GCTM State Math Tournament 2016 — TEAM ROUND

Please write your answers on the answer sheet.

1. Today is the 40th annual State Mathematics Tournament. Like today's event, the first State Mathematics Tournament also took place in Macon (at Northeast High School). More interestingly, like today's event, the first State Mathematics Tournament also took place on Saturday, April 30 (but, of course, in 1977, not 2016). Between 1977 and 2016 compute the sum of the years in which April 30 fell on a Saturday. (Do *not* include 1977 and 2016 in your sum.)

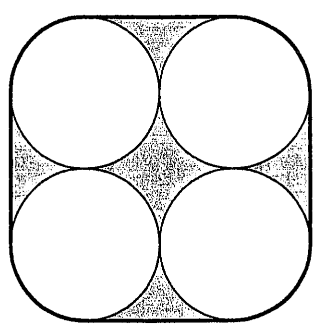
2. Compute the area enclosed by the curves $y - 2 = -\frac{1}{3}(x+1)$ and $y = |x-1|$.

3. The line $y = x + m$ and the parabola $y = x^2 + n$ have one and only one point in common whenever m and n have the relationship $4m - 4n = k$. Compute k.

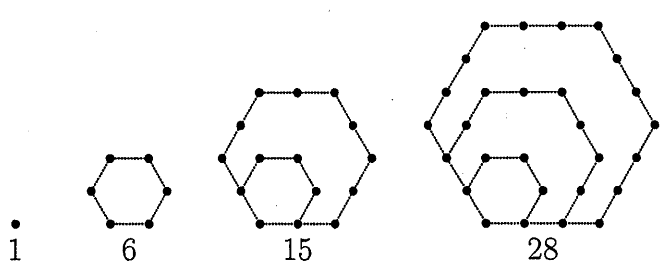
4. Given the function $f(n) = (n+1)!(n-1)!$, where n is a positive integer, compute the smallest value of N such that $f(N)$ is divisible by 2187.

5. Angela is in a room with three identical doors. She has equal probability of opening each of the doors. Behind the first door is a tiger. If she opens this door, there is a $\frac{4}{5}$ probability that the tiger will be hungry and eat her. Behind the second door is a giant snake. If she opens this door, there is a $\frac{2}{3}$ probability of being strangled by the snake. Behind the third door, there is a chocolate cake! If Angela opens this door, there is a 100% probability that she will eat the chocolate cake and survive. Given that Angela survives, the probability that she ate the cake is the reduced fraction $\frac{m}{n}$. Compute $m + n$.

6. The diagram below shows four tangent unit circles with a rubber band stretched around them. Compute the area of the shaded region.

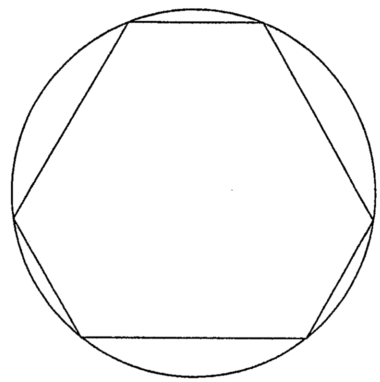


7. The first four hexagonal numbers are 1, 6, 15, and 28. Compute the 20th hexagonal number.



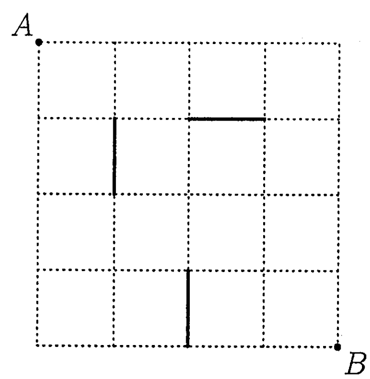
Problems continue on the next page.

8. A hexagon with alternating side lengths of 3 and 5 is inscribed in a circle as shown. The radius of the circle can be written as the reduced fraction $\frac{m\sqrt{n}}{p}$, where n is not divisible by the square of any prime. Compute $m+n+p$.



9. Completely simplify $\sqrt{15+6\sqrt{6}}+\sqrt{15-6\sqrt{6}}$.

10. The following diagram represents a network of roads. Suppose one starts at point A, and randomly chooses to travel either down or to the right at each intersection until arriving at point B. However, there are three roads which must be avoided, shown in solid lines. The probability the three roads drawn with solid lines below are avoided can be written as the reduced fraction $\frac{p}{q}$, where p and q are integers. Compute $p+q$.



11. A digit x is called *twitchy* in base b if $x \neq 1$ and x divides an integer m whenever x divides the sum of the digits of m written in base b. As an example, both 3 and 9 are twitchy in base 10. What is the sum, in base 10, of all twitchy digits in base 7?

12. A game of Craps is played in the following way: A pair of dice is rolled. If the roll results in a 7 or 11, the player wins. If the roll results in a 2, 3, or 12, the player loses. If any other roll results, the value of that roll is recorded as the *point* and then the dice are rolled repeatedly until either a 7 or the *point* value is rolled. If the *point* value is rolled before a 7 is rolled, the player wins. If a 7 is rolled before the *point* value was rolled, the player loses. If the probability of winning this game is the reduced fraction $\frac{p}{q}$, compute $p+q$.