g Identities Quiz Review WS #3

Key

eciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Verify the Identity

1) $\sec^2\theta(1-\cos^2\theta) = \tan^2\theta$

$1-\cos^2\theta = \sin^2\theta$

$\sec^2\theta \cdot \sin^2\theta$

$\frac{1}{\cos^2\theta} \cdot \frac{\sin^2\theta}{1} \rightarrow \frac{\sin^2\theta}{\cos^2\theta} \rightarrow \boxed{\tan^2\theta}$

2) $\cot^2\theta\csc^2\theta - \cot^2\theta = \cot^4\theta$

$\cot^2\theta(\csc^2\theta - 1)$

$\cot^2\theta \cdot \cot^2\theta \rightarrow \boxed{\cot^4\theta}$

3) $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

$\frac{\frac{1}{\cos\theta}}{\sin\theta} \rightarrow \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} \rightarrow \frac{1}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cdot\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta\sin\theta} - \frac{\sin^2\theta}{\cos\theta\sin\theta} \rightarrow \frac{\cos^2\theta}{\cos\theta\sin\theta}$

$\frac{\cos\theta}{\sin\theta} \rightarrow \boxed{\cot\theta}$

4) $\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\csc\theta$

$\frac{\sin^2\theta}{(1-\cos\theta)(\sin\theta)} + \frac{1-\cos\theta-\cos\theta+\cos^2\theta}{(1-\cos\theta)(\sin\theta)}$

$\frac{\sin^2\theta+1-\cos\theta-\cos\theta+\cos^2\theta}{(1-\cos\theta)(\sin\theta)} \rightarrow \frac{2-2\cos\theta}{(1-\cos\theta)(\sin\theta)}$

$\frac{2(1-\cos\theta)}{(1-\cos\theta)(\sin\theta)} \rightarrow \frac{2}{\sin\theta} \rightarrow \boxed{2\csc\theta}$

5) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=2\sec\theta$

$(1-\sin\theta)$ $(1+\sin\theta)$

$$\frac{\cos\theta-\cos\theta\sin\theta}{(1+\sin\theta)(1-\sin\theta)}+\frac{\cos\theta+\cos\theta\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{\cos\theta-\cos\theta\sin\theta+\cos\theta+\cos\theta\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{2\cos\theta}{1-\sin^2\theta}\rightarrow\frac{2\cos\theta}{\cos^2\theta}\rightarrow\frac{2}{\cos\theta}$$

$$\rightarrow\frac{2}{\frac{1}{\sec\theta}}\rightarrow 2\cdot\frac{\sec\theta}{1}=\boxed{2\sec\theta}$$

6) $\csc^4\theta-\cot^4\theta=2\cot^2\theta+1$

* difference of squares

$$(\csc^2\theta-\cot^2\theta)(\csc^2\theta+\cot^2\theta)$$

$$(1)\cdot(\csc^2\theta+\cot^2\theta)$$

$$\csc^2\theta+\cot^2\theta$$

$$1+\cot^2\theta+\cot^2\theta$$

$$\boxed{1+2\cot^2\theta}$$

7) $\dfrac{\csc^2\theta+2\csc\theta-3}{\csc^2\theta-1}=\dfrac{\csc\theta+3}{\csc\theta+1}$

think x^2+2x-3

$(x+3)(x-1)$

$(\csc\theta+3)(\csc\theta-1)$

$$\frac{(\csc\theta+3)(\csc\theta-1)}{(\csc\theta-1)(\csc\theta+1)}\rightarrow\boxed{\frac{\csc\theta+3}{\csc\theta+1}}$$

8) $\dfrac{1+\cos x}{\sin x}=\csc x+\cot x$

$$\frac{1}{\sin x}+\frac{\cos x}{\sin x}$$

$$\boxed{\csc x+\cot x}$$