

**Test For
FINAL EXAM
Chapters P-5**

Name _____ Date _____
Class _____ Section _____

1. Find the limit: $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - 9 - \sqrt{x - 9}}{\Delta x}$

- (a) $\frac{1}{\sqrt{x - 9}}$ (b) $\sqrt{x - 9}$ (c) $\frac{1}{2\sqrt{x - 9}}$
 (d) 0 (e) None of these

2. Find the limit: $\lim_{x \rightarrow 4^+} \frac{x^2 - x}{(x - 4)^2}$

- (a) 0 (b) $+\infty$ (c) $-\infty$
 (d) 4 (e) None of these

3. Find an equation of the tangent line to the graph of $f(x) = \frac{1}{x - 1}$ at the point (2, 1).

- (a) $x + y + 3 = 0$ (b) $x - y = 1$ (c) $y - 1 = -\frac{(x - 2)}{(x - 1)^2}$
 (d) $x + y = 6$ (e) None of these

4. Find $f'(x)$: $f(x) = \frac{x^2 - 4x}{\sqrt{x}}$

- (a) $\frac{3x^{3/2} - 4}{2x^{1/2}}$ (b) $\frac{2x - 4}{\sqrt{x}}$ (c) $\frac{2x - 4}{1/(2\sqrt{x})}$
 (d) $x^{3/2} - 4x^{1/2}$ (e) None of these

5. The position function for a particular object is $s = -12t^2 + 51t + 38$. Which statement is true?

- (a) The velocity at time $t = 1$ is 27. (b) The velocity is a constant.
 (c) The initial position is 51. (d) The initial velocity is -24 .
 (e) None of these

6. Find the derivative: $s(t) = \sec \sqrt{t}$.

- (a) $\tan^2 \sqrt{t}$ (b) $\frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$ (c) $\sec \frac{1}{2\sqrt{t}} \tan \frac{1}{2\sqrt{t}}$
 (d) $\sec \sqrt{t} \tan \sqrt{t}$ (e) None of these

7. Find $\frac{dy}{dx}$ for $5x^2 - 2xy + 7y^2 = 0$.

- (a) $\frac{5x + 7y}{x}$ (b) $\frac{y - 5x}{7y}$ (c) $10x - 2y + 14y$
 (d) $\frac{y - 5x}{7y - x}$ (e) None of these

8. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 2.5 inches? [Hint: $V = \pi r^2 h$]

- (a) 20.48 in/sec (b) $-.8192$ in/sec (c) -16.38 in/sec
 (d) .8192 in/sec (e) None of these

9. Determine whether the Mean Value Theorem applies to $f(x) = -\frac{1}{x}$ on the interval $[-3, -\frac{1}{2}]$. If the Mean Value

Theorem applies, find all values of c in the interval such that $f(c) = \frac{f(-1/2) - f(-3)}{-1/2 - (-3)}$. If the Mean Value Theorem does not apply, state why.

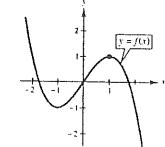
- (a) Mean Value Theorem applies; $c = -\sqrt{\frac{2}{3}}$.
 (b) Mean Value Theorem applies; $c = \pm \sqrt{\frac{2}{3}}$.
 (c) The Mean Value Theorem does not apply because f is not continuous at $x = 0$.
 (d) The Mean Value Theorem does not apply because $f'(-\frac{1}{2}) \neq f(-3)$.
 (e) None of these

10. Given that $f(x) = -x^2 + 12x - 34$ has a relative maximum at $x = 6$, choose the correct statement.

- (a) f' is positive on the interval $(6, \infty)$. (b) f' is positive on the interval $(-\infty, \infty)$.
 (c) f' is negative on the interval $(6, \infty)$. (d) f' is negative on the interval $(-\infty, 6)$.
 (e) None of these

11. Give the sign of the second derivative of f at the indicated point.

- (a) Positive
 (b) Negative
 (c) Zero
 (d) The sign cannot be determined.
 (e) None of these



12. Find the horizontal asymptotes (if any) for $f(x) = \frac{ax^3}{b + cx + dx^2}$.

- (a) $y = \frac{a}{b}$ (b) $y = 0$ (c) $y = \frac{a}{d}$
 (d) There are no horizontal asymptotes. (e) None of these

13. Find the derivative: $g(x) = \text{arcsec } \frac{x}{2}$.

- (a) $\frac{1}{\sqrt{4 - x^2}}$ (b) $\frac{1}{\sqrt{x^2 - 4}}$ (c) $\frac{2}{x\sqrt{x^2 - 4}}$
 (d) $\frac{4}{x\sqrt{x^2 - 4}}$ (e) None of these

