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Trig Unit — Derivatives of Inverse Trig Functions (Ch. 5.6)

Rules:		
1) switch x and y	$y = \sin x$	$f^{-1}(x) = \arcsin x$ *
2) Solve for y	$x = \sin y$	or
	$\arcsin x = y$	$f^{-1}(x) = \sin^{-1}(x)$

Since $f(f^{-1}(x)) = x$, then $\sin(\arcsin x) = x$

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \frac{d}{dx}\text{arccot}\, u = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}\text{arcsec}\, u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\text{arccsc}\, u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

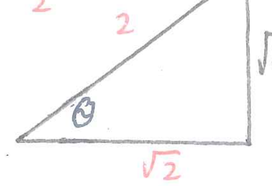
$\boxed{\text{Ex. 1}}$ Find $\tan\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$.

This means "find the tangent of the angle whose cosine is $\frac{\sqrt{2}}{2}$"

Steps:

1) Draw triangle and label angle θ
2) Label sides according to arccos given
3) Use pythagorean theorem to find third side
4) Find $\tan\theta$

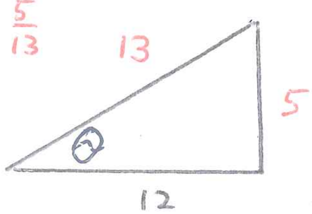
$\cos\theta = \frac{\sqrt{2}}{2}$



$$\tan\left(\arccos\frac{\sqrt{2}}{2}\right) = \frac{o}{a} = \frac{\sqrt{2}}{\sqrt{2}} = \boxed{1}$$

$\boxed{\text{Ex. 2}}$ Find $\cos\left(\arcsin\frac{5}{13}\right)$

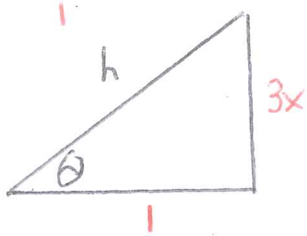
$\sin\theta = \frac{5}{13}$



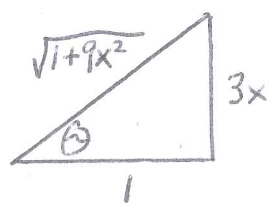
$$\cos\left(\arcsin\frac{5}{13}\right) = \frac{a}{h} = \boxed{\frac{12}{13}}$$

$\boxed{\text{Ex. 3}}$ Write the expression $\sec(\arctan 3x)$ in algebraic form

$\tan\theta = \frac{3x}{1}$



$$1^2 + (3x)^2 = h^2$$
$$1 + 9x^2 = h^2$$
$$\sqrt{1+9x^2} = h$$



$$\sec(\arctan 3x) = \frac{h}{a} = \frac{\sqrt{1+9x^2}}{1} = \boxed{\sqrt{1+9x^2}}$$

$\boxed{\text{Ex. 4}}$ Find derivative of $y = \arcsin x$

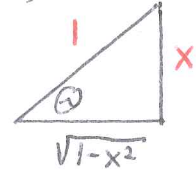
Steps:

1) Take the sine of both sides
2) Differentiate implicitly
3) Solve for dy/dx
4) Rewrite right side of equation in terms of x.

$$\sin y = \sin(\arcsin x)$$
$$\sin y = x$$
$$\cos y\left(\frac{dy}{dx}\right) = 1$$
$$\frac{dy}{dx} = \frac{1}{\cos y}$$
$$\frac{dy}{dx} = \sec y$$

$$\frac{dy}{dx} = \sec(\arcsin x)$$

$\sin\theta = \frac{x}{1}$



$$\sec(\arcsin x) = \frac{h}{a} = \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

