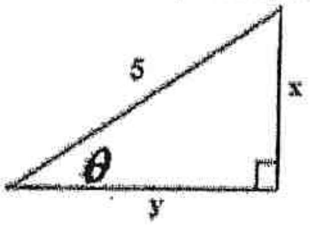
Trig Unit Test Review Session #1



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is y decreasing in units per minute when x equals 3 units?

2. A particle moves along a horizontal line so that at any time t its position is given by $x(t) = 2\pi t + \cos(2\pi t)$.

(a) Find the velocity at time t.

(b) Find the acceleration at time t.

(c) What are all values of t for $0 \le t \le 3$, for which the particle is at rest. Justify your answer.

d) When is particle moving left? Moving right? Justify with because statement

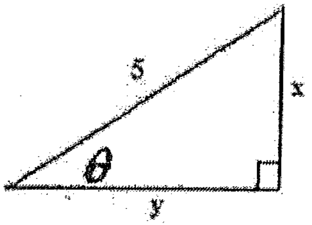
②

3) Find dy/dx $y\sec x = 12 - 3y + 5x^2$

4) MVT: $f(x) = x - 2\sin x \quad [-\pi, \pi]$

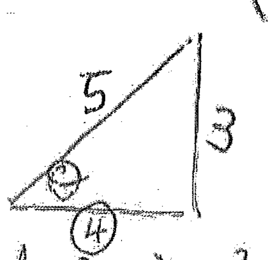
Trig Unit Test Review Session #1

1)



$\cos\theta = \frac{y}{5} = \frac{1}{5}y$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dy}{dt}\right)$



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is y decreasing in units per minute when x equals 3 units?

$\frac{d\theta}{dt} = 3$ rad/min

$\frac{dy}{dt} =$ ___

$x = 3$

$-\left(\frac{3}{5}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$-\frac{3}{5}(3) = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$-\frac{9}{5} = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$\frac{5}{1} \cdot -\frac{9}{\cancel{5}} = \frac{dy}{dt}$

$a^2 + 3^2 = 5^2$

$a^2 = 16$

$a = 4$

$\sin\theta = \frac{3}{5}$

$\boxed{\frac{dy}{dt} = -9 \text{ units/min}}$

2. A particle moves along a horizontal line so that at any time t its position is given by $x(t) = 2\pi t + \cos(2\pi t)$.

(a) Find the velocity at time t.

$v(t) = 2\pi + -\sin(2\pi t) \cdot 2\pi$

$v(t) = 2\pi - 2\pi\sin(2\pi t)$

(b) Find the acceleration at time t.

$a(t) = 0 - 2\pi\cos(2\pi t)2\pi$

$a(t) = -4\pi^2\cos(2\pi t)$

(c) What are all values of t for $0 \le t \le 3$, for which the particle is at rest. Justify your answer.

$\sin\theta = 1$

$v(t) = 0$

*set $v(t) = 0$

$2\pi - 2\pi\sin(2\pi t) = 0$

$\frac{-2\pi\sin(2\pi t)}{-2\pi} = \frac{-2\pi}{-2\pi}$

$\sin(2\pi t) = 1$

$2\pi t = \sin^{-1}(1)$ ← Add 2π $\left(+\frac{4\pi}{2}\right)$

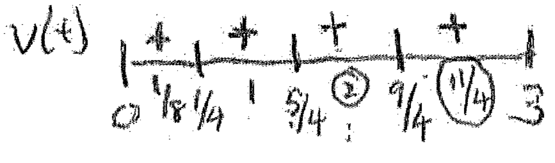
$\frac{1}{2\pi}\, 2\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2} \,\frac{1}{2\pi}$

$\boxed{t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \cancel{\frac{13}{4}} \text{ b/c } v(t) = 0}$

d) When is particle moving left? Moving right? Justify with because statement

$v(t) = 2\pi(1 - \sin 2\pi t)$

$2\pi(1 - \sin(\pi/4))$



moving right $(0, 1/4) \cup (1/4, 5/4) \cup (5/4, 9/4) \cup (9/4, 3)$ b/c $v(t) > 0$

moving left: none.

