Ch. 4 Trig Integrals Notes and WS

Trig Integral Rules

1) $\int \sin u \, du = -\cos u + C$	2) $\int \cos u \, du = \sin u + C$
3) $\int sec^2 u \, du = \tan u + C$	4) $\int csc^2 u \, du = -\cot u + C$
5) $\int \sec u \tan u \, du = \sec u + C$	6) $\int \csc u \cot u \, du = -\csc u + C$

Find the Indefinite Integral below

25. $\int (5 \cos x + 4 \sin x) \, dx$

26. $\int (t^2 - \cos t) \, dt$

27. $\int (1 - \csc t \cot t) \, dt$

28. $\int (\theta^2 + \sec^2 \theta) \, d\theta$

29. $\int (\sec^2 \theta - \sin \theta) \, d\theta$

30. $\int \sec y (\tan y - \sec y) \, dy$

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Finding an Indefinite Integral **In Exercises 33–42, find the indefinite integral.**

34. $\int \sin 4x \, dx$	**36.** $\int \csc^2\left(\frac{x}{2}\right) dx$
37) $\int \frac{1}{x^2} cos \frac{1}{x} \, dx$	**40.** $\int \sqrt{\tan x} \sec^2 x \, dx$
41. $\int \frac{\csc^2 x}{\cot^3 x} dx$	**42.** $\int \frac{\sin x}{\cos^3 x} dx$

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Key

Trig Integral Rules

1) $\int \sin u \, du = -\cos u + C$	2) $\int \cos u \, du = \sin u + C$
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5) $\int \sec u \tan u \, du = \sec u + C$	6) $\int \csc u \cot u \, du = -\csc u + C$

Find the Indefinite Integral below

25. $\int (5\cos x + 4\sin x)\, dx$

$5\sin x + 4(-\cos x) + C$

$5\sin x - 4\cos x + C$

26. $\int (t^2 - \cos t)\, dt$

$\frac{t^3}{3} - \sin(t) + C$

27. $\int (1 - \csc t \cot t)\, dt$

$t - (-\csc t) + C$

$t + \csc(t) + C$

28. $\int (\theta^2 + \sec^2\theta)\, d\theta$

$\frac{\theta^3}{3} + \tan\theta + C$

29. $\int (\sec^2\theta - \sin\theta)\, d\theta$

$\tan\theta - (-\cos\theta) + C$

$\tan\theta + \cos\theta + C$

30. $\int \sec y(\tan y - \sec y)\, dy$

$\int \sec y \tan y - \sec^2 y \, dy$

$\sec y - \tan y + C$

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Finding an Indefinite Integral **In Exercises 33–42, find the indefinite integral.**

34. $\int \sin 4x\, dx$

$u = 4x$ | $dx = \frac{du}{4}$ | $\frac{1}{4}\int \sin u\, du$

$\frac{du}{dx} = 4$ | $\int \sin u \frac{du}{4}$ | $-\frac{1}{4}\cos u + C$

$-\frac{1}{4}\cos(4x) + C$

36. $\int \csc^2\left(\frac{x}{2}\right) dx$

$u = \frac{x}{2} = \frac{1}{2}x$ | $dx = 2du$ | $2\int \csc^2 u\, du$

$\frac{du}{dx} = \frac{1}{2}$ | $\int \csc^2 u \cdot 2\, du$ | $-2\cot u + C$

$-2\cot\left(\frac{x}{2}\right) + C$

37) $\int \frac{1}{x^2}\cos\frac{1}{x}\, dx$

$u = \frac{1}{x} = x^{-1}$ | $dx = -x^2 du$ | $\int \frac{1}{x^2}\cos u \cdot -x^2\, du$

$\frac{du}{dx} = -1x^{-2}$ | | $-\int \cos u\, du$

$\frac{du}{dx} = \frac{-1}{x^2}$ | | $-\sin u + C$

$-dx = x^2 du$ | | $-\sin\left(\frac{1}{x}\right) + C$

40. $\int \sqrt{\tan x}\sec^2 x\, dx$

$\int (\tan x)^{1/2}\sec^2 x\, dx$ | $\int u^{1/2}\sec^2 x \cdot \frac{du}{\sec^2 x}$

$u = \tan x$ | $\int u^{1/2} du = \frac{u^{3/2}}{3/2} + C$

$\frac{du}{dx} = \sec^2 x$ | $\frac{2}{3}(\tan x)^{3/2} + C$

$dx = \frac{du}{\sec^2 x}$

41. $\int \frac{\csc^2 x}{\cot^3 x}\, dx$

$\int \frac{\csc^2 x}{(\cot x)^3} dx$ | $\int \frac{\csc^2 x}{u^3} \cdot \frac{du}{-\csc^2 x}$

$u = \cot x$ | $-\int u^{-3} du = -\frac{u^{-2}}{-2} + C$

$\frac{du}{dx} = -\csc^2 x$ | $\frac{1}{2u^2} + C$

$dx = \frac{du}{-\csc^2 x}$ | $\frac{1}{2(\cot x)^2} + C$

42. $\int \frac{\sin x}{\cos^3 x}\, dx$

$\int \frac{\sin x}{(\cos x)^3} dx$ | $\int \frac{\sin x}{u^3} \cdot \frac{du}{-\sin x}$

$u = \cos x$ | $-\int u^{-3} du = \frac{-u^{-2}}{-2} + C$

$\frac{du}{dx} = -\sin x$ | $\frac{1}{2u^2} + C$

$dx = \frac{du}{-\sin x}$ | $\frac{1}{2(\cos x)^2} + C$

Chapter 4 Integral Rules and Formulas

Power Rule (Integrals)

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Power Rule (Derivatives)

$$\frac{d}{dx}\, u^n = nu^{n-1} * u'$$

Average Value Theorem

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Trig Derivatives Rule

13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

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