AP Calculus AB **Trig Day 3: Trig Derivatives** Sections 1-3, 1-4, 3-5

Trig Derivative Rules: $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

Ex. 1: Show that the following are true using the above rules and product/quotient rules:

a) $\frac{d}{dx}\tan x = \sec^2 x$

b) $\frac{d}{dx}\cot x = -\csc^2 x$

c) $\frac{d}{dx}\sec x = \sec x \tan x$

d) $\frac{d}{dx}\csc x = -\csc x \cot x$

Ex 2: if y = sin(5x), find y'

Ex 3: if y = $\sqrt{\cos(x^2)}$, find y'

Ex. 4: if = $\sqrt[3]{\tan(6x^4 + 3x - 2)}$, find y '

Ex. 5: if y = $\sec^2(\pi x)$, find y''

Ex. 6: if sin(x + y) = 3x + 2y, find $\dfrac{dy}{dx}$

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Trig Derivative Rules: $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

Ex. 1: Show that the following are true using the above rules and product/quotient rules:

a) $\frac{d}{dx}\tan x = \sec^2 x$

* use quotient rule

$$\frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\frac{\sin x}{\cos x}$$

$$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

b) $\frac{d}{dx}\cot x = -\csc^2 x$

$$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$$

c) $\frac{d}{dx}\sec x = \sec x \tan x$

$$f(x) = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$f'(x) = -1(\cos x)^{-2}(-\sin x)$$

$$= \frac{+\sin x}{\cos^2 x} = \frac{\sin x}{(\cos x)(\cos x)}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

$$= \boxed{\sec x \tan x}$$

d) $\frac{d}{dx}\csc x = -\csc x \cot x$

$$f(x) = \frac{1}{\sin x} = (\sin x)^{-1}$$

$$f'(x) = -1(\sin x)^{-2}(\cos x)$$

$$= \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x}$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x(\csc x)$$

$$= \boxed{-\csc x \cot x}$$

Ex 2: if y = sin(5x), find y'

* chain rule

$$y' = \cos(5x) \cdot 5$$

$$\boxed{y' = 5\cos(5x)}$$

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$ 2) $\frac{d}{dx}\cos u = -\sin u \cdot u'$

3) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$ 4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$ 6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

Ex 3: if y = $\sqrt{\cos(x^2)}$, find y'

$$y = [\cos(x^2)]^{1/2}$$

$$y' = \frac{1}{2}[\cos(x^2)]^{-1/2} \cdot -\sin(x^2) \cdot 2x$$

$$y' = \frac{-x\sin(x^2)}{[\cos(x^2)]^{1/2}}$$

$$= \boxed{\frac{-x\sin(x^2)}{\sqrt{\cos(x^2)}}}$$

Ex. 4: $y = \sqrt[3]{\tan(6x^4 + 3x - 2)}$, find y '

$$y = [\tan(6x^4+3x-2)]^{1/3}$$

$$y' = \frac{1}{3}[\tan(6x^4+3x-2)]^{-2/3} \sec^2(6x^4+3x-2) \cdot (24x^3+3)$$

$$y' = \frac{(8x^3+1)(\sec^2(6x^4+3x-2))}{[\tan(6x^4+3x-2)]^{2/3}}$$

Ex. 5: if y = $\sec^2(\pi x)$, find ~~y''~~ $\boxed{y'}$

$$y = [\sec(\pi x)]^2$$

$$y' = 2[\sec(\pi x)] \cdot \sec(\pi x)\tan(\pi x) \cdot (\pi)$$

$$\boxed{y' = 2\pi\sec^2(\pi x)\tan(\pi x)}$$

Ex. 6: if sin(x + y) = 3x + 2y, find $\frac{dy}{dx}$

$$\cos(x+y) \cdot \left[1 + \frac{dy}{dx}\right] = 3 + 2\left(\frac{dy}{dx}\right)$$

$$\cos(x+y) + \cos(x+y)\left(\frac{dy}{dx}\right) = 3 + 2\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\cos(x+y) - 2\left(\frac{dy}{dx}\right) = 3 - \cos(x+y)$$

$$\frac{dy}{dx}\left(\cos(x+y) - 2\right) = 3 - \cos(x+y)$$

$$\boxed{\frac{dy}{dx} = \frac{3-\cos(x+y)}{\cos(x+y) - 2}}$$