3) Find dy/dx $\overset{f}{y}\,\overset{g}{\sec x} = 12 - 3y + 5x^2$

*product Rule
*implicit

$$\overset{f'}{\left(\frac{dy}{dx}\right)}\overset{g}{\sec x} + \overset{f}{y} \cdot \overset{g'}{\sec x \tan x} = 0 - 3\left(\frac{dy}{dx}\right) + 10x$$

$$\frac{dy}{dx}(\sec x) + 3\left(\frac{dy}{dx}\right) = 10x - y\sec x\tan x$$

$$\frac{dy}{dx}(\sec x + 3) = 10x - y\sec x\tan x$$

$$\boxed{\frac{dy}{dx} = \frac{10x - y\sec x\tan x}{\sec x + 3}}$$

MVT: $f(x) = x - 2\sin x \quad [-\pi, \pi]$

4) i) $f(x)$ continuous $[-\pi, \pi]$

$f(x)$ differentiable $(-\pi, \pi)$

MVT: $f'(c) = \underbrace{\frac{f(b) - f(a)}{b - a}}_{\text{find ordered pairs first}}$

$f(-\pi) = -\pi - \cancel{2\sin(-\pi)} = -\pi$

$f(\pi) = \pi - \cancel{2\sin(\pi)} = \pi$

slope: $m = \frac{\pi - (-\pi)}{\pi - (-\pi)} = \frac{2\pi}{2\pi} = 1$

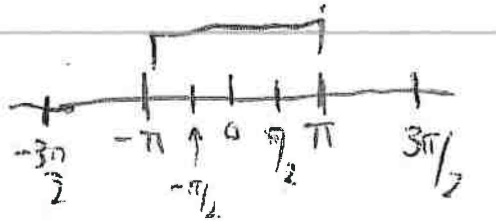
$f'(x) = 1 - 2\cos x$

$1 = 1 - 2\cos x$

$\frac{2\cos x}{2} = \frac{0}{2}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$



$$\boxed{c = -\frac{\pi}{2}, \frac{\pi}{2}}$$

WS #3 Trig Unit Review WS #3

6) $f(x) = \frac{1}{2}x - \cos x \quad [-\pi, \pi]$

$f'(x) = \frac{1}{2} - (-\sin x)$

$f'(x) = \frac{1}{2} + \sin x$

$0 = \frac{1}{2} + \sin x$

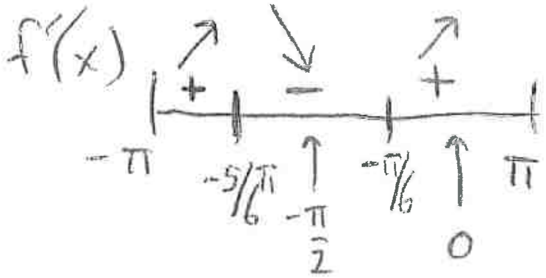
$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$-\frac{12\pi}{6} \quad -\frac{12\pi}{6}$

$x = -\frac{5\pi}{6}, -\frac{\pi}{6}$

$f'(x) = \frac{1}{2} + \sin x$



$f'(-\frac{\pi}{2}) = \frac{1}{2} + \sin(-\frac{\pi}{2})$

$f'(0) = \frac{1}{2} + \sin(0)$

S	A
T	C

$f''(x) = \cos x$

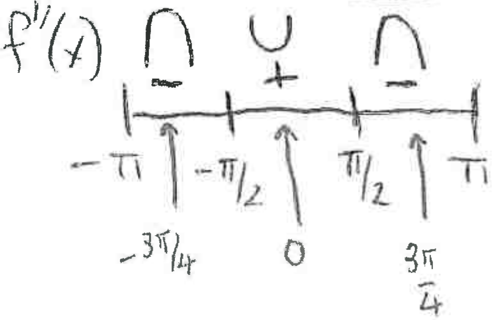
$0 = \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$

