



Solve each equation for $0 \leq x < 2\pi$.

1) $(2 \sin x - 1)(2 \cos^2 x - 1) = 0$

$2 \sin x - 1 = 0$	$2 \cos^2 x - 1 = 0$
$\sin x = 1/2$	$\cos^2 x = 1/2$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$	$\cos x = \pm \sqrt{1/2}$
	$\cos x = \frac{\sqrt{2}}{2}$
	$x = \frac{\pi}{4}, \frac{5\pi}{4}$
	$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

2) $4 \sin^2 x + 1 = -4 \sin x$

$4 \sin^2 x + 4 \sin x + 1 = 0$ * let $x = \sin x$

$4x^2 + 4x + 1 = 0$

$(2x+1)(2x+1) = 0$

$2x+1 = 0$

$x = -1/2$

$\sin x = -1/2$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

3) $\sqrt{2} \tan x = 2 \sin x$

$\frac{\sqrt{2} \sin x}{\cos x} = 2 \sin x$	$\sin x = 0$	$\sqrt{2} - 2 \cos x = 0$
$\sqrt{2} \sin x = 2 \sin x \cos x$	$x = 0, \pi$	$\cos x = \frac{\sqrt{2}}{2}$
		$x = \frac{\pi}{4}, \frac{7\pi}{4}$

$\sqrt{2} \sin x - 2 \sin x \cos x = 0$

$\sin x (\sqrt{2} - 2 \cos x) = 0$

4) $\sin x = \cos 2x - 1$ * $\cos 2x = 1 - 2 \sin^2 x$

$\sin x = 1 - 2 \sin^2 x - 1$	$\sin x = 0$	$\sin x = -1/2$
$2 \sin^2 x + \sin x = 0$	$x = 0, \pi$	$x = \frac{7\pi}{6}, \frac{11\pi}{6}$
$\sin x (2 \sin x + 1) = 0$		
$\sin x = 0$ $2 \sin x = -1$		

5) $\cot^2 x - \csc x = 1$

$\csc^2 x - 1 - \csc x = 1$ * $1 + \cot^2 x = \csc^2 x$

$\csc^2 x - \csc x - 2 = 0$ * let $y = \csc x$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$\csc x - 2 = 0$	$\csc x + 1 = 0$
$\csc x = 2$	$\csc x = -1$
$\sin x = 1/2$	$x = 3\pi/2$
$x = \pi/6, 5\pi/6$	

6) $\sin x + \cos x = 0$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = \frac{-\cos x}{\cos x}$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

7) Find all values of θ between 0 and 2π that satisfy $-1 - 3 \sin \theta = \cos 2\theta$

* $\cos 2\theta = 1 - 2 \sin^2 \theta$

$-1 - 3 \sin \theta = 1 - 2 \sin^2 \theta$

$2 \sin^2 \theta - 3 \sin \theta - 2 = 0$

* let $x = \sin \theta$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$2x+1 = 0$	$x-2 = 0$	$\sin \theta = -1/2$	$\sin \theta = 2$
$x = -1/2$	$x = 2$	$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$	none

check for extraneous solution

8) $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

9) $\cos x \tan x - 2 \cos^2 x = -1$

$\cos x \left(\frac{\sin x}{\cos x}\right) - 2(1 - \sin^2 x) + 1 = 0$

$\sin x - 2 + 2\sin^2 x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

* let $y = \sin x$ $(2y - 1)(y + 1) = 0$

$2y^2 + y - 1 = 0$ $y = \frac{1}{2}, y = -1$

$\sin x = -1$
 $x = \frac{3\pi}{2}$

$\sin x = \frac{1}{2}$
 $\frac{\pi}{6}, \frac{5\pi}{6}$

10) $3 \tan^2 x = \sqrt{3} \tan x$

$3 \tan^2 x - \sqrt{3} \tan x = 0$

$\tan x (3 \tan x - \sqrt{3}) = 0$

$\tan x = 0$ $\tan x = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$x = 0, \pi$ $x = \frac{\pi}{6}, \frac{7\pi}{6}$

