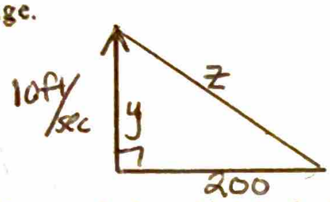
Calculus Trig Unit **2.6 Trig Related Rates Practice Problems WS**

Key

1) https://www.showme.com/sh/?h=OJRDzfs

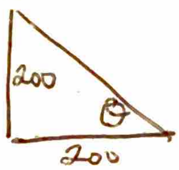
A hot air balloon rises vertically at a constant rate of 10 feet per second. An observer is lying on the ground 200 feet from the spot directly below the balloon watching the balloon rise.

a) Draw a diagram of the situation. Label all fixed quantities with their value, and label all changing distances with variables. Draw arrows to show the direction that the distances will change.



b) At 20 seconds, how high is the balloon? What is the angle of elevation from the observer to the balloon?

$\frac{10 \text{ ft}}{\text{sec}} \cdot 20 \text{ secs} = \boxed{200 \text{ ft}}$ off the ground

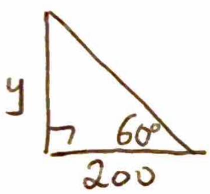


$\tan\theta = \frac{200}{200} = 1$

$\tan\theta = 1$

$\boxed{\theta = 45°}$

c) If the angle of elevation is 60 degrees, how high is the balloon? To the nearest second, how much time is needed for the balloon to reach this height?



$\tan 60° = \frac{y}{200}$

$200 \tan 60° = y$

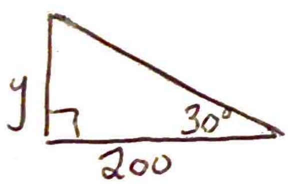
$y = 346.4$ ft

$y = 200\sqrt{3}$ ft

$200\sqrt{3}\,\cancel{\text{ft}} \cdot \frac{1 \text{ sec}}{10\,\cancel{\text{ft}}} =$

$\boxed{\text{34.64 seconds for balloon to reach height.}}$

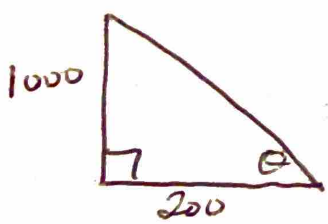
d) If the angle of elevation from the observer to the balloon is 30 degrees, how high is the balloon? To the nearest second, in how much time did the balloon reach this height?



$y = \frac{200}{\sqrt{3}}$ ft.

$\frac{200}{\sqrt{3}} \text{ ft} \cdot \frac{1 \text{ sec}}{10 \text{ ft}} = 11.547$ seconds

e) When the balloon is 1000 feet above the ground, what is the angle of elevation from the observer to the balloon? To the nearest second, in how much time did the balloon reach this height?



$\tan\theta = \frac{1000}{200}$

$\tan\theta = 5$

$\theta = 78.69°$

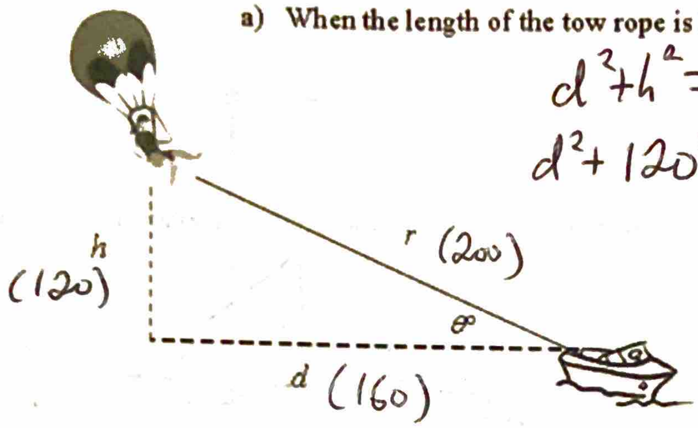
$1000 \text{ ft} \cdot \frac{1 \text{ sec}}{10 \text{ ft}} =$

$\boxed{\text{100 seconds to reach height}}$

2) https://www.showme.com/sh/?h=FGbdclk

A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow rope, r. As the boat's speed increases, the rider's distance above the water increases. The length of the tow rope is controlled by a winch mounted on the back of the deck so that the rope forms an angle with the deck.

a) When the length of the tow rope is 200 ft and $h = 120$ feet, what is d?