$-\frac{4\pi}{2} \quad -\frac{4\pi}{2}$

$x = \cancel{-\frac{3\pi}{2}}, -\frac{\pi}{2}$



$+\frac{8\pi}{4}$

$f''(-\frac{3\pi}{4}) = \cos(-\frac{3\pi}{4}) = \cos(\frac{5\pi}{4})$

$f''(0) = \cos(0) = 1$

$f''(\frac{3\pi}{4}) = \cos(\frac{3\pi}{4}) < 0$

⑥

6 continued | Sketch graph | $f(x) = \frac{1}{2}x - \cos x \quad [-\pi, \pi]$

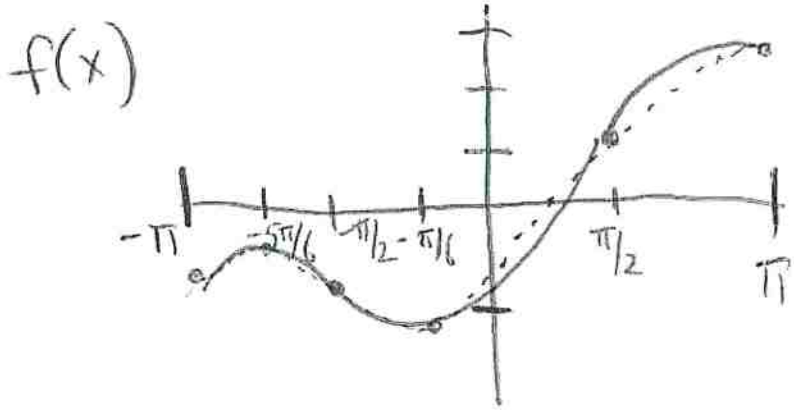
$\sqrt{3} = 1.7$
$\sqrt{2} = 1.4$

$$f(-\pi) = \frac{-\pi}{2} - \cos(-\pi) = -1.5 + 1 \approx \boxed{-0.5}$$

$$f(\pi) = \frac{\pi}{2} - \cos(\pi) = 1.5 - (-1) \approx \boxed{2.5}$$

$$f\left(\frac{-5\pi}{6}\right) = \frac{1}{2} \cdot \frac{-5\pi}{6} - \cos\left(\frac{-5\pi}{6}\right) = -1.2 + \frac{\sqrt{3}}{2} \simeq -1.2 + 0.9 \simeq \boxed{-0.3}$$

$$f(-\pi/6) = \frac{1}{2}\left(\frac{-\pi}{6}\right) - \cos(-\pi/6) = -0.2 - \frac{\sqrt{3}}{2} = -0.2 - 0.9 \simeq \boxed{-1.1}$$



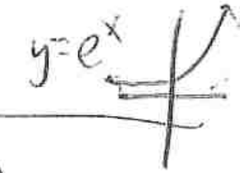
Abs max ≃ y = 2.5

Abs min ≃ y = −1.1

2) Particle motion problem
position function $x(t) = e^{\csc(t)}$ on $\left(\pi/6, 5\pi/6\right)$

a) Find $v(t)$ b) Find where particle is at rest

c) Find interval particle is moving left, moving right. Justify with b/c statement.



---

a) * $\frac{d}{dx}e^u = e^u \cdot u'$ $v(t) = e^{\csc(t)} \cdot (-\csc t)(\cot t)$

b) $0 = e^{\csc(t)} \cdot (-\csc(t))(\cot(t))$

$e^{\csc(t)} \neq 0$

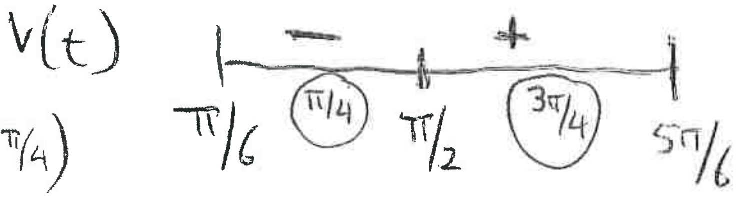
$-\csc(t) = 0$
$\frac{-1}{\sin(t)} = 0$
$\csc(t) \neq 0$

