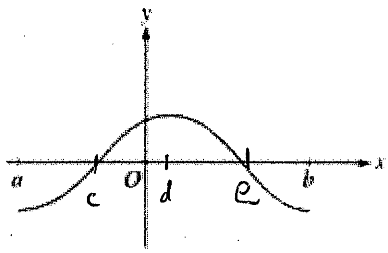
(D) f has a local minimum at $x = 0$. ✗

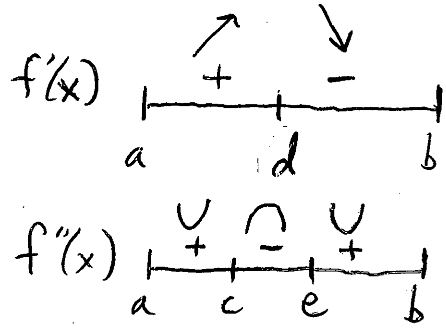
(E) f is not differentiable at $x = -1$ and $x = 1$. ✗

*$f'(x)$ is not differentiable but $f(x)$ is differentiable

8)

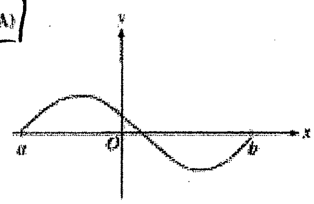
$\boxed{A}$



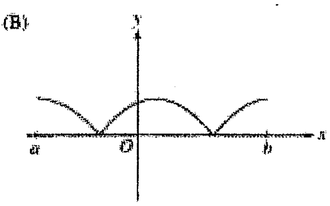


The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

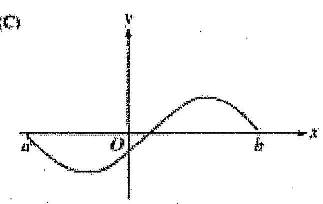
(A)



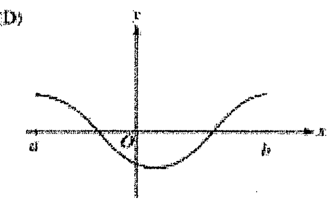
(B)



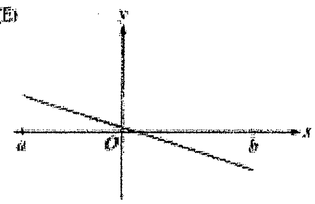
(C)

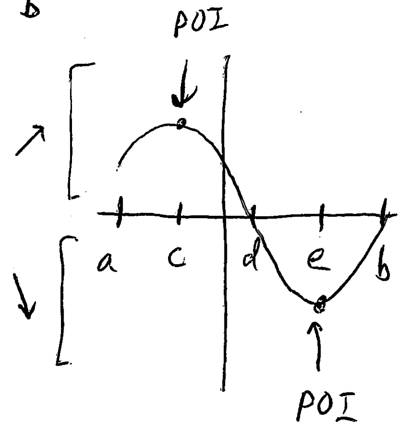


(D)



(E)





$\boxed{A}$

9)

* derivative of inverse at a point

$f(a)=b$	$g(b)=a$
$f'(a)=n$	$g'(b)=\frac{1}{n}$

Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

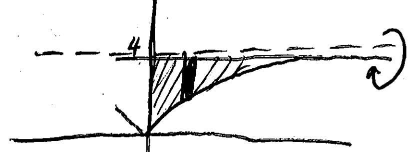
$f(\ )=3$	$g(3)=$___
$f'(\ )=$_	$g'(3)=$___

$f(6)=3$	$g(3)=6$
$f'(6)=-2$	$g'(3)=\boxed{\frac{-1}{2}}$

(E) The value of $g'(3)$ cannot be determined from the information given.

10)

The region enclosed by the graphs of $y = x^{2/3}$, $y = 4$, and the y-axis is rotated about the line $y = 4$. The volume of the solid generated can be represented by the integral



$\boxed{B}$

(A) $2\pi \int_0^8 (4 - x^{2/3})^2\, dx$

(B) $\pi \int_0^8 (4 - x^{2/3})^2\, dx$

(C) $2\pi \int_0^4 (4 - x^{2/3})^2\, dx$

(D) $\pi \int_0^4 (16 - x^{4/3})\, dx$

(E) $\pi \int_0^8 (16 - x^{4/3})\, dx$

* Disc Method

Top/Bottom

$y = \underline{x^{2/3}}$

$y = \underline{4}$

$R(x) = 4 - x^{2/3}$

$V = \pi \int_{x_1}^{x_2} [R(x)]^2\, dx$

*bounds

$x^{2/3} = 4$

$(x^{2/3})^{3/2} = (4)^{3/2}$

$x = 2^3 = 8$

$$\boxed{V = \pi \int_0^8 [4 - x^{2/3}]^2\, dx}$$

11)

A solid is formed that has the region R as its base and cross sections perpendicular to the x-axis that are squares. Find the value of k so that the volume of the solid on the interval $[0, k]$ is half the total volume of the solid.