$d^2 + h^2 = r^2$

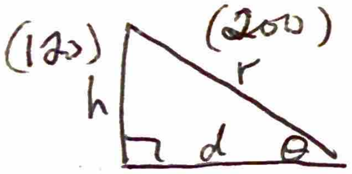
$d^2 + 120^2 = 200^2$

$d^2 = \sqrt{25600}$

$\underline{d = 160 \text{ ft}}$

A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow

b) When the length of the tow rope is 200 ft, what is the measure of the angle formed by the rope and the deck of the boat?



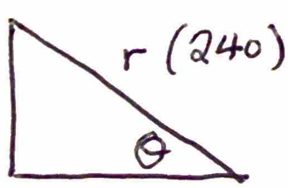
$\sin\theta = \frac{120}{200}$

$\sin\theta = 0.6$

$\theta = \sin^{-1}(0.6)$

$\boxed{\theta = 36.870°}$

c) As more rope is released, the angle between the tow rope and the deck of the boat increases at 2 degrees per 5 feet of rope. If the total length of the tow rope increases to 240 feet, what is the measure of the angle?



$\frac{2 \text{ degrees}}{5 \text{ ft}} = \frac{x \text{ degrees}}{40}$

$240 - 200 = 40$

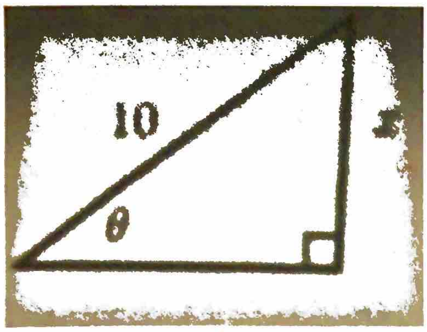
$5x = 80$

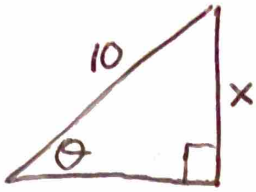
$x = 16$ degrees

original angle + newly added angle = new angle

$36.870° + 16° = \boxed{52.870° \text{ degrees}}$

3) The angle θ is increasing at a constant rate of 6 radians per hour. At what rate is the side of length x increasing when x = 6 feet?





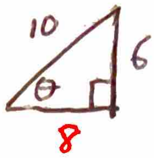
$x = 6$ | $\frac{dx}{dt} = \_\_\_?$

$\frac{d\theta}{dt} = 6$

$\sin\theta = \frac{x}{10}$

$\sin\theta = \frac{1}{10}x$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{10}\left(\frac{dx}{dt}\right)$



