1. Solve: $x! = \frac{(5!)!}{5!}$.

State Individual Ciphering 2016

#1.

Name:_____

School:_____

Answer:

5 points 11 points

2. Suppose f and g are inverse functions, and $g(x) = x^3 - x$. Compute A so that $f(A) = 7$.

#2.

Name:_____

School:_____

Answer:

5 points 11 points

State Individual Ciphering 2016

3. Let k and m be real numbers. The lines $y = kx + 1$ and $y = mx + 11$ intersect at the point $(5, 7)$. Compute the area of the region enclosed by the two lines and the y-axis.

#3.

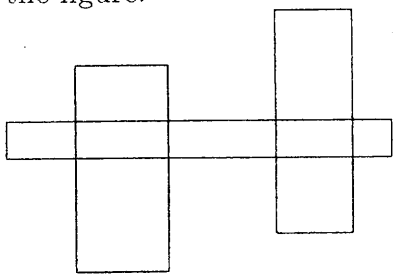
Name:_____

School:_____

Answer:

5 points 11 points

4. Three rectangles overlap, as shown below. Compute the number of rectangles of all sizes which can be found in the figure.



#4.

Name:_____

School:_____

Answer:

5 points 11 points

5. Consider the function

$$f(x) = \frac{2x^2 + x - 7}{x - 5}.$$

The oblique asymptote is $y = mx + b$ and the vertical asymptote is $x = c$. Compute $m + b - c$.

#5.

Name:_____

School:_____

Answer:

5 points 11 points

6. Convert the hexadecimal number 2016_{16} into base-10.

#6.

Name:_____

School:_____

Answer:

5 points 11 points

7. A regular hexadecagon (16-sided polygon) with a perimeter of 2016 is inscribed by a circle of radius r and is circumscribed by a circle of radius R. Compute the positive difference of the areas of the two circles.

State Individual Ciphering 2016

#7.

Name:_____

School:_____

Answer:

5 points 11 points

8. How many unique, nontrivial right triangles with integral side lengths have a leg of length 10?

#8.

School:_____

Name:_____

Answer:

5 points 11 points

9. Let k be an integer which satisfies the equation $\log_2(k-3)-\log_2(k+3) = -2$.

Compute $\sum_{n=1}^{k} n^3$.

#9.

Name:_____

School:_____

Answer:

5 points 11 points

10. Given $\sqrt{5+2\sqrt{6}} = \sqrt{x}+\sqrt{y}$, compute $|x-y|$.

#10.

School:_____

Name:_____

Answer:

5 points 11 points