$\boxed{3.1}$ Trig Unit p.169

17) Find critical number of function.

$h(x) = \sin^2 x + \cos x \qquad 0 < x < 2\pi$

$h(x) = [\sin x]^2 + \cos x$

$h'(x) = 2\sin x \cos x + (-\sin x)$

$0 = \sin x(2\cos x - 1)$

$\sin x = 0$ | $2\cos x - 1 = 0$

$x = \cancel{0}, \pi, \cancel{2\pi}$ | $\cos x = 1/2$

$x = \pi$ | $x = \pi/3, 5\pi/3$

$\boxed{x = \pi, \pi/3, 5\pi/3}$

33) Locate Absolute Extrema on closed interval

$f(x) = \cos(\pi x) \quad [0, 1/6]$

*EVT applies since f(x) is continuous on closed interval. EVT guarantees Absolute max and Absolute Min at either endpoints or critical points.

$f'(x) = -\sin(\pi x) \cdot \pi = -\pi \sin(\pi x)$

$0 = -\pi \sin(\pi x)$ | $\sin^{-1}(0) = \pi x$ | $0 = \pi x \rightarrow x = 0$

$0 = \sin(\pi x)$ | $0, \pi, 2\pi = \pi x$ | $\pi = \pi x \rightarrow \cancel{x = 1}$

$2\pi = \pi x \rightarrow \cancel{x = 2}$

$f(0) = \cos(0) = 1$

$f(1/6) = \cos(\pi/6) = \frac{\sqrt{3}}{2}$

$\boxed{\begin{array}{l}\text{Max Value is 1 at } x = 0 \\ \text{Min value is } \sqrt{3}/2 \text{ at } x = 1/6\end{array}}$

$\boxed{p.176}$ Trig Unit $\boxed{3.2}$

Determine whether Rolle's Theorem applies on $[a,b]$.
If so, find value(s) of c.

19) $f(x) = \sin x \quad [0, 2\pi]$

Rolle's theorem conditions:
1) $f(x)$ continuous on $[0, 2\pi]$
2) $f(x)$ differentiable on $(0, 2\pi)$
3) Test if $f(0) = f(2\pi)$

Sine and cosine curves are always continuous and differentiable for all real numbers, so conditions 1 and 2 passes.

$f(0) = \sin(0) = 0$
$f(2\pi) = \sin(2\pi) = 0$ ✓ $\rightarrow$

set $f'(x) = 0$

$f'(x) = \cos x$

$0 = \cos x$

$x = \pi/2, 3\pi/2$, so $\boxed{c = \pi/2, 3\pi/2}$

21) Rolle's theorem

$f(x) = \frac{6x}{\pi} - 4\sin^2 x \quad [0, \pi/6]$

$f(x)$ continuous on $[0, \pi/6]$ and differentiable on $(0, \pi/6)$.

$f(0) = \frac{6(0)}{\pi} - 4(\sin 0)^2 = 0$

$f(\pi/6) = \frac{6(\pi/6)}{\pi} - 4(\sin(\pi/6))^2 = 1 - 4(1/2)^2 = 1 - \frac{4}{4} = 0$ ✓

$f(0) = f(\pi/6) = 0$

set $f'(x) = 0$

$f(x) = \frac{6}{\pi}x - 4[\sin x]^2$

$f'(x) = \frac{6}{\pi} - 4 \cdot 2(\sin x) \cdot \cos x$

$0 = \frac{6}{\pi} - 8\sin x \cos x$

$\frac{6}{\pi} = 8\sin x \cos x$

$\frac{6}{\pi} = 4(2\sin x \cos x)$

$\frac{6}{\pi} = 4(\sin 2x)$

$\frac{3}{2\pi} = \sin(2x)$

Radian Mode

$\sin^{-1}\left(\frac{3}{2\pi}\right) = 2x$

$0.4978 = 2x$

$0.249 = x$

$\boxed{c = 0.249}$

27) Rolle's. $f(x) = 4x - \tan(\pi x)$ $[-1/4, 1/4]$

$f(x)$ continuous on $[-1/4, 1/4]$ and differentiable on $(-1/4, 1/4)$

* Note that Vertical asymptotes for $\tan x$ are at $-\pi/2$ and $\pi/2$ which are outside of our intervals.

$f(-1/4) = 4(-1/4) - \tan(-\pi/4) = -1 - (-1) = 0$

$f(1/4) = 4(1/4) - \tan(\pi/4) = 1 - (1) = 0$

set $f'(x) = 0$	$\pi \sec^2(\pi x) = 4$	$\sqrt{\frac{\pi}{4}} = \sqrt{\cos^2(\pi x)}$
$f'(x) = 4 - \sec^2(\pi x) \cdot \pi$	$\frac{\pi}{\cos^2(\pi x)} = 4$	$\pm\sqrt{\frac{\pi}{4}} = \cos(\pi x)$
$0 = 4 - \pi \sec^2(\pi x)$	$\pi = \cos^2(\pi x) \cdot 4$	$\pm\cos^{-1}\left(\sqrt{\frac{\pi}{4}}\right) = \pi x$

$\pi x = \cos^{-1}\left(\sqrt{\pi/4}\right)$	$\pi x = -\cos^{-1}\left(\sqrt{\pi/4}\right)$	$\boxed{c = \pm 0.153}$
$\pi x = 0.4816$	$\pi x = -0.4816$	
$x = \frac{0.4816}{\pi} = \boxed{0.153}$	$x = -0.153$	

3.2 p.177 MVT Trig Unit

45) Determine whether MVT can be applied. If so, find value of c.

$f(x) = \sin x \quad [0, \pi]$

$f(x)$ continuous on $[0, \pi]$ and differentiable on $(0, \pi)$

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

$f(0) = \sin(0) = 0$

$f(\pi) = \sin(\pi) = 0$

$f'(x) = \cos x$

$\frac{f(b)-f(a)}{b-a} = \frac{0-0}{\pi-0} = 0$

$\cos x = 0$

$x = \pi/2$

$\boxed{c = \pi/2}$

Determine whether MVT can be applied. If so, find value of c.

*48) MVT $f(x) = x - 2\sin x \quad [-\pi, \pi]$

$f(x)$ continuous on $[-\pi, \pi]$ and differentiable on $(-\pi, \pi)$

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

$f(-\pi) = -\pi - 2\sin(-\pi)$

$= -\pi - 0 = -\pi$

$f(\pi) = \pi - 2\sin(\pi)$

$= \pi - 0 = \pi$

$\frac{f(\pi)-f(-\pi)}{\pi-(-\pi)} = \frac{\pi-(-\pi)}{\pi-(-\pi)}$

$= \frac{2\pi}{2\pi} = \boxed{1}$

$f'(x) = 1 - 2\cos x$

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

$1 - 2\cos x = 1$

$-2\cos x = 0$

$\cos x = 0$

$x = \pi/2, -\pi/2$

$\boxed{c = -\pi/2, \pi/2}$