Trig Curve Sketch Notes (2021)

3) $f(x) = \sin(2x) + x$ $\quad [0, 2\pi]$

$f(0) = \sin(2 \cdot 0) + 0 = 0$

$f(2\pi) = \sin(4\pi) + 2\pi = 2\pi$

slope: $m = \dfrac{2\pi - 0}{2\pi - 0} = 1$

$f'(x) = \cos(2x) \cdot 2 + 1$

$f'(x) = 2\cos(2x) + 1$

$2\cos(2x) + 1 = 1$

$\dfrac{2\cos(2x)}{2} = \dfrac{0}{2}$

$\cos(2x) = 0$

$\cos(2x) = 0$

$\cancel{\cos^{-1}}(\cos(2x)) = \cos^{-1}(0)$

$\cos\theta = 0?$

$2x = \cos^{-1}(0)$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

Add 2π

$+\frac{4\pi}{2}$

$\frac{1}{2}\left(\cancel{2}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}\right) \cdot \frac{1}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \cancel{\frac{9\pi}{4}}, \cancel{\frac{11\pi}{4}}$

$\boxed{c = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

$\frac{8\pi}{4}$

4) $f(x) = x - 2\cos x \quad [-\pi, \pi]$

$f'(x) = 1 - 2(-\sin x) \quad \boxed{f'(x) = 1 + 2\sin x}$

$1 + 2\sin x = 0$

S | A
T | C

$2\sin x = -1$

Q3
Q4

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\frac{7\pi}{6} - \frac{12\pi}{6} = \frac{-5\pi}{6}$

$\frac{11\pi}{6} - \frac{12\pi}{6} = \frac{-\pi}{6}$

$f'(x)$: $-\pi$ (+) $-5\pi/6$ (−) $-\pi/6$ (+) π; $\frac{-\pi}{2}$, 0

Inc: $\left(-\pi, \frac{-5\pi}{6}\right) \cup \left(\frac{-\pi}{6}, \pi\right)$ b/c $f'(x) > 0$

Dec: $\left(\frac{-5\pi}{6}, \frac{-\pi}{6}\right)$ b/c $f'(x) < 0$

Rel. max $\left(\frac{-5\pi}{6}, \frac{-5\pi}{6} + \sqrt{3}\right)$ b/c $f'(x)$ changes from + to − $\boxed{-0.3}$

Rel. min $\left(-\pi/6, \frac{-\pi}{6} - \sqrt{3}\right)$ b/c $f'(x)$ changes from − to + $\boxed{-2}$

$f\left(\frac{-5\pi}{6}\right) = \frac{-5\pi}{6} - 2\cos\left(\frac{-5\pi}{6}\right)$

$= \frac{-5\pi}{6} - 2\left(\frac{-\sqrt{3}}{2}\right)$

$\sqrt{3} \approx 1.7$

$= -\frac{5\pi}{6} + \sqrt{3} \rightarrow \frac{-15}{6} + 1.7$

$-2 + 1.7 \approx -0.3$

$f\left(\frac{-\pi}{6}\right) = \frac{-\pi}{6} - 2\cos\left(\frac{-\pi}{6}\right) = \frac{-\pi}{6} - 2\left(\frac{\sqrt{3}}{2}\right) = \frac{-\pi}{6} - \sqrt{3} \approx -0.5 - 1.7$

$f'(x) = 1 + 2\sin x$

$\boxed{f''(x) = 2\cos x}$

$2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} - \frac{4\pi}{2} = \frac{-\pi}{2}$

$f''(x)$: $-\pi$ (−) $\frac{-\pi}{2}$ (+) $\pi/2$ (−) π; $\frac{-3\pi}{4}$, 0, $3\pi/4$

$f\left(\frac{-\pi}{2}\right) = \frac{-\pi}{2} - 2\cos\left(\frac{-\pi}{2}\right)$

$f(\pi/2) = \frac{\pi}{2} - 2\cos(\pi/2)$

POI at $\left(\frac{-\pi}{2}, \frac{-\pi}{2}\right)$ $\boxed{-1.5}$ and $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\boxed{1.5}$

b/c $f''(x)$ change signs

Concave up $\left(-\pi/2, \pi/2\right)$ b/c $f''(x) > 0$

Concave down $\left(-\pi, -\pi/2\right) \cup \left(\pi/2, \pi\right)$ b/c $f''(x) < 0$

sketch graph $[-\pi, \pi]$

$f(x) = x - 2\cos x$

$f(-\pi) = -\pi - 2\cos(-\pi) = -\pi + 2 \approx -3+2 \approx \boxed{-1}$

$f(\pi) = \pi - 2\cos(\pi) = \pi + 2 \approx \boxed{5}$

$f(x)$