Key

ıg Identities Quiz Review WS #2

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Verify the Identity

1) $\frac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$

$\frac{\sec^2\theta}{\tan^2\theta} \rightarrow \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$

$\frac{1}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{\sin^2\theta} \rightarrow \frac{1}{\sin^2\theta} \rightarrow \boxed{\csc^2\theta}$

2) $\frac{(1+\cos x)}{\sin x} + \frac{(\sin x)}{1+\cos x} = 2\csc x$ $\quad (1+\cos x)$, $(\sin x)$

$\frac{1+\cos x+\cos x+\cos^2 x}{\sin x(1+\cos x)} + \frac{\sin^2 x}{\sin x(1+\cos x)}$

$\frac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$

$\frac{1+2\cos x+1}{\sin x(1+\cos x)}$

$\frac{2+2\cos x}{\sin x(1+\cos x)}$

$\frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$\frac{2}{\sin x} \rightarrow \frac{2}{\frac{1}{\csc x}}$

$2 \cdot \csc x \rightarrow \boxed{2\csc x}$

3) $(\cot^2 x+1)(\sin^2 x-1) = -\cot^2 x$

$(\csc^2 x)(-\cos^2 x)$

$\frac{1}{\sin^2 x} \cdot \frac{-\cos^2 x}{1} \rightarrow \frac{-\cos^2 x}{\sin^2 x}$

$\rightarrow \boxed{-\cot^2 x}$

4) $\frac{\sin^2\theta - 2\sin\theta + 1}{\sin\theta - 1} \stackrel{?}{=} \sin\theta - 1$

*hint: factor $x^2 - 2x + 1$

$(x-1)(x-1)$

$(\sin\theta - 1)(\sin\theta - 1)$

$\frac{(\sin\theta-1)(\sin\theta-1)}{(\sin\theta-1)} \rightarrow \boxed{\sin\theta - 1}$

5) $\dfrac{\csc x - \cot x}{\sec x - 1} = \cot x$

$\dfrac{\csc x - \cot x}{\sec x - 1} \cdot \dfrac{\sec x + 1}{\sec x + 1}$

$\dfrac{(\csc x - \cot x)(\sec x + 1)}{\sec^2 x - 1}$

$\dfrac{\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x} + 1\right)}{\sec^2 x - 1}$

$\dfrac{\frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{\cos x}}{\tan^2 x}$

$\dfrac{\frac{1-\cos^2 x}{\sin x \cos x}}{\frac{\sin^2 x}{\cos^2 x}}$

$\dfrac{\sin^2 x}{\sin x \cos x} \cdot \dfrac{\cos^2 x}{\sin^2 x}$

$\dfrac{\cos x}{\sin x} \rightarrow \boxed{\cot x}$

6) $\sin^2\theta(1 + \cot^2\theta) = 1$

$\sin^2\theta \cdot \csc^2\theta$

$\sin^2\theta \cdot \dfrac{1}{\sin^2\theta} = \boxed{1}$ ✓

7) $\dfrac{1 + \sec^2\theta}{\sec^2\theta} = 1 + \cos^2\theta$

$\dfrac{1}{\sec^2\theta} + \dfrac{\sec^2\theta}{\sec^2\theta}$

$\boxed{\cos^2\theta + 1}$ ✓

8) $\dfrac{\tan\theta}{\sec\theta} + \dfrac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{1} + \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{1}$

$\boxed{\sin\theta + \cos\theta}$ ✓