Review Test Form A Answer Key

$$\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+4x-9} - \sqrt{x-9}}{\Delta x} \text{ This is the same as } \frac{d}{dx} \sqrt{x-9} = \frac{1}{2}(x-9)^{-\frac{1}{2}} \quad \boxed{C}$$

$$\begin{aligned} \text{OR } \lim_{\Delta x \rightarrow 0} \frac{\sqrt{(x+4x-9)} - \sqrt{x-9}}{\Delta x}, \frac{\sqrt{(x+4x-9)} + \sqrt{x-9}}{\sqrt{x+4x-9} + \sqrt{x-9}} &= \lim_{\Delta x \rightarrow 0} \frac{x+4x-9-(x-9)}{\Delta x(\sqrt{x+4x-9} + \sqrt{x-9})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x+4x-9} + \sqrt{x-9})} = \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+4x-9} + \sqrt{x-9}} = \frac{1}{2\sqrt{x-9}} \end{aligned}$$

1) $\lim_{x \rightarrow 4^+} \frac{x^2-x}{(x-4)^2}$ Since there is a V.A. at $x=4$, check values from the right like 4.001. This yields $\frac{+}{+} = +$, so $\lim_{x \rightarrow 4^+} \frac{x^2-x}{(x-4)^2} = \infty$. b

$$f(x) = \frac{1}{x-1} / f'(x) = -(x-1)^{-2} = \frac{-1}{(x-1)^2} / f'(2) = -1 / y-1 = -(x-2) \quad \boxed{e}$$

$$f(x) = \frac{x^2-4x}{\sqrt{x}} = x^{\frac{3}{2}} - 4x^{\frac{1}{2}} \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = \frac{3\sqrt{x}}{2} - \frac{2}{\sqrt{x}} = \frac{3x-4}{2\sqrt{x}} \quad \boxed{e}$$

$$\begin{aligned} s &= -12t^2 + 51t + 38 \\ v(t) &= -24t + 51 \quad \boxed{a} \\ v(1) &= 27 \end{aligned}$$

$$s(t) = \sec \sqrt{t} \quad s'(t) = \sec \sqrt{t} \tan \sqrt{t} \cdot \frac{1}{2}t^{-\frac{1}{2}} = \frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}} \quad \boxed{b}$$

$$\begin{aligned} 5x^2 - 2xy + 7y^2 &= 0 \\ 10x - 2y - 2x \frac{dy}{dx} + 14y \frac{dy}{dx} &= 0 \quad \Rightarrow \frac{dy}{dx}(14y - 2x) = 2y - 10x \\ \frac{dy}{dx} &= \frac{2y - 10x}{14y - 2x} = \frac{y - 5x}{7y - x} \quad \boxed{d} \end{aligned}$$

$$\begin{aligned} \text{B) } \frac{dr}{dt} &= -.05 & 128\pi &= \pi r^2 h \\ V &= 128\pi r & 0 &= 2\pi r \frac{dr}{dt} \cdot h + \pi r^2 \frac{dh}{dt} \\ &\text{constant} & 2\pi(2.5)(-.05)(20.48) + \pi(2.5)^2 \frac{dh}{dt} &= 0 \\ && -5.12\pi + 6.25\pi \frac{dh}{dt} &= 0 \\ && \frac{dh}{dt} &= \frac{5.12\pi}{6.25\pi} = .8192 \frac{\text{in}}{\text{sec}} \quad \boxed{d} \end{aligned}$$

$$9) f(x) = -\frac{1}{x} \text{ on } [-3, -\frac{1}{2}] \text{ contr diff} \checkmark$$

$$\text{slope of secant} = \frac{f(-\frac{1}{2}) - f(-3)}{-\frac{1}{2} - (-3)} = \frac{\frac{2}{3} - \frac{1}{3}}{\frac{5}{2}} = \frac{\frac{1}{3}}{\frac{5}{2}} = \frac{2}{15}$$

$$f'(x) = x^{-2} = \frac{1}{x^2} \quad \frac{1}{x^2} = \frac{2}{3} \quad x^2 = \frac{3}{2} \quad x = \pm \sqrt{\frac{3}{2}} \quad c = -\sqrt{\frac{3}{2}}$$

$$10) f(x) = -x^2 + 12x - 34, \text{ rel max @ } x=6 \quad \boxed{c}$$

11) concave down so b

$$12) f(x) = \frac{ax^3}{b+cx+dx^2} \quad \text{no H.A. b/c degree of numerator is greater than degree of denominator} \quad \boxed{d}$$

$$13) g(x) = \arccos \frac{x}{2} \quad y = \arccos \frac{x}{2} \quad \sec y = \frac{x}{2}$$

$$\begin{aligned} \sec y \tan y \frac{dy}{dx} &= \frac{1}{2} \\ \frac{dy}{dx} &= \frac{1}{2 \sec y \tan y} = \frac{1}{2(\frac{x}{2})(\frac{\sqrt{x^2-4}}{2})} = \frac{1}{\frac{1}{2}|x|\sqrt{x^2-4}} \\ &= \frac{2}{|x|\sqrt{x^2-4}} \end{aligned}$$

E