$\rightarrow \cos\theta = \frac{8}{10} = \frac{4}{5}$

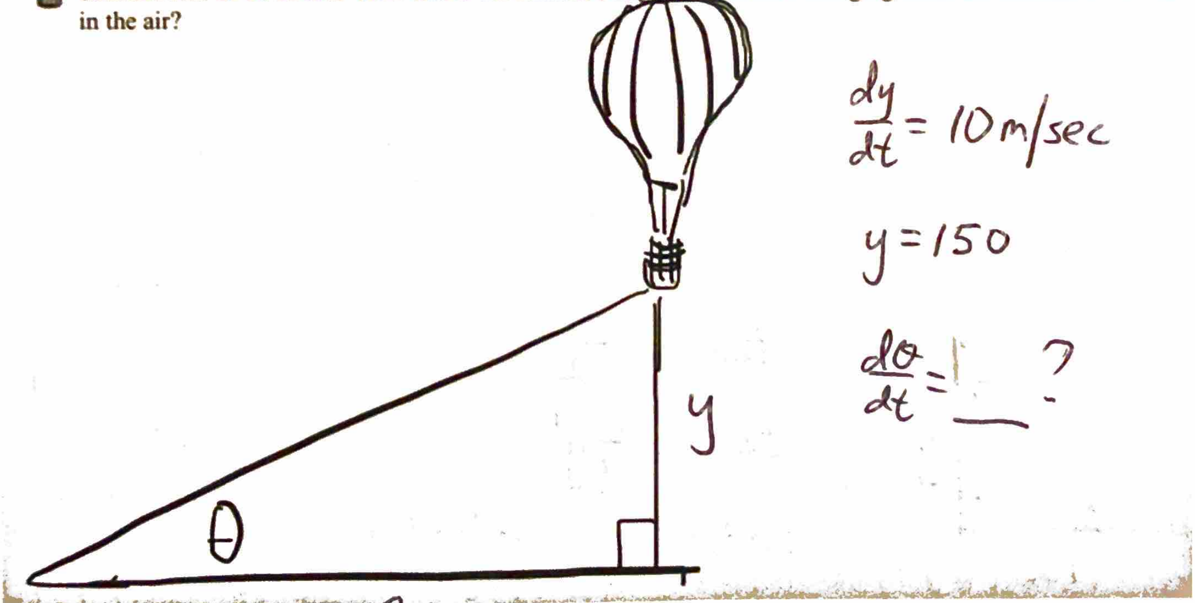
$\left(\frac{4}{5}\right)(6) = \frac{1}{10}\left(\frac{dx}{dt}\right)$

$\left(\frac{4}{5}\right)(6)\left(\frac{10}{1}\right) = \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 48 \text{ ft/hr.}}$

4) https://www.youtube.com/watch?v=v1Qy7Rj9L0A

A camera on the ground 200 meters away from a hot air balloon records the balloon rising into the sky at a constant rate of 10 m/sec. How fast is the camera's angle of elevation changing when the balloon is 150 m in the air?



$\frac{dy}{dt} = 10 \text{ m/sec}$

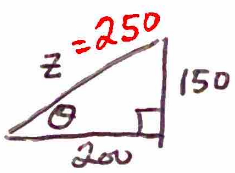
$y = 150$

$\frac{d\theta}{dt} = \_\_\_?$

$\tan\theta = \frac{y}{200}$

$\tan\theta = \frac{1}{200}(y)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{200}\left(\frac{dy}{dt}\right)$



$200^2 + 150^2 = z^2$

$z = 250$

$\sec\theta = \frac{hyp}{adj} = \frac{250}{200} = \frac{5}{4}$

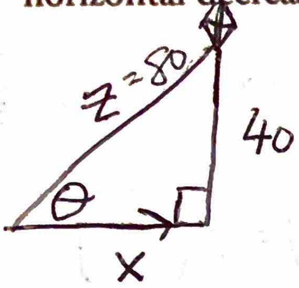
$\left(\frac{5}{4}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{200}(10)$

$\frac{d\theta}{dt} = \left(\frac{4}{5}\right)^2\left(\frac{10}{200}\right) = 0.032$

$\boxed{\frac{d\theta}{dt} = 0.032 \text{ rad/sec}}$

5) https://www.youtube.com/watch?v=kXLKmovhSP8

A kite 40 m above the ground moves horizontally at the rate of $3\frac{m}{s}$. At what rate is the angle between the string and the horizontal decreasing when 80 m of string has been let out?



$\frac{dx}{dt} = 3 m/s$

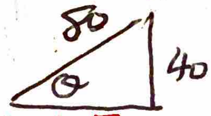
$x = 80$

$\frac{d\theta}{dt} = \_\_ ?$

$\tan\theta = \frac{40}{x}$

$\tan\theta = 40x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -40x^{-2}\left(\frac{dx}{dt}\right)$



$x = 40\sqrt{3}$

$x^2 + 40^2 = 80^2$

$x = 40\sqrt{3}$

$\sec\theta = \frac{80}{40\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-40}{(40\sqrt{3})^2}(3)$

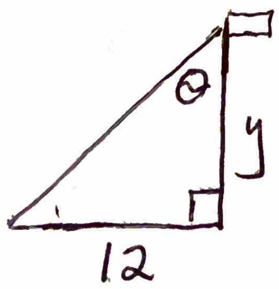
$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{-40}{}\right)(3)$

