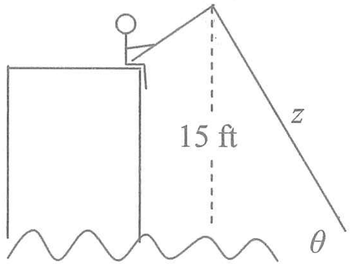
AP Calculus **Trig Related Rates Section 2-6** Class Notes

Notes: Steps:

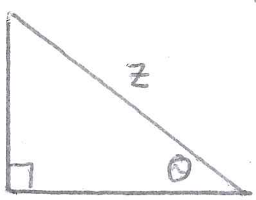
1) List known and unknown variables
2) Write trig equation relating variables
3) Differentiate with respect to time (t)
4) Use trig ratios to substitute in values and solve for variable

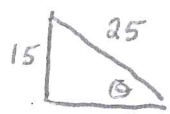
$dz/dt = -1$ ft/s

Example 1: A fish is reeled in at a rate of 1 foot per second from a point 15 feet above the water (see figure below). At what rate is the angle between the line and the water changing when there is a total of 25 feet of line out? $\to \frac{d\theta}{dt}$



unchanging → 15





$x^2 + 15^2 = 25^2 \longrightarrow 20$

$\frac{dz}{dt} = -1$ ft/s

$z = 25$ ft

Find $\frac{d\theta}{dt} =$ ____

$\sin\theta = \frac{15}{z}$

$\sin\theta = 15z^{-1}$

$\cos\theta\left(\frac{d\theta}{dt}\right) = -15z^{-2}\left(\frac{dz}{dt}\right)$

$\frac{d\theta}{dt} = \frac{-15}{z^2\cos\theta}\left(\frac{dz}{dt}\right)$

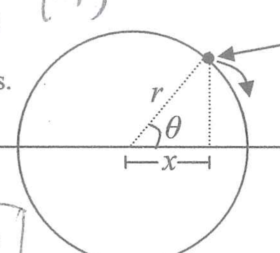
$= \frac{-15}{25^2(4/5)}(-1)$

$\cos\theta = \frac{a}{h} = \frac{20}{25} = \boxed{\frac{4}{5}}$

$= \frac{-15\cdot(-1)}{(25)(25)(4/5)}$

$= \frac{-3(-1)}{5\cdot 20} = \frac{3}{100}$ rad/s

$\boxed{\frac{d\theta}{dt} = \frac{3}{100} \text{ rad/s}}$



Ex. 2 A carousel has a radius of 20 feet and completes one rotation every 30 seconds.

* Remember 2π radians is one rotation around circle

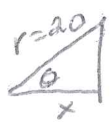
a) Determine the angular velocity of the carousel in radians per second. Call this $d\theta/dt$.

$\frac{d\theta}{dt} = \frac{\text{rotation}}{\text{second}} \quad \frac{2\pi}{30} = \frac{\pi}{15}$ rad/s $\quad \boxed{\frac{d\theta}{dt} = \frac{\pi}{15} \text{ rad/s}}$

b) If you were to ride on the carousel for 10 seconds, what angle would you be at compared to where you started?

$10 \text{ seconds} \cdot \frac{\pi \text{ rad}}{15 \text{ sec}} = \boxed{\frac{2\pi}{3} \text{ radians}} \qquad \theta = \frac{2\pi}{3}$ radians

c) Find dx/dt as a function of θ.



$\cos\theta = \frac{x}{20} \to -\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{20}\left(\frac{dx}{dt}\right) \qquad \boxed{-20\sin\theta\frac{d\theta}{dt} = \frac{dx}{dt}}$

d) Find dx/dt when t = 10 seconds.

$\frac{dx}{dt} = -20\sin\theta\left(\frac{d\theta}{dt}\right)$

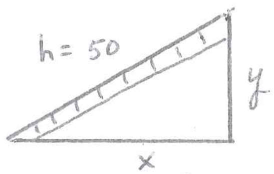
$= -20\sin\left(\frac{2\pi}{3}\right)\cdot\frac{\pi}{15}$

$(-20)\cdot\left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{\pi}{15}\right)$

$= \frac{-20\sqrt{3}\cdot\pi}{30} = \boxed{\frac{-2\sqrt{3}\pi}{3} \text{ ft/s}}$

A.P. Calculus AB **Trig Related Rates Worksheet** Class Examples

1. A ladder, 50 ft long, is being pushed against the wall at a rate of 5 ft / sec. When the bottom of the ladder is 30 ft from the wall:
   a) What is the velocity at the top of the ladder?
   b) At what rate is the area of the triangle enclosed by the ladder, wall, and floor changing?
   c) At what rate is the angle formed by the ladder and the floor changing at that time?



