2017

Trig Curve Sketch

Ch. 3.1 p. 167 #15, 33

15) Find critical numbers: $h(x) = \sin^2 x + \cos x \quad 0 < x < 2\pi$

$h(x) = (\sin x)^2 + \cos x$

$h'(x) = 2(\sin x)'\cos x + (-\sin x)$

$0 = \sin x(2\cos x - 1)$

$\sin x = 0$	$2\cos x - 1 = 0$
$\boxed{x = \phi, \pi}$	$\cos x = 1/2$
	$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

33) $f(x) = \sin x \quad \left[\frac{5\pi}{6}, \frac{11\pi}{6}\right]$ Find Absolute Extrema on closed interval

*EVT, test critical points and endpoints.

$f'(x) = \cos x$

$0 = \cos x$

$x = \cancel{\pi/2}, 3\pi/2$

$f\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

$f\left(\frac{3\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

$f\left(\frac{11\pi}{6}\right) = \sin\left(\frac{11\pi}{6}\right) = -1/2$

$$\boxed{\begin{array}{l}\text{Max is } 1/2 \text{ at } x = \frac{5\pi}{6} \\ \text{Min is } -1 \text{ at } x = 3\pi/2\end{array}}$$

3.2 (Trig) p.174-175 #19, 21, 25, 45, 48

Rolle's Theorem

i) continuous on $[a,b]$
ii) differentiable on (a,b)
iii) $f(a)=f(b)$

$*$set $f'(c)=0$

$x=c$ in (a,b)

19) Apply Rolle's Theorem: $f(x)=\sin 3x$ $[0, \pi/3]$

$f'(x)=\cos(3x)\cdot 3$

$f(0)=0$
$f(\pi/3)=\sin\pi=0$ ✓

$f(x)$ continuous on $[0,\pi/3]$, differentiable on $(0,\pi/3)$

$3\cos(3x)=0$
$\cos(3x)=0$
$3x=\cos^{-1}(0)$
$3x=\frac{\pi}{2}, 3\pi/2$
$x=\frac{\pi}{6}, \cancel{\frac{3\pi}{6}}$

$\boxed{c=\pi/6 \text{ in interval } [0,\pi/3]}$

21) $f(x)=\tan x$ $[0,\pi]$

$~~f'(x)=\sec^2 x~~$
$~~0=\sec^2 x~~$
$~~x=~~$

$f(0)=\tan 0=0$
$f(\pi)=\tan\pi=0$

$f(x)$ not continuous on $[0,\pi]$, not differentiable on $(0,\pi)$ since $\tan\pi/2=$ DNE

25) Determine if Rolle's Theorem applies $f(x)=x-\tan\pi x$ $[-1/4, 1/4]$

$~~f'(x)=~~$

$f(x)$ continuous on $[-1/4, 1/4]$
$f(x)$ differentiable on $(-1/4, 1/4)$
$f(-1/4)=-1/4-\tan\pi/4=-1/4-1=-5/4$
$f(1/4)=1/4-\tan(\pi/4)=1/4-1=-3/4$ $\neq$

Rolle's theorem does not apply since $f(-1/4)\neq f(1/4)$

45) Apply MVT

$f(x)=\sin x$ on $[0,\pi]$

$f(x)$ continuous on $[0,\pi]$, differentiable on $(0,\pi)$

$f(0)=\sin 0=0$
$f(\pi)=\sin\pi=0$

$\frac{f(b)-f(a)}{b-a}=\frac{0-0}{\pi-0}=0$

$f'(x)=\cos x$
$\cos x=0$
$\boxed{x=\pi/2}, \cancel{3\pi/2}$

$\boxed{c=\pi/2 \text{ on } [0,\pi]}$

MVT

i) continuous on $[a,b]$
ii) differentiable on (a,b)
iii) set $f'(x)=\frac{f(b)-f(a)}{b-a}$

3.2 (Trig)

48) MVT: $f(x) = x - 2\sin x \quad [-\pi, \pi]$

$f(x)$ continuous on $[-\pi, \pi]$, differentiable on $(-\pi, \pi)$

$f(-\pi) = -\pi - 2\sin(-\pi) = -\pi$

$f(\pi) = \pi - 2\sin(\pi) = \pi$

$m_{Avg} = \frac{f(b)-f(a)}{b-a} = \frac{f(\pi)-f(-\pi)}{\pi-(-\pi)} = \frac{\pi-(-\pi)}{\pi-(-\pi)} = \frac{2\pi}{2\pi} = 1$

set $f'(x) = m_{Avg}$

$f'(x) = 1 - 2\cos x$

$1 - 2\cos x = 1$

$-2\cos x = 0$

$\cos x = 0$

$x = \pi/2, -\pi/2, 3\pi/2, -3\pi/2$

$\boxed{c = -\pi/2, \pi/2 \text{ on } [-\pi, \pi]}$

b) secant line: (Find equation of line through the endpts)

points: $(-\pi, -\pi)$ and (π, π)

slope: $m = 1$

$y - y_1 = m(x - x_1)$

$\boxed{y - \pi = 1(x - \pi)}$

c) Find equations of tangent lines: (using results from MVT:)