$\frac{d\theta}{dt} = \frac{3}{4}\cdot\frac{-40}{4800}\cdot 3$

$\boxed{\frac{d\theta}{dt} = \frac{-3}{160} \text{ rad/sec}}$

6) https://www.youtube.com/watch?v=mbmaW7X54To

A man stands 12 metres away from a flagpole. He holds onto a long rope attached to the flag. As the flag is raised at a rate of 10 metres per minute, the rope runs tautly through the man's hands (so that it is always kept straight). Find the rate of change of the angle between the rope and the flagpole, at the moment when there is 24 metres of rope between the flag and the man.



$\frac{d\theta}{dt} = \_\_ ?$

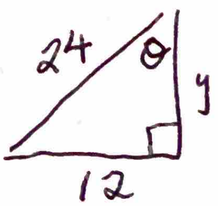
$\frac{dy}{dt} = 10 m/min$

$z = 24$

$\tan\theta = \frac{12}{y}$

$\tan\theta = 12y^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -12y^{-2}\left(\frac{dy}{dt}\right)$



$12^2 + y^2 = 24^2$

$y = \sqrt{432}$

$y = 12\sqrt{3}$

$\sec\theta = \frac{24}{12\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{-12}{y^2}\left(\frac{dy}{dt}\right)$

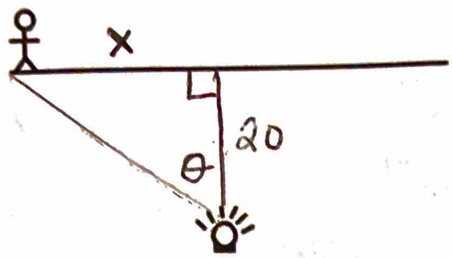
$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-12}{(12\sqrt{3})^2}(10)$

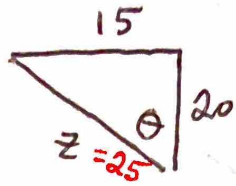
$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{-12}{432}\right)(10)$

$\boxed{\frac{d\theta}{dt} = \frac{-5}{24} \text{ rad/min.}}$

7) https://www.youtube.com/watch?v=LWJYL513Rm8

A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path, and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?





$15^2 + 20^2 = z^2$

$z = 25$

$\sec\theta = \frac{25}{20} = \frac{5}{4}$

$\frac{dx}{dt} = -4$ ft/s

$x = 15$

$\frac{d\theta}{dt} = \_\_\,?$

$\tan\theta = \frac{x}{20}$

$\tan\theta = \frac{1}{20}x$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{20}\left(\frac{dx}{dt}\right)$

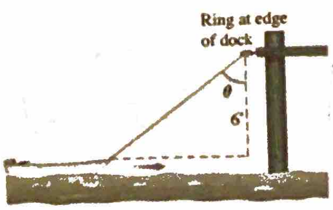
$\left(\frac{5}{4}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{20}(-4)$

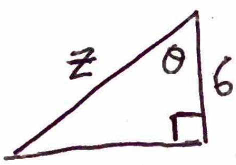
$\frac{d\theta}{dt} = \left(\frac{4}{5}\right)^2\left(\frac{1}{20}\right)(-4)$

$\boxed{\frac{d\theta}{dt} = \frac{-16}{125}\ \text{rad/sec}}$

https://math.stackexchange.com/questions/1826233/at-what-rate-is-the-angle-theta-changing-when-10-ft-of-rope-is-out

8) A rowboat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft. above the bow. The rope is hauled in a rate of 2 ft/s. At what rate is the angle θ changing when 10 ft. of rope is out?