$x = 30 \qquad \frac{dx}{dt} = -5$ ft/s

$y = 40 \qquad \frac{dy}{dt} = \_\_\_\_$

$h = 50 \qquad \frac{dh}{dt} = 0$

$x^2 + y^2 = h^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2h\left(\frac{dh}{dt}\right)$

$2(30)(-5) + 2(40)\left(\frac{dy}{dt}\right) = 0$

$\boxed{\frac{dy}{dt} = 3.75 \text{ ft/s}}$

b) $A = \frac{1}{2}xy \qquad f'g + fg'$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right)y + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$= \frac{1}{2}(-5)(40) + \frac{1}{2}(30)(3.75)$

$= -100 + 56.25$

$= \boxed{-43.75 \text{ ft}^2/\text{s}}$

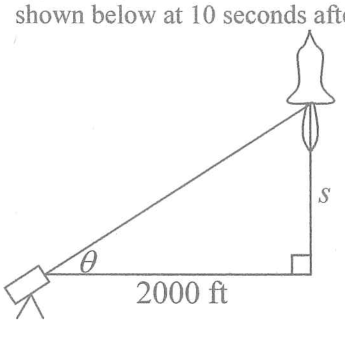
$\sin\theta = \frac{y}{50} \qquad \sin\theta = \frac{1}{50}y$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dy}{dt}\right)$

$\frac{d\theta}{dt} = \frac{(dy/dt)}{50 \cdot \cos\theta} = \frac{3.75}{50 \cdot \left(\frac{30}{50}\right)}$

$= \frac{3.75}{30} = \boxed{\frac{1}{8} \text{ rad/s}}$

2. A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to $s(t) = 50t^2$, where s is in feet and t is in seconds. Find the rate of change in the angle of elevation of the camera shown below at 10 seconds after lift-off.



$\tan\theta = \frac{s}{2000}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{2000}\left(\frac{ds}{dt}\right)$

$\frac{d\theta}{dt} = \frac{1}{2000 \cdot \sec^2\theta}\left(\frac{ds}{dt}\right)$

$= \frac{\cos^2\theta}{2000}\left(\frac{ds}{dt}\right)$

$\left(\frac{2000}{\sqrt{29{,}000{,}000}}\right)^2 \cdot \frac{1000}{2000} = \frac{4{,}000{,}000}{29{,}000{,}000}\left(\frac{1}{2}\right)$

$\frac{d\theta}{dt} = \frac{4}{29} \cdot \frac{1}{2} = \boxed{\frac{2}{29} \text{ rad/s}}$

$s(10) = 50(10)^2 = 5000$

$\frac{ds}{dt} = 100t$

$\frac{ds}{dt} = 100(10) = 1000$ ft/s

$2000^2 + 5000^2 = h^2$

$\sqrt{29{,}000{,}000} = h$

$\cos\theta = \frac{a}{h} = \frac{2000}{\sqrt{29{,}000{,}000}}$

3. A light source is located over the center of a circular table of diameter 4 feet. Find the height, h, of the light source such that the illumination, I at the perimeter of the table is maximum if $I = \frac{k(\sin\theta)}{\ldots}$ where s is the

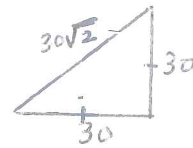
Ch. 2.6 Trig Related Rates Homework

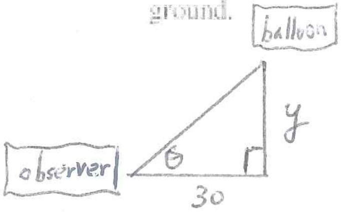
p. 156-157 #43, 45, 47

43. *Angle of Elevation* A balloon rises at a rate of 3 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

$\frac{dy}{dt}$

$\frac{d\theta}{dt}$





Find $\frac{d\theta}{dt}$ $\qquad \frac{dy}{dt} = 3\,m/s$

$y = 30\,m$

$\tan\theta = \frac{y}{30}$

$\tan\theta = \frac{1}{30}y$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{30}\cdot\frac{dy}{dt}$

$\cos\theta = \frac{a}{h} = \frac{30}{30\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{d\theta}{dt} = \frac{1}{30\cdot\sec^2\theta}\cdot\frac{dy}{dt}$

$= \frac{\cos^2\theta}{30}\cdot\frac{dy}{dt}$

$= \cos^2\theta\cdot\frac{1}{30}\cdot\frac{dy}{dt}$

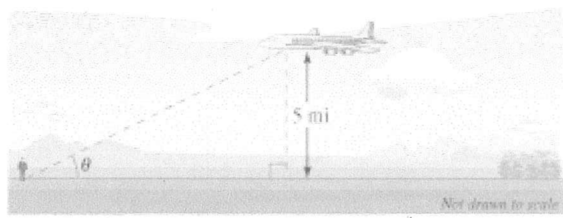
$= \left(\frac{1}{\sqrt{2}}\right)^2\cdot\frac{1}{30}\cdot(3)$

$= \frac{1}{2}\cdot\frac{1}{30}\cdot 3 = \frac{3}{60} = \frac{1}{20}$

$\boxed{\frac{d\theta}{dt} = \frac{1}{20}\,rad/s}$

45. *Angle of Elevation* An airplane flies at an altitude of 5 miles toward a point directly over an observer (see figure). The speed of the plane is 600 miles per hour. Find the rates at which the angle of elevation θ is changing when the angle is (a) $\theta = 30°$, (b) $\theta = 60°$, and (c) $\theta = 75°$.