$f(-\pi/2) = -\pi/2 - 2\sin(-\pi/2) = -\pi/2 - 2(-1) = -\frac{\pi}{2} + 2 = \frac{-\pi+4}{2}$

$f(\pi/2) = \pi/2 - 2\sin(\pi/2) = \pi/2 - 2(1) = \frac{\pi}{2} - 2 = \frac{\pi-4}{2}$

tangent line #1

point: $(-\pi/2, \frac{-\pi+4}{2})$ slope: $m = 1 \rightarrow \boxed{y - \frac{-\pi+4}{2} = 1(x + \pi/2) \rightarrow y = x + 2}$

tangent line #2

point: $(\pi/2, \frac{\pi-4}{2})$ slope: $m = 1 \rightarrow \boxed{y - \frac{\pi-4}{2} = 1(x - \pi/2) \rightarrow y = x - 2}$

$\boxed{3.3}$ Trig First Derivative Test p. 183-184 # 13, 41, 43

13) $f(x) = \sin x - 1$ $\quad 0 < x < 2\pi$ Determine interval where function is increasing, decreasing

$f'(x) = \cos x$

$0 = \cos x$

$x = \pi/2, 3\pi/2$

$\nearrow \quad \searrow \quad \nearrow$

$+ \quad - \quad +$

$0 \quad \pi/2 \quad 3\pi/2 \quad 2\pi$

$f'(x) > 0$

Increasing on $(0, \pi/2) \cup (3\pi/2, 2\pi)$

Decreasing on $(\pi/2, 3\pi/2)$ b/c $f'(x) < 0$

41) 1st Derivative Test $f(x) = \frac{x}{2} + \cos x$ on interval $(0, 2\pi)$

$f(x) = \frac{1}{2}x + \cos x$

$f'(x) = \frac{1}{2} - \sin x$

$0 = \frac{1}{2} - \sin x$

$\sin x = 1/2$

$x = \frac{5\pi}{6}, \frac{\pi}{6}$

$\nearrow \quad \searrow \quad \nearrow$

$+ \quad - \quad +$

$0 \quad \pi/6 \quad 5\pi/6 \quad 2\pi$

$f(x)$ increasing $(0, \pi/6) \cup (5\pi/6, 2\pi)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(\pi/6, 5\pi/6)$ b/c $f'(x) < 0$

Rel. max at $\left(\pi/6, \frac{\pi}{12} + \frac{\sqrt{3}}{2}\right)$ b/c $f'(x)$ changes from + to −

Rel. min at $\left(\frac{5\pi}{6}, \frac{5\pi}{12} - \frac{\sqrt{3}}{2}\right)$ b/c $f'(x)$ changes from − to +

43) $f(x) = \sin x + \cos x$ $\quad 0 < x < 2\pi$

$f'(x) = \cos x - \sin x$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \pi/4, 5\pi/4$