$\cot(t) = 0$
$\frac{\cos(t)}{\sin(t)} = 0$
$\cos(t) = 0$
$t = \frac{\pi}{2}, \cancel{3\pi/2}$

$$\boxed{\text{particle at rest when } t = \pi/2 \text{ b/c } v(t) = 0}$$

$v(t) = -e^{\csc(t)} \csc(t)\cot(t)$

c) $v(t)$ sign line



$v(\pi/4) = -e^{\csc(\pi/4)} \csc(\pi/4)\cot(\pi/4)$

$v(3\pi/4) = -e^{\csc(3\pi/4)} \csc(3\pi/4)\cot(3\pi/4)$

particle moving left $\left(\frac{\pi}{6}, \pi/2\right)$ b/c $v(t) < 0$

moving right $(\pi/2, 5\pi/6)$ b/c $v(t) > 0$

Help session #2 (Trig unit) Calc AB

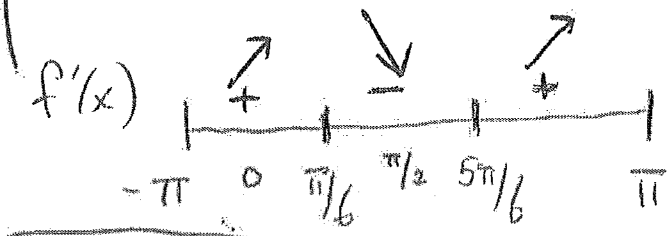
1) Curve sketch: $f(x) = x + 2\cos x$ $[-\pi, \pi]$

$\boxed{f'(x) = 1 - 2\sin x}$ | $2\sin x = 1$ $x = \pi/6, 5\pi/6,$

$-13\pi/6$ $-12\pi/6$

$0 = 1 - 2\sin x$ | $\sin x = 1/2$

~~$-11\pi/6$~~ ~~$-7\pi/6$~~



$f'(0) = 1 - 2\sin 0 =$

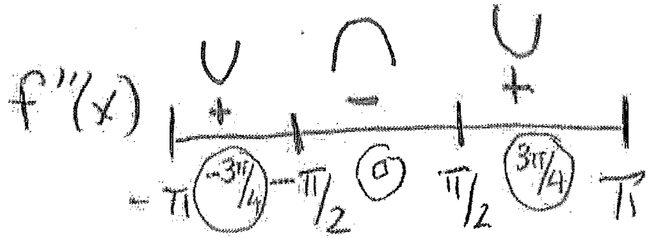
$f'(\pi/2) = 1 - 2\sin(\pi/2) =$

$\boxed{f''(x) = -2\cos x}$ | $\cos x = 0$

$x = \pi/2, \cancel{3\pi/2}, -\pi/2, \cancel{-3\pi/2}$

$-2\cos x = 0$

$\cos x = \frac{0}{-2} = 0$



$f''(-3\pi/4) = -2\cos(-3\pi/4)$

$= -2\cos(5\pi/4)$

$f''(3\pi/4) = -2\cos(3\pi/4)$

⑩

1b) $f(x) = x + 2\cos x \quad [-\pi, \pi]$

$\sqrt{3} = 1.7$
$\sqrt{2} = 1.2$

$f(-\pi) = -\pi + 2\cos(-\pi) \approx -5 \quad f(\pi) = \pi + 2\cos(\pi) = 1$

$f(\pi/6) = \frac{\pi}{6} + 2\cos(\pi/6) = 0.5 + \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right) \approx 0.5 + 1.7 \approx 2.2$