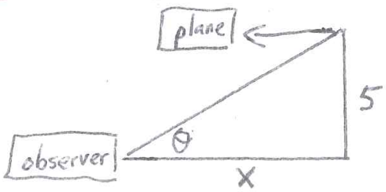
Find $\frac{d\theta}{dt}$



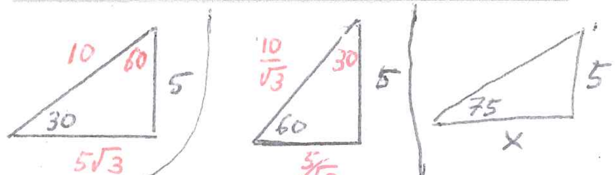
$\tan 75 = \frac{5}{x}$

$x = \frac{5}{\tan 75}$

$x = 1.3397$



$\frac{dx}{dt} = -600\,mph$



$\tan\theta = \frac{5}{x} = 5x^{-1}$

$\sec^2\theta\cdot\frac{d\theta}{dt} = -5x^{-2}\cdot\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-5}{x^2\cdot\sec^2\theta}\cdot\frac{dx}{dt}$

$\boxed{\frac{d\theta}{dt} = \frac{-5}{x^2}\cdot\cos^2\theta\cdot\frac{dx}{dt}}$

a) $\theta = 30°$, $x = 5\sqrt{3}$

$\frac{d\theta}{dt} = \frac{-5}{(5\sqrt{3})^2}\cdot\left[\cos\left(\frac{\pi}{6}\right)\right]^2\cdot\frac{dx}{dt}$

$= \frac{-5}{25\cdot 3}\cdot\left(\frac{\sqrt{3}}{2}\right)^2\cdot(-600)$

$= \frac{-5\cdot 3\cdot -600}{25\cdot \cancel{3}\cdot 4} =$

$\boxed{\frac{d\theta}{dt} = 30\,rad/hr.}$

b) $\theta = 60°$, $x = \frac{5}{\sqrt{3}}$

$\frac{d\theta}{dt} = \frac{-5}{\left(\frac{5}{\sqrt{3}}\right)^2}\left[\cos\left(\frac{\pi}{3}\right)\right]^2\cdot\frac{dx}{dt}$

$= \frac{-5}{25/3}\cdot\left(\frac{1}{2}\right)^2\cdot(-600)$

$= \frac{-5\cdot 3\cdot -600}{25\cdot 4}$

$\boxed{\frac{d\theta}{dt} = 90\,rad/hr}$

c) $\theta = 75°$, $x = 1.3397$

$\frac{d\theta}{dt} = \frac{-5}{(1.3397)^2}\left(\cos^2(75)\right)\cdot -600$

$= 111.96\,rad/hr$

$\boxed{\frac{d\theta}{dt} = 111.96\,rad/hr}$

47. *Linear vs. Angular Speed* A wheel of radius 30 centimeters revolves at a rate of 10 revolutions per second. A dot is painted at a point P on the rim of the wheel (see figure).

(a) Find dx/dt as a function of θ.

(b) Use a graphing utility to graph the function in part (a).

(c) When is the absolute value of the rate of change of x greatest? When is it least?

(d) Find dx/dt when $\theta = 30°$ and $\theta = 60°$.

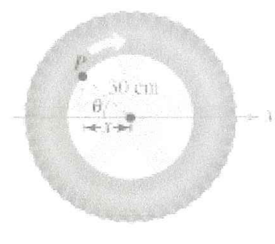
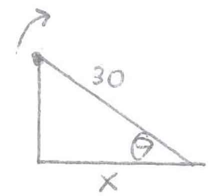


Figure for 47



$$\frac{d\theta}{dt} = \frac{10 \text{ revolution}}{\text{sec}} \cdot \frac{2\pi \text{ rad.}}{\text{revolution}} = 20\pi \text{ rad}/s$$

a) $\cos\theta = \frac{x}{30} = \frac{1}{30}x$

$$-\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{30}\left(\frac{dx}{dt}\right)$$

$$-30\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$$

$$\frac{dx}{dt} = -30\sin\theta \cdot \frac{d\theta}{dt}$$

$$= -30 \cdot \sin\theta \cdot 20\pi$$

$$\boxed{\frac{dx}{dt} = -600\pi \sin\theta}$$

c) $|-600\pi \sin\theta|$ is greatest when $\sin\theta = 1$

$$\boxed{\theta = \frac{\pi}{2} + n\pi}$$

$|-600\pi \sin\theta|$ is least when $\sin\theta = 0$

$$\boxed{\theta = \pi + n\pi}$$

d) when $\theta = 30°$

$$\frac{dx}{dt} = -600\pi \sin(\pi/6)$$

$$= -600\pi \cdot \left(\frac{1}{2}\right)$$

$$= \boxed{-300\pi \text{ cm/sec}}$$

when $\theta = 60°$

$$\frac{dx}{dt} = -600\pi \sin(\pi/3)$$

$$= -600\pi \cdot \left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{-300\pi\sqrt{3} \text{ cm/sec.}}$$