$\frac{dz}{dt} = -2$

$z = 10$

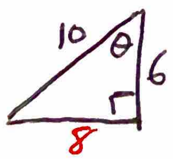
$\frac{d\theta}{dt} = \_\_\,?$

$\cos\theta = \frac{6}{z}$

$\cos\theta = 6z^{-1}$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = -6z^{-2}\left(\frac{dz}{dt}\right)$

$-\sin\theta\left(\frac{d\theta}{dt}\right) - \frac{6}{z^2}\left(\frac{dz}{dt}\right)$



$6^2 + x^2 = 10^2$

$x = 8$

$\sin\theta = \frac{8}{10}$

$\sin\theta = \frac{4}{5}$

$-\left(\frac{4}{5}\right)\left(\frac{d\theta}{dt}\right) = \frac{-6}{10^2}(-2)$

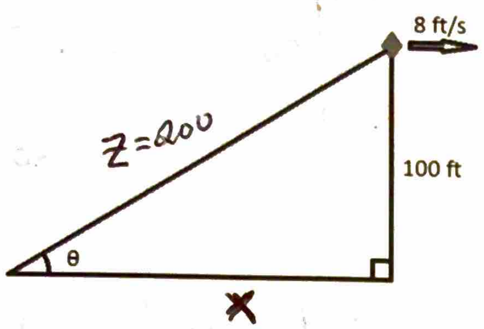
$\frac{d\theta}{dt} = \frac{-5}{4} \cdot \frac{6}{100} \cdot 2$

$\frac{d\theta}{dt} = \frac{-15}{100} = \frac{-3}{20}$

or -0.15 rad/sec

https://jakesmathlessons.com/derivatives/solution-a-kite-100-ft-above-the-ground-moves-horizontally-at-a-speed-of-8-ft-s-at-what-rate-is-the-angle-between-the-string-and-the-horizontal-decreasing-when-200-ft-of-string-has-been-let-out/

9) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?



$Z = 200$ ft

$\frac{dx}{dt} = 8$ ft/s

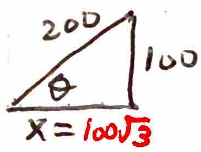
$\frac{d\theta}{dt} = \_\_\_$?

$\tan\theta = \frac{100}{x}$

$\tan\theta = 100x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -100x^{-2}\left(\frac{dx}{dt}\right)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{-100}{x^2}\left(\frac{dx}{dt}\right)$



$x^2 + 100^2 = 200^2 \quad x = 100\sqrt{3}$

$\sec\theta = \frac{200}{100\sqrt{3}} = \frac{2}{\sqrt{3}}$

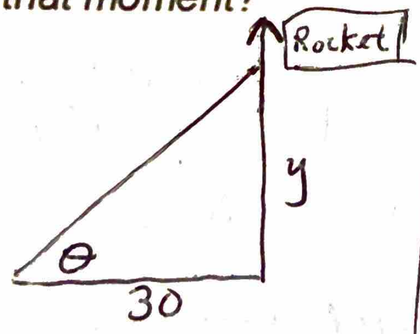
$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-100}{(100\sqrt{3})^2}(8)$

$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \frac{-100}{30{,}000} \cdot 8$

$\boxed{\frac{d\theta}{dt} = \frac{-1}{50} \text{ rad/sec}}$

10) https://www.youtube.com/watch?v=o_k-i7pJgRA

A model rocket is launched 30 feet from Maria, and is rising vertically at a constant rate of 20 ft/s when the rocket has an elevation of 40 feet. How fast is the angle of elevation from Maria to the rocket changing at that moment?



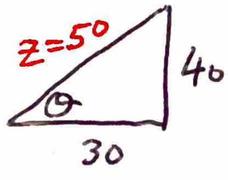
$\frac{dy}{dt} = 20$ ft/s

$y = 40$ ft

$\frac{d\theta}{dt} = \_\_\_$?

$\tan\theta = \frac{y}{30} = \frac{1}{30}(y)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{30}\left(\frac{dy}{dt}\right)$



$\sec\theta = \frac{50}{30} = \frac{5}{3}$

$\left(\frac{5}{3}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{30}(20)$

$\frac{d\theta}{dt} = \left(\frac{3}{5}\right)^2\left(\frac{1}{30}\right)(20)$

$\frac{d\theta}{dt} = \frac{9}{25} \cdot \frac{2}{3}$

$\boxed{\frac{d\theta}{dt} = \frac{6}{25} \text{ rad/sec}}$