$\boxed{\text{Ex. 4b}}$ Find $\frac{dy}{dx}$ for $y = \arcsin x$

derivative rules

$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

$$\frac{dy}{dx} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

$\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$

$\frac{d}{dx} \operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$

$\boxed{\text{Ex. 5}}$ Find y' for $y = \arctan(4x)$

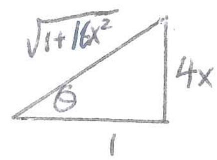
$\tan y = \tan(\arctan(4x))$

$\tan y = 4x$

$\sec^2 y \left(\frac{dy}{dx}\right) = 4$

$\frac{dy}{dx} = \frac{4}{\sec^2 y} = 4\cos^2 y$

$= 4\cos^2(\arctan(4x))$



$\tan\theta = \frac{4x}{1}$

$\cos\theta = \frac{1}{\sqrt{1+16x^2}}$

$= 4\left(\frac{1}{\sqrt{1+16x^2}}\right)^2 = \boxed{\frac{4}{1+16x^2}}$

OR use derivative rule above

$u = 4x$, $u' = 4$

$\frac{dy}{dx} = \frac{4}{1+(4x)^2}$

$= \boxed{\frac{4}{1+16x^2}}$

$\boxed{\text{Ex. 6}}$ Find y' for $y = 2\operatorname{arcsec}(3x^2)$

$\frac{y}{2} = \operatorname{arcsec}(3x^2)$

$\sec(y/2) = \sec(\operatorname{arcsec}(3x^2))$

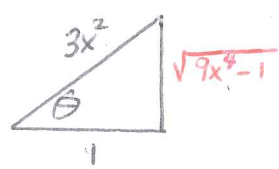
$\sec(y/2) = 3x^2$

$\sec(y/2)\tan(y/2) \cdot \frac{1}{2}\left(\frac{dy}{dx}\right) = 6x$

$\frac{dy}{dx} = \frac{12x}{\sec(y/2)\tan(y/2)}$

$= 12x\cos(y/2)\cot(y/2)$

$= 12x\cos(\operatorname{arcsec} 3x^2)\cot(\operatorname{arcsec} 3x^2)$



$\cos\theta = \frac{a}{h} = \frac{1}{3x^2}$ $\cot\theta = \frac{a}{o} = \frac{1}{\sqrt{9x^4-1}}$

$= 12x \cdot \left(\frac{1}{3x^2}\right) \cdot \left(\frac{1}{\sqrt{9x^4-1}}\right)$

$= \boxed{\frac{4}{|x|\sqrt{9x^4-1}}}$

OR use derivative rule

$u = 3x^2$ $u' = 6x$

$= \frac{2 \cdot 6x}{|3x^2|\sqrt{9x^4-1}}$

$= \boxed{\frac{4}{|x|\sqrt{9x^4-1}}}$

Ex. 7 find y' for $y = x\arccos(2x)$

* product rule!! $f'g + fg'$

$$y' = \underbrace{(1)}_{f'}\underbrace{\arccos 2x}_{g} + \underbrace{x}_{f}\underbrace{\left(\frac{d}{dx}\arccos(2x)\right)}_{g'}$$

$$y' = \arccos 2x + x\left(\frac{-2}{\sqrt{1-4x^2}}\right)$$

$$\boxed{y' = \arccos 2x - \frac{2x}{\sqrt{1-4x^2}}}$$

$y = \arccos(2x)$

$\cos y = 2x$

$-\sin y\left(\frac{dy}{dx}\right) = 2$

$$\frac{dy}{dx} = \frac{-2}{\sin y} = \frac{-2}{\sin(\arccos(2x))}$$

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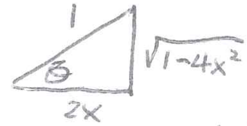


$\sin\theta = \frac{\sqrt{1-4x^2}}{1}$

$$= \boxed{\frac{-2}{\sqrt{1-4x^2}}}$$

OR use derivative rule

$$\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$= \boxed{\frac{-2}{\sqrt{1-4x^2}}}$$

Inverse Trig Derivatives Homework (5.6)

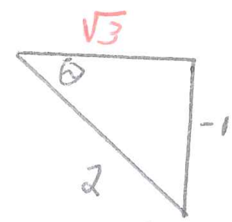
p.377-378 #5-15 odd, 17-27 odd, 41-46 all, 49, 55, 59, 61, 63

#19, 42, 44, 46, 49, 55, 59, 61, 63

19) Evaluate expression. a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

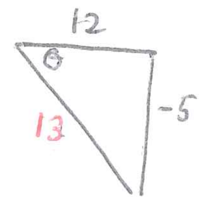
$\cot\left[\arcsin\left(-\frac{1}{2}\right)\right] = \frac{a}{o} = \frac{\sqrt{3}}{-1}$

$= \boxed{-\sqrt{3}}$



b) $\csc\left[\arctan\left(\frac{-5}{12}\right)\right]$

$\csc\left[\arctan\left(\frac{-5}{12}\right)\right] = \frac{h}{o} = \boxed{\frac{-13}{5}}$



42) $f(t) = \arcsin(t^2)$ find $f'(t)$

$y = \arcsin(t^2)$

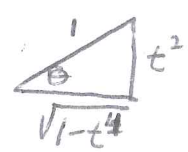
$\sin y = \sin(\arcsin(t^2))$

$\sin y = t^2$

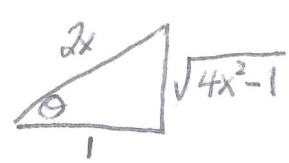
$\cos y\left(\frac{dy}{dt}\right) = 2t$

$\frac{dy}{dt} = \frac{2t}{\cos y} = \frac{2t}{\cos[\arcsin t^2]}$

$= \frac{2t}{\sqrt{1-t^4}}$



OR $\frac{u'}{\sqrt{1-u^2}} = \boxed{\frac{2t}{\sqrt{1-t^4}}}$



44) $f(x) = \operatorname{arcsec}(2x)$

$y = \operatorname{arcsec}(2x)$

$\sec y = 2x$

$\sec y \tan y\left(\frac{dy}{dx}\right) = 2$

$\frac{dy}{dx} = \frac{2}{\sec y \tan y}$

$= 2\cos y \cot y$

$\frac{dy}{dx} = 2\cos[\operatorname{arcsec} 2x]\cot[\operatorname{arcsec} 2x]$

$= 2\left(\frac{1}{2x}\right)\left(\frac{1}{\sqrt{4x^2-1}}\right)$

$= \boxed{\frac{1}{|x|\sqrt{4x^2-1}}}$

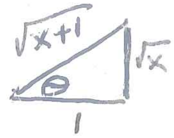
Find $f'(x)$

46) $f(x) = \arctan\sqrt{x}$

$y = \arctan(x^{1/2})$

$\tan y = x^{1/2}$

$\sec^2 y\left(\frac{dy}{dx}\right) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$