11) $2 \cos^2 x = 3 \sin x$

* $\cos^2 x = 1 - \sin^2 x$

$2(1 - \sin^2 x) = 3 \sin x$

$2 - 2 \sin^2 x = 3 \sin x$

$2 \sin^2 x + 3 \sin x - 2 = 0$

* let $y = \sin x$

$2y^2 + 3y - 2 = 0$

$(2y - 1)(y + 2) = 0$

$y = \frac{1}{2}, y = 2$

$\sin x = \frac{1}{2}$ $\sin x = -2$
 $x = \frac{\pi}{6}, \frac{5\pi}{6}$ none

12) $\sin x \cos x = \frac{1}{2}$

$\frac{1}{2} \sin(2x) = \frac{1}{2}$

$\sin(2x) = 1$

$2x = \sin^{-1}(1)$

$2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

* $2 \sin x \cos x = \sin(2x)$
 $\sin x \cos x = \frac{1}{2} \sin(2x)$

13) $\cos^2 x - \sin^2 x = \frac{\sqrt{3}}{2}$

* $\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

Solve each equation

14) $\log_2 x = 5$

$2^5 = x$

$x = 32$

15) $\log_7 n = \frac{2}{3} \log_7 8$

$\log_7 n = \log_7 8^{2/3}$

$n = 8^{2/3}$

$n = 2^2 = 4$

16) $\log_6 (4x + 4) = \log_6 64$

$4x + 4 = 64$

$4x = 60$

$x = 15$

17) $2 \log_6 4 - \frac{1}{4} \log_6 16 = \log_6 x$

$\log_6 4^2 - \log_6 16^{1/4} = \log_6 x$

$\log_6 \left(\frac{16}{2}\right) = \log_6 x$

$8 = x$

Use natural logarithms to solve each equation

18) $9^{x-4} = 7.13$

$\ln 9^{x-4} = \ln 7.13$

$(x-4)(\ln 9) = \ln 7.13$

$x - 4 = \frac{\ln 7.13}{\ln 9}$

$x = \frac{\ln 7.13}{\ln 9} + 4$

$x = 4.8940$

19) $25e^x = 1000$

$e^x = 40$

$\ln e^x = \ln 40$

$x = \ln 40$

$x = 3.6889$

20) $5^x = 4^{x+3}$

$\ln 5^x = \ln 4^{x+3}$

$x(\ln 5) = (x+3)(\ln 4)$

$x = 18.6377$

$x \ln 5 = x \ln 4 + 3 \ln 4$

$x \ln 5 - x \ln 4 = 3 \ln 4$

$x(\ln 5 - \ln 4) = 3 \ln 4$

$x = \frac{3 \ln 4}{\ln 5 - \ln 4}$

21) $\frac{1}{3} \log x = \log 8$

$\log x^{1/3} = \log 8$

$x^{1/3} = 8$

$x = 8^3$

$x = 512$

22) $4 \log(x+3) = 9$

$\log(x+3) = \frac{9}{4}$

$\log_{10}(x+3) = \frac{9}{4}$

$10^{9/4} = x+3$

$x = 10^{9/4} - 3$

$x = 174.828$

22b) $0.25 = \log 16^x$

$\frac{1}{4} = \log_{10} 16^x$

$10^{1/4} = 16^x$

$\ln 10^{1/4} = \ln 16^x$

$\ln 10^{1/4} = (x)(\ln 16)$

$\frac{\ln 10^{1/4}}{\ln 16} = x$

$x = 0.2076$

Solve each equation

23) $\log_5(x+4) + \log_5 8 = \log_5 64$

$\log_5 8(x+4) = \log_5 64$

$8(x+4) = 64$

$x+4 = 8$

$x = 4$

24) $\log_4(x-3) + \log_4(x+3) = 2$

$\log_4(x-3)(x+3) = 2$

$\log_4(x^2-9) = 2$

$4^2 = x^2 - 9$

$25 = x^2$

$x = 5, -5$

$x = 5$

25) $\frac{1}{2}(\log_7 x + \log_7 8) = \log_7 16$

$\frac{1}{2}[\log_7(8x)] = \log_7 16$

$\log_7(8x)^{1/2} = \log_7 16$

$(8x)^{1/2} = 16$

$8x = 16^2$

$x = \frac{16^2}{8}$

$x = 32$

26) $2 \log_5(x-2) = \log_5 36$

$\log_5(x-2)^2 = \log_5 36$

$(x-2)^2 = 36$

$(x-2) = \pm\sqrt{36}$

$x-2 = 6$ | $x-2 = -6$

$x = 8$

$x = -4$

$x = 8$

27) $\log_3 3 + \log_3 x = \log_3 45$

$\log_3(3x) = \log_3 45$

$3x = 45$

$x = 15$

28) $2 \log_6 4 - \frac{1}{3} \log_6 8 = \log_6 x$

$\log_6 4^2 - \log_6 8^{1/3} = \log_6 x$

$\log_6\left(\frac{16}{2}\right) = \log_6 x$

$8 = x$