

AP Calculus AB

Trig Quiz #2 Review Worksheet 1

1) If $y = \cos^3(\pi x^2)$, find $\frac{dy}{dx}$

2) Given $y = \frac{\sin(\pi x)}{\cos(2\pi x)}$

Evaluate $y'(0)$

3) If $\frac{\sin(x)}{3\cos(y)} = 2x$ find $\frac{dy}{dx}$

4) If $[\sin(\pi x) + \cos(\pi y)]^2 = 2$ find $\frac{dy}{dx}$

5) If velocity of a particle is $v(t) = \sin(2t)\tan(t)$,

a) Find $a(t)$ b) find acceleration at $t = \pi$

6. Find the equation of the tangent line at $x = \frac{\pi}{4}$ for $y = 2\tan^3 x$

1) If $y = \cos^3(\pi x^2)$, find $\frac{dy}{dx}$

$$y = [\cos(\pi x^2)]^3$$

$$y' = 3[\cos(\pi x^2)]^2 \cdot \sin(\pi x^2) \cdot \pi(2x)$$

$$y' = -6\pi x [\cos(\pi x^2)]^2 \sin(\pi x^2)$$

or

$$y' = -6\pi x \cos^2(\pi x^2) \sin(\pi x^2)$$

3) If $\frac{\sin(x)}{3\cos(y)} = 2x$ find $\frac{dy}{dx}$

* quotient
* implicit

$$\frac{\cos x \cdot 3\cos y - \sin x \cdot (-3\sin y) \left(\frac{dy}{dx}\right)}{(3\cos y)^2} = 2$$

$$3\cos x \cos y + 3\sin x \sin y \left(\frac{dy}{dx}\right) = 2 \cdot 9\cos^2 y$$

$$\frac{dy}{dx} (3\sin x \sin y) = 18\cos^2 y - 3\cos x \cos y$$

$$\frac{dy}{dx} = \frac{18\cos^2 y - 3\cos x \cos y}{3\sin x \sin y} = \frac{6\cos^2 y - \cos x \cos y}{\sin x \sin y}$$

5) If velocity of a particle is $v(t) = \sin(2t)\tan(t)$,

a) Find $a(t)$

$$a(t) = \cos(2t) \cdot 2 \tan(t) + \sin(2t) \sec^2(t) \cdot (1)$$

$$a(t) = 2\cos(2t)\tan(t) + \sin(2t)\sec^2(t)$$

b) find acceleration at $t = \pi$

$$a(\pi) = 2\cos(2\pi)\tan(\pi) + \sin(2\pi)\sec^2(\pi)$$

$$= 2(1)(0) + (0)(-1)^2 = 0$$

6) Find equation of tangent line at $x = \pi/4$ for $y = 2\tan^3 x$

$$y = 2[\tan x]^3$$

$$y' = 2 \cdot 3[\tan x]^2 \sec^2 x$$

$$y'(\pi/4) = 6[\tan(\pi/4)]^2 \sec^2(\pi/4) = 6(1)(\sqrt{2})^2 = 12$$

point: $y(\pi/4) = 2[\tan(\pi/4)]^3 = 2(1)^3 = 2$
 point: $(\pi/4, 2)$ slope: $m = 12$

$$y - 2 = 12(x - \pi/4)$$

Name: Key

* chain rule
* quotient rule
 $f'g - fg'$

2) Given $y = \frac{\sin(\pi x)}{\cos(2\pi x)}$ Evaluate $y'(0)$

$$y' = \frac{[\cos(\pi x) \cdot \pi] \cos(2\pi x) - \sin(\pi x) \cdot [-\sin(2\pi x) \cdot 2\pi]}{[\cos(2\pi x)]^2}$$

$$y'(0) = \frac{\pi \cos 0 \cdot \cos 0 - \sin(0) \cdot (-\sin 0) \cdot 2\pi}{(\cos 0)^2}$$

$$= \frac{\pi}{1} = \boxed{\pi}$$

4) If $[\sin(\pi x) + \cos(\pi y)]^2 = 2$ find $\frac{dy}{dx}$

* chain
* implicit

$$2[\sin(\pi x) + \cos(\pi y)] \cdot [\cos(\pi x) \cdot \pi - \sin(\pi y) \cdot \pi \left(\frac{dy}{dx}\right)] = 0$$

$$\pi \cos(\pi x) - \frac{dy}{dx} (\pi \sin(\pi y)) = 0$$

$$-\frac{dy}{dx} (\pi \sin(\pi y)) = -\pi \cos(\pi x)$$

$$\frac{dy}{dx} = \frac{-\pi \cos(\pi x)}{-\pi \sin(\pi y)} = \frac{\cos(\pi x)}{\sin(\pi y)}$$