OR $\frac{u'}{1+u^2}$

$= \frac{\frac{1}{2}x^{-1/2}}{1+(\sqrt{x})^2} = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\sec^2 y} = \frac{\cos^2[\arctan\sqrt{x}]}{2\sqrt{x}}$ $\left(\frac{1}{\sqrt{x+1}}\right)^2 \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}(x+1)}}$

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49) $h(t) = \sin(\arccos t)$. Find $h'(t)$

$h'(t) = \cos(\arccos t) \cdot \frac{d}{dx}(\arccos t)$

chain Rule $= t \cdot \frac{1}{-\sqrt{1-t^2}} = \boxed{\frac{-t}{\sqrt{1-t^2}}}$

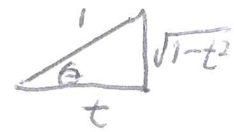
$y = \arccos t$

$\cos y = t$

$-\sin y\left(\frac{dy}{dt}\right) = 1$

$\frac{dy}{dt} = \frac{1}{-\sin y} = \frac{1}{-\sin(\arccos t)}$

$= \frac{1}{-\sqrt{1-t^2}}$



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55) $y = x\arcsin x + \sqrt{1-x^2}$

$+ (1-x^2)^{1/2}$

product rule

chain rule

$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$

$\frac{dy}{dx} = (1)(\arcsin x) + (x)\left(\frac{1}{\sqrt{1-x^2}}\right) + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \arcsin x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}} = \boxed{\arcsin x}$

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59) $y = \arctan x + \frac{x}{1+x^2}$

quotient rule

$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$

$y' = \frac{1}{1+x^2} + \frac{(1)(1+x^2) - (x)(+2x)}{(1+x^2)^2}$

$= \frac{1}{1+x^2} + \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1}{1+x^2} + \frac{1-x^2}{(1+x^2)^2} = \frac{1+x^2}{(1+x^2)^2} + \frac{1-x^2}{(1+x^2)^2} = \boxed{\frac{2}{(1+x^2)^2}}$

Find equation of tangent line to graph.

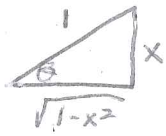
61) $y = 2\arcsin x \quad (\tfrac{1}{2}, \tfrac{\pi}{3})$

$\frac{y}{2} = \arcsin x$

$\sin(\tfrac{y}{2}) = \sin(\arcsin x) = x$

$\cos(\tfrac{y}{2}) \cdot \frac{1}{2}\left(\frac{dy}{dx}\right) = 1$

$\frac{dy}{dx} = \frac{2}{\cos(y/2)} = \frac{2}{\cos(\arcsin x)} = \frac{2}{\sqrt{1-x^2}}$



$y'(x) = \frac{2}{\sqrt{1-x^2}}$

$y'(x) = \frac{2}{\sqrt{1-x^2}}$

$y'(\tfrac{1}{2}) = \frac{2}{\sqrt{1-(\frac{1}{2})^2}}$

$= \frac{2}{\sqrt{\frac{4}{4}-\frac{1}{4}}} = \frac{2}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}}$

$m = \frac{4}{\sqrt{3}}$

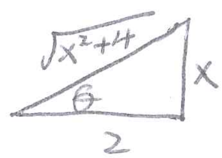
$\boxed{y - \frac{\pi}{3} = \frac{4}{\sqrt{3}}(x - \tfrac{1}{2})}$

63) $y = \arctan(\tfrac{x}{2})$ at $(2, \tfrac{\pi}{4})$

$\tan y = \frac{x}{2}$

$\sec^2 y \left(\frac{dy}{dx}\right) = \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2\sec^2 y} = \frac{\cos^2 y}{2} = \frac{\cos^2[\arctan(\frac{x}{2})]}{2}$



$= \left(\frac{2}{\sqrt{x^2+4}}\right)^2 \cdot \frac{1}{2} = \frac{4}{x^2+4} \cdot \frac{1}{2} = \frac{2}{x^2+4}$

$y'(x) = \frac{2}{x^2+4}$

$y'(2) = \frac{2}{(2)^2+4} = \frac{2}{8} = \frac{1}{4}$

$y - y_1 = m(x - x_1)$

$\boxed{y - \frac{\pi}{4} = \frac{1}{4}(x - 2)}$