$\nearrow \quad \searrow \quad \nearrow$

$+ \quad - \quad +$

$0 \quad \pi/4 \quad 5\pi/4 \quad 2\pi$

Increasing on $(0, \pi/4) \cup (5\pi/4, 2\pi)$ b/c $f'(x) > 0$

Decreasing on $(\pi/4, 5\pi/4)$ b/c $f'(x) < 0$

Rel. max at $(\pi/4, \sqrt{2})$ b/c $f'(x)$ changes from + to −

Rel. min at $(5\pi/4, -\sqrt{2})$ b/c $f'(x)$ changes from − to +

$\boxed{3.4}$ Trig p.192 #25, 29, 41, 42 Concavity, POI

25) $f(x) = \sin\frac{x}{2}$ $[0, 4\pi]$ Find POI, determine concavity

$f'(x) = \cos(\frac{1}{2}x)\cdot\frac{1}{2}$

$f'(x) = \frac{1}{2}\cos(\frac{x}{2})$

$f''(x) = -\frac{1}{2}\sin(\frac{x}{2})\cdot\frac{1}{2}$

$0 = -\frac{1}{4}\sin(\frac{x}{2})$

$\sin(\frac{x}{2}) = 0$

$\frac{x}{2} = \sin^{-1}(0)$

$\frac{1}{2}x = 0, \pi, 2\pi, 4\pi$

$x = 0, 2\pi, 4\pi$

∩ ∪

− +

0 2π 4π

$f(x)$ is concave up $(2\pi, 4\pi]$ b/c $f''(x) > 0$

Concave down $[0, 2\pi)$ b/c $f''(x) < 0$

POI at $(2\pi, 0)$ b/c $f''(x)$ changes sign

29) $f(x) = 2\sin x + \sin 2x$ $[0, 2\pi]$

$f'(x) = 2\cos x + \cos(2x)\cdot 2$

$f''(x) = -2\sin x - 2\sin(2x)\cdot 2$

$0 = -2\sin x - 4\sin(2x)$

$0 = -2\sin x - 4(2\sin x\cos x)$

$0 = -2\sin x(1 + 4\cos x)$

$x = 0, \pi, 1.823, 4.460$

$1 + 4\cos x = 0$

$\cos x = -\frac{1}{4}$

∩ ∪ ∩ ∪

− + − +

0 1.823 π 4.46 2π

Concave up $(1.823, \pi) \cup (4.46, 2\pi)$ b/c $f''(x) > 0$

Concave down $(0, 1.823) \cup (\pi, 4.46)$ b/c $f''(x) < 0$

POI at $(1.823, 1.452)$, $(\pi, 0)$, $(4.46, -1.45)$ b/c $f''(x)$ change signs

41) Use 2nd Derivative Test: * Take critical pt from $f'(x) = 0$, plug into $f''(x)$ to determine if critical pt is max or min

$f(x) = \cos x - x$ $[0, 4\pi]$

$f'(x) = -\sin x - 1$ $\quad -1 = +\sin x$ ~~$x = 0, \pi, 2\pi, 3\pi, 4\pi$~~

$f''(x) = -\cos x - 0$

always decreasing, so no relative extrema exist.

42) 2^{nd} Derivative Test $f(x) = 2\sin x + \cos 2x \quad [0, 2\pi]$

$f'(x) = 2\cos x - \sin 2x \cdot 2$

$f'(x) = 2\cos x - 2\sin 2x$

$= 2\cos x - 2(2\sin x \cos x)$

$= 2\cos x - 4\sin x \cos x$

$f'(x) = 2\cos x(1 - 2\sin x)$

$0 = 2\cos x(1 - 2\sin x)$

$2\cos x = 0$

$x = \pi/2, 3\pi/2$

$1 - 2\sin x = 0$

$\sin x = 1/2$

$x = \pi/6, 5\pi/6$

* plug these critical points into $f''(x)$

$f''(x) = -2\sin x - 2\cos 2x \cdot 2$

$f''(x) = -2\sin x - 4\cos 2x$

$f''(\pi/6) = -3 < 0$ Rel. max at $(\pi/6, 3/2)$

$f''(\pi/2) = 2 > 0$ Rel. min at $(\pi/2, 1)$

$f''(\frac{5\pi}{6}) = -3 < 0$ Rel. max at $(5\pi/6, \frac{3}{2})$

$f''(3\pi/2) = 6 > 0$ Rel. min at $(3\pi/2, -3)$

3.6 (Trig) p.212 #32, 33 Curve Sketching

Find Rel. extrema, POI, asymptotes

32) $y = 2x - \tan x \quad -\pi/2 < x < \pi/2$ V.A. $x = \pi/2, x = -\pi/2$

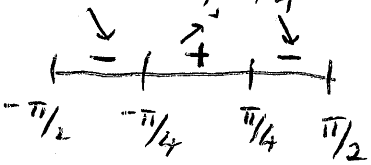
$y' = 2 - \sec^2 x$

$0 = 2 - \sec^2 x$

$\sec^2 x = 2$

$\cos^2 x = \frac{1}{2} \quad x = \pm\frac{\sqrt{2}}{2}$

$x = -\pi/4, \pi/4$



Rel. min $(-\pi/4, 1-\pi/2)$ b/c $f'(x)$ changes from $-$ to $+$

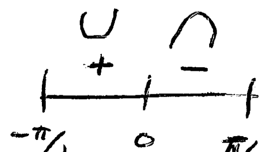
Rel. max at $(\pi/4, \pi/2 - 1)$ b/c $f'(x)$ changes from $+$ to $-$

$y'' = 0 - 2[\sec x]' \sec x \tan x$

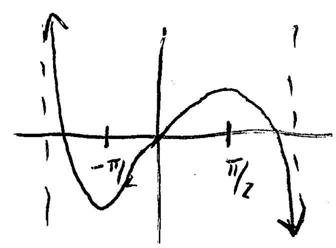
$y'' = -2\sec^2 x \tan x$

$y'' = \frac{-2\sin x}{\cos^3 x} = 0$

$x = 0, \pi$



POI at $(0,0)$ b/c $f''(x)$ changes signs.



33) $y = 2(\csc x + \sec x) \quad 0 < x < \pi/2$ V.A. $x = 0, \pi/2$

$y' = -2\csc x \cot x + 2\sec x \tan x = 0$

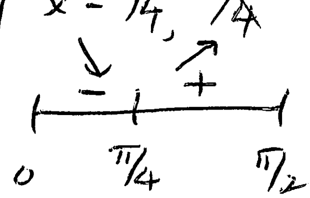
$\cancel{2}\csc x \cot x = \cancel{2}\sec x \tan x$

$\frac{\cos x}{\sin^2 x} = \frac{\sin x}{\cos^2 x}$

$\cos^3 x = \sin^3 x$

$1 = \tan^3 x$

$x = \pi/4, \cancel{5\pi/4}$



Rel. min at $(\pi/4, 4\sqrt{2})$ b/c $f'(x)$ changes from $-$ to $+$.