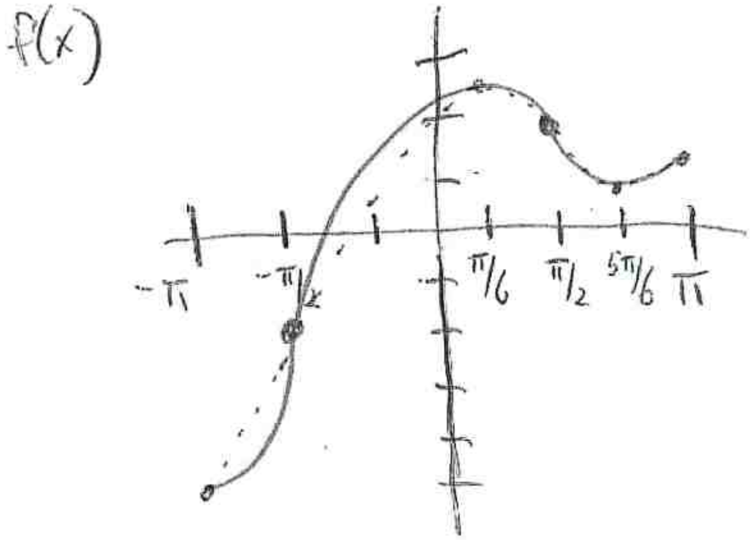
$f(5\pi/6) = \frac{5\pi}{6} + 2\cos(5\pi/6) = 2.5 - 2\left(\frac{\sqrt{3}}{2}\right) = 2.5 - 1.7 = 0.8$

$f(-\pi/2) = -\pi/2 + 2\cos(-\pi/2) = -1.5 + 0 = -1.5$

$f(\pi/2) = \pi/2 + 2\cos(\pi/2) = 1.5$

Abs max: $y = 2.2$

Abs min: $y \approx -5$



$$2)\ f(x) = \overbrace{x^2}^{f}\overbrace{\arccos(5-x^2)}^{g} - e^{x^2-3x}$$

* product Rule

$$\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}e^u = e^u \cdot u'$$

$$f'(x) = \overbrace{(2x)}^{f'}\overbrace{(\arccos(5-x^2)}^{g} + \overbrace{x^2}^{f} \cdot \overbrace{\frac{-(-2x)}{\sqrt{1-(5-x^2)^2}}}^{g'} - e^{x^2-3x} \cdot (2x-3)$$

$$\boxed{f'(x) = 2x\arccos(5-x^2) + \frac{2x^3}{\sqrt{1-(5-x^2)^2}} - (2x-3)e^{x^2-3x}}$$

$$3)\ \text{Find } \frac{dy}{dx} \quad \overbrace{y^2}^{f}\overbrace{\cot(2y)}^{g} - 3x^2 = y + 4$$

* product Rule
* implicit
* trig derivative

$$\overbrace{2y\left(\frac{dy}{dx}\right)}^{f'}\overbrace{\cot(2y)}^{g} + \overbrace{y^2}^{f} \cdot \overbrace{-\csc^2(2y)\left(2\left(\frac{dy}{dx}\right)\right)}^{g'} - 6x = 1\left(\frac{dy}{dx}\right) + 0$$

$$\frac{dy}{dx}2y\cot(2y) - 2y^2\csc^2(2y)\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 6x$$

$$\frac{dy}{dx}\left(2y\cot(2y) - 2y^2\csc^2(2y) - 1\right) = 6x$$

$$\boxed{\frac{dy}{dx} = \frac{6x}{2y\cot(2y) - 2y^2\csc^2(2y) - 1}}$$

Morning Help session (Trig Unit)

Curve sketching

1) $f(x) = x - 2\sin x$ $\quad [-\pi, \pi]$

$\boxed{f'(x) = 1 - 2\cos x}$

$0 = 1 - 2\cos x$

$2\cos x = 1$

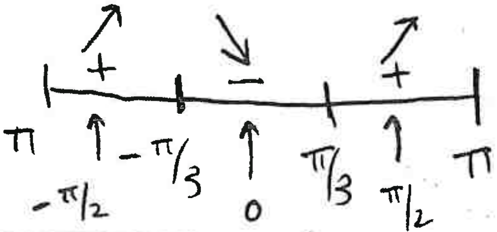
$\cos x = 1/2$

$x = \boxed{\pi/3}$ $\quad \cancel{5\pi/3}$

$-\frac{6\pi}{3} \quad -\frac{6\pi}{3}$

$\cancel{-\frac{5\pi}{3}} \quad \boxed{-\frac{\pi}{3}}$

f'(x)