* product rule $f'g + fg'$
* chain rule

1) If $y = \cos^3(\pi x^2)$, find $\frac{dy}{dx}$

$$y = [\cos(\pi x^2)]^3$$

$$y' = 3[\cos(\pi x^2)]^2 \cdot -\sin(\pi x^2) \cdot 2\pi x$$

$$= \boxed{-6\pi x \cos^2(\pi x^2) \sin(\pi x^2)}$$

2) Given $y = \frac{\sin(\pi x)}{\cos(2\pi x)}$ Evaluate $y'(0)$

$$y' = \frac{[\cos(\pi x) \cdot \pi][\cos(2\pi x)] - [\sin(\pi x)][-\sin(2\pi x) \cdot 2\pi]}{[\cos(2\pi x)]^2}$$

$$y'(0) = \frac{[\cos(0) \cdot \pi][\cos(0)] - [\sin(0)][-\sin(0) \cdot 2\pi]}{(\cos(0))^2}$$

$$= \frac{\pi - 0}{1} = \boxed{\pi}$$

3) If $\frac{\sin(x)}{3\cos(y)} = 2x$ find $\frac{dy}{dx}$

$$\frac{[\cos(x)][3\cos(y)] - [\sin(x)][-3\sin(y)\frac{dy}{dx}]}{(3\cos(y))^2} = 2$$

$$3\cos x \cos y + \frac{dy}{dx}(3\sin x \sin y) = 2 \cdot (3\cos y)^2$$

$$\frac{dy}{dx}(3\sin x \sin y) = 18\cos^2 y - 3\cos x \cos y$$

$$\frac{dy}{dx} = \frac{18\cos^2 y - 3\cos x \cos y}{3\sin x \sin y} = \boxed{\frac{6\cos^2 y - \cos x \cos y}{\sin x \sin y}}$$

4) If $(\sin(\pi x) + \cos(\pi y))^2 = \pi$ find $\frac{dy}{dx}$

$$2[\sin(\pi x) + \cos(\pi y)] \cdot [\cos(\pi x) \cdot \pi + -\sin(\pi y) \cdot \pi \frac{dy}{dx}] = 0$$

$$\pi \cos(\pi x) - \frac{dy}{dx} \pi \sin(\pi y) = 0$$

$$-\frac{dy}{dx} \pi \sin(\pi y) = -\pi \cos(\pi x)$$

$$\frac{dy}{dx} = \frac{-\pi \cos(\pi x)}{-\pi \sin(\pi y)} = \boxed{\frac{\cos(\pi x)}{\sin(\pi y)}}$$

5) 5. If velocity of a particle is $v(t) = \sin(2t)\tan(t)$.a) Find $a(t)$

$$a(t) = [\cos(2t) \cdot 2][\tan t] + [\sin(2t)] \sec^2 t$$

b) find acceleration at $t = \pi$

$$a(\pi) = 2\cos(2\pi)\tan(\pi) + \sin(2\pi)\sec^2(\pi) \\ = 2(1)(0) + (0)(-1)^2$$

$$2[\sin(\pi x) + \cos(\pi y)] \cdot \pi \cos(\pi x) - 2[\sin(\pi x) + \cos(\pi y)] \pi \sin(\pi y) \frac{dy}{dx} = 0$$

$$-2[\sin(\pi x) + \cos(\pi y)] \pi \sin(\pi y) \frac{dy}{dx} = -2[\sin(\pi x) + \cos(\pi y)] \pi \cos(\pi x)$$

$$\frac{dy}{dx} = \frac{-2[\sin(\pi x) + \cos(\pi y)] \pi \cos(\pi x)}{-2[\sin(\pi x) + \cos(\pi y)] \pi \sin(\pi y)} = \frac{\cos(\pi x)}{\sin(\pi y)}$$