$\boxed{B}$

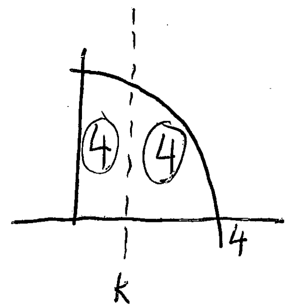
(A) 0.568

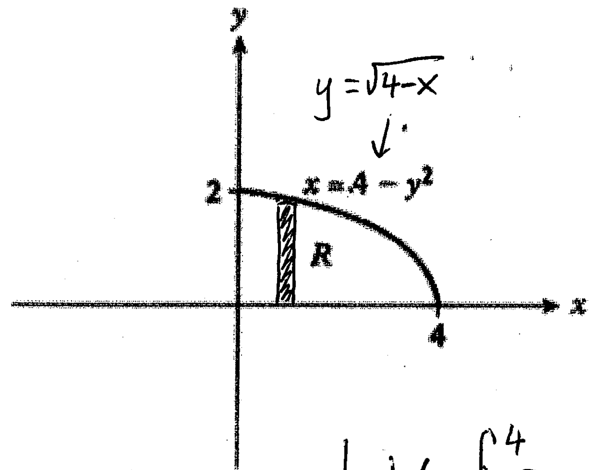
(B) 1.172

(C) 2.201

(D) 3.2

(E) 3.567





Top/Bottom:

$base = \sqrt{4-x^2} - 0$

$Area = [base]^2$

$Area = [\sqrt{4-x^2}]^2$

$V = \int_0^4 [\sqrt{4-x}]^2\, dx$

$V = \int_0^4 4 - x\, dx = 8$

$\int_0^k [\sqrt{4-x}]^2\, dx = \frac{8}{2}$

$\int_0^k 4 - x\, dx = 4$

$4x - \frac{x^2}{2}\Big]_0^k = 4k - \frac{k^2}{2} = 4$

$-2\left(4k - \frac{k^2}{2} - 4 = 0\right)$

$-8k + k^2 + 8 = 0$

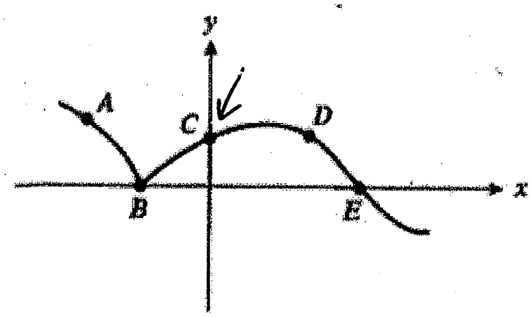
$k^2 - 8k + 8 = 0$

$$\boxed{k \approx 1.172}$$

12)

In the graph shown, at which point is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

point has positive slope and part of concave down graph.



$\boxed{C}$

(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

---

13)

If $f'(x) = \sqrt{1+x^3}$ and $f(1) = 0.5$, then $f(4) =$

$\boxed{D}$

(A) 7.562 (B) 8.062 (C) 12.871 (D) 13.371 (E) 17.871

* final position = initial position + displacement

$x(b) = x(a) + \int_a^b x'(t)\,dt$

$f(b) = f(a) + \int_a^b f'(x)\,dx$

$f(4) = f(1) + \int_1^4 f'(x)\,dx$

$f(4) = 0.5 + \int_1^4 \sqrt{1+x^3}\,dx$

$= 0.5 + 12.8714$

$\boxed{f(4) = 13.371}$

14)

If the region enclosed by the y-axis and the graph of $x = 4 - y^2$ is revolved about the y-axis, the volume of the solid generated is

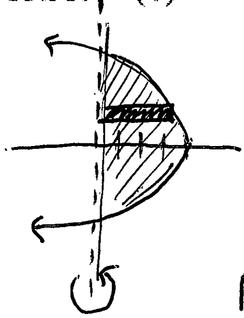
$\boxed{D}$

(A) 25.133 (B) 33.510 (C) 53.617 (D) 107.233 (E) 214.466

$x = 4 - y^2$

$y^2 = 4 - x$

$y = \pm\sqrt{4-x}$



* Disc Method

Right/Left

$x = \underline{4-y^2}$

$x = \underline{0}$

$R(y) = 4 - y^2 - (0)$

$\text{Volume} = \pi \int_{y_1}^{y_2} [R(y)]^2\,dy$

* bounds:

set $4 - y^2 = 0$

$4 = y^2$

$y = \pm 2$

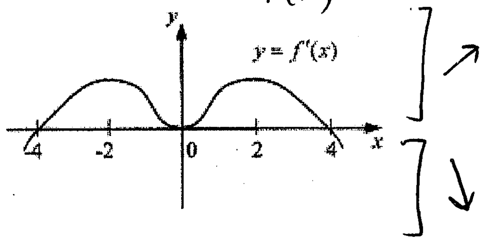
$V = \pi \int_{-2}^{2} [4 - y^2]^2\,dy$

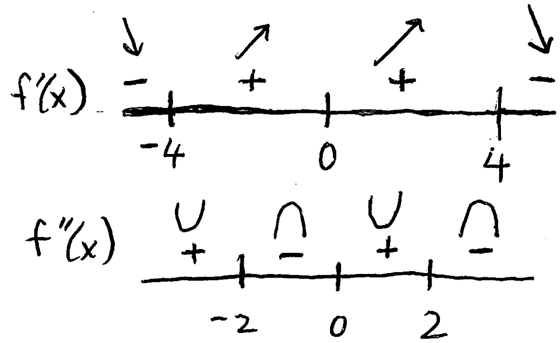
$V = 34.133\pi \approx \boxed{107.233}$

15)

$\boxed{B}$

$f'(x)$





characteristics of $f(x)$ graph:

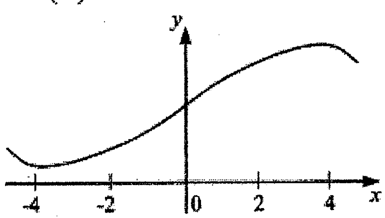
i) Rel. min at $x=-4$

ii) Rel. max at $x=4$

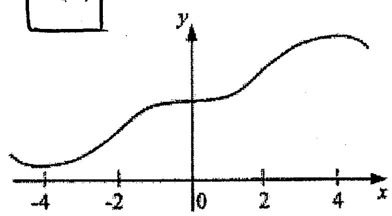
iii) POI at $x=-2, 0, 2$

The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

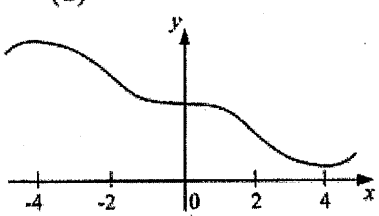
(A)



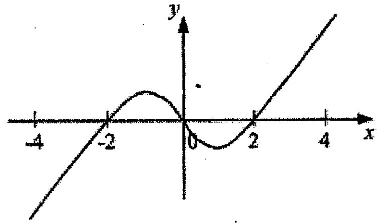
$\boxed{\text{(B)}}$



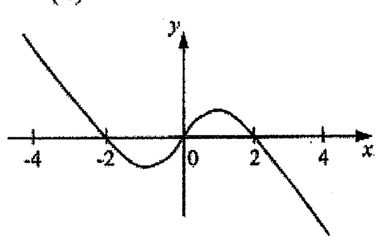
(C)



(D)



(E)



16)

$\boxed{E}$

At how many points on the interval $[0,\pi]$ does $f(x) = 2\sin x + \sin 4x$ satisfy the Mean Value Theorem?

(A) none (B) 1 (C) 2 (D) 3 $\boxed{\text{(E)}}$ 4

$f(\pi) = 2\sin(\pi) + \sin 4(\pi) = 0$

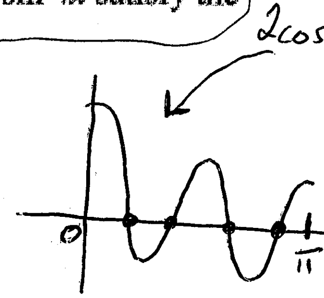
$f(0) = 2\sin(0) + \sin(0) = 0$

$\frac{f(b)-f(a)}{b-a} = \frac{f(\pi)-f(0)}{\pi - 0} = \frac{0-0}{\pi} = 0$

set $f'(x) = \frac{f(\pi)-f(0)}{\pi-0} = 0$

$f'(x) = 2\cos x + 4\cos(4x)$

$2\cos x + 4\cos(4x) = 0$



$f(x)$ continuous $[0,\pi]$ ✓

$f(x)$ differentiable $(0,\pi)$ ✓

$f'(x) = 2\cos x + \cos(4x)\cdot 4$

MVT:

i) $f(x)$ continuous $[a,b]$

ii) $f(x)$ differentiable on (a,b)

iii) set $f'(c) = \frac{f(b)-f(a)}{b-a}$

set derivative function equal to slope between endpoints.