$f'(-\pi/2) = 1 - 2\cos(-\pi/2)$

$f'(0) = 1 - 2\cos(0)$

$f'(\pi/2)\ 1 - 2\cos(\pi/2)$

$f''(x) = -2(-\sin x)$

$\boxed{f''(x) = 2\sin x}$

$0 = 2\sin x$

$2\sin x = 0$

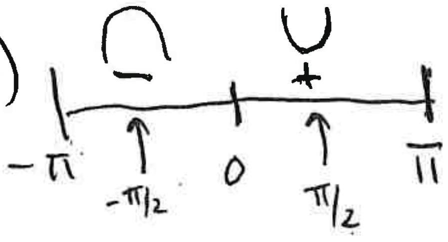
$\sin x = 0$

$x = \boxed{0}, \pi, 2\pi, -\pi, -2\pi$

$f''(-\pi/2) = 2\sin(-\pi/2) < 0$

$f''(\pi/2) = 2\sin(\pi/2) > 0$

f''(x)



1) (Continued) * sketch graph $f(x) = x - 2\sin x$

$\sqrt{3} = 1.7$

$f(-\pi) = -\pi - \cancel{2\sin(-\pi)} \approx -3$

$f(\pi) = \pi - 2\cancel{\sin}(\pi) \approx 3$

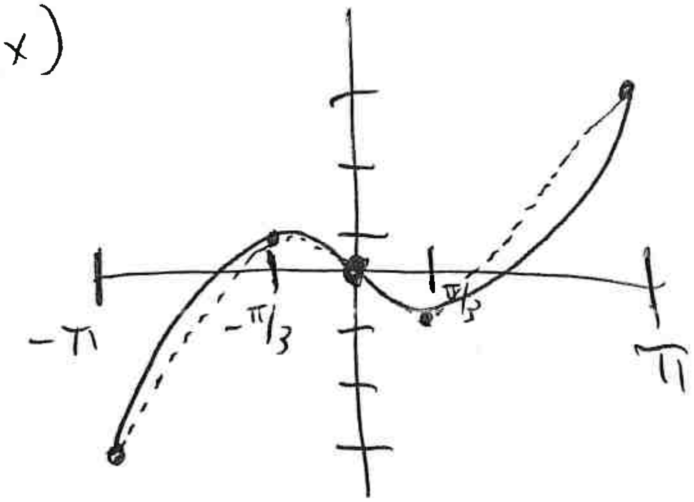
$f(-\pi/3) = -\frac{\pi}{3} - \underbrace{2\sin(-\pi/3)}_{\sin(5\pi/3)} \to -\frac{3}{3} - (2)\left(-\frac{\sqrt{3}}{2}\right) = -1 + \frac{1.7}{[illegible]}$

$-1 + 1.7 = \boxed{0.7}$

$f(\pi/3) = \frac{\pi}{3} - 2\sin(\pi/3) \to \frac{3}{3} - \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$ $1 - 1.7 = \boxed{-0.7}$

$f(0) = 0 - 2\sin(0) = 0$

$f(x)$



3) $\overbrace{y^2}^{f}\overbrace{\sec(2x)}^{g} - e^{3\pi x} = \arctan(3y) - 4\pi$ Find $\frac{dy}{dx}$

$$\overbrace{2y\left(\frac{dy}{dx}\right)}^{f'}\overbrace{\sec(2x)}^{g} + \overbrace{y^2}^{f}\cdot\overbrace{\sec(2x)\tan(2x)\cdot 2}^{g'} - e^{3\pi x}\cdot(3\pi) = \frac{3\left(\frac{dy}{dx}\right)}{1+(3y)^2} - 0$$

* implicit

* product Rule

* Arctrig

$$2y\sec(2x)\frac{dy}{dx} - \frac{3}{1+9y^2}\left(\frac{dy}{dx}\right) = 3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)$$

$$\frac{dy}{dx}\left[2y\sec(2x) - \frac{3}{1+9y^2}\right] = 3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)$$

$$\boxed{\frac{dy}{dx} = \frac{3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)}{2y\sec(2x) - \frac{3}{1+9y^2}}}$$