Key

Trig Identities Quiz Review WS #1

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Verify the Identity

1) $\sin\theta\cot\theta\sec\theta = 1$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = 1$

2) $\frac{\csc^2\theta}{\csc^2\theta} - \frac{\cos^2\theta\csc^2\theta}{\csc^2\theta} = 1$

$\csc^2\theta(1 - \cos^2\theta)$

$\csc^2\theta(\sin^2\theta)$

$\frac{1}{\sin^2\theta} \cdot \sin^2\theta = \boxed{1}$

3) $\tan\theta\csc\theta\cos\theta = 1$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} \cdot \cos\theta = \boxed{1}$

4) $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta) = 1 - 2\cos^2\theta$

$\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta - \cos^2\theta$

$\sin^2\theta - \cos^2\theta$

$\downarrow$

$(1 - \cos^2\theta) - \cos^2\theta$

$1 - \cos^2\theta - \cos^2\theta$

$\boxed{1 - 2\cos^2\theta}$

5) $\frac{\sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

$\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\frac{\sin\theta(1-\cos\theta)}{1+\cos\theta-\cos\theta-\cos^2\theta}$

$\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} \rightarrow \boxed{\frac{1-\cos\theta}{\sin\theta}}$

6) $\frac{\sec\theta-\cos\theta}{\sec\theta} = \sin^2\theta$

$\frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{1}}{\frac{1}{\cos\theta}} \rightarrow \frac{\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$ (multiply by $\cos\theta$)

$\frac{1-\cos^2\theta}{\cos\theta} \rightarrow \frac{\frac{\sin^2\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\frac{\sin^2\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} \rightarrow \boxed{\sin^2\theta}$

7) $\frac{1+\sec^2\theta}{\sec^2\theta} = 1+\cos^2\theta$

$\frac{1}{\sec^2\theta} + \frac{\sec^2\theta}{\sec^2\theta}$

$\boxed{\cos^2\theta + 1}$ ✓

8) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$ (multiply by $(1+\cos x)$ and $(1-\cos x)$)

$\frac{1+\cos x}{(1-\cos x)(1+\cos x)} + \frac{1-\cos x}{(1-\cos x)(1+\cos x)}$

$\frac{1+\cos x+1-\cos x}{1-\cos^2 x} \rightarrow \frac{2}{\sin^2 x}$

$\frac{2}{\frac{1}{\csc^2 x}} \rightarrow 2 \cdot \frac{\csc^2 x}{1} \rightarrow \boxed{2\csc^2 x}$