AP Calculus AB | **Trig Quiz #2 (Chapter 2)** | **Name:** Key A **Period:** 30 pts

No Calculators. | Leave all answers reasonably simplified | Each question is worth 5 points.

1. If $y = 4sin^3(2x^3)$, find $\frac{dy}{dx}$

$y = 4\left[\sin(2x^3)\right]^3$

$y' = 12\left[\sin(2x^3)\right]^2 \cdot \cos(2x^3) \cdot 6x^2$

$\boxed{y' = 72x^2\left[\sin(2x^3)\right]^2 \cos(2x^3)}$

2. If $y = \frac{\tan(3x)}{\cot(2x)}$ find $\frac{dy}{dx}$.

$y' = \frac{\sec^2(3x) \cdot 3\cot(2x) - \tan(3x)(-\csc^2(2x)) \cdot 2}{[\cot(2x)]^2}$

$\boxed{y' = \frac{3\sec^2(3x)\cot(2x) + 2\tan(3x)\csc^2(2x)}{\cot^2(2x)}}$

3. If $\csc(y) = 3xtan(2x)^3$ find $\frac{dy}{dx}$.

$3x\tan(8x^3)$

$-\csc y \cot y \cdot \frac{dy}{dx} = \underbrace{3}_{f'}\,\underbrace{\tan(8x^3)}_{g} + \underbrace{3x}_{f} \cdot \underbrace{\sec^2(8x^3) \cdot 24x^2}_{g'}$

$\frac{dy}{dx} = \frac{3\tan(8x^3) + 72x^3\sec^2(8x^3)}{-\csc y \cot y}$

4. $y = \frac{3}{2}\cot(5\theta)$ find $\frac{dy}{d\theta}$.

$y' = \frac{3}{2}(-\csc^2(5\theta)) \cdot 5$

$\boxed{y' = \frac{-15}{2}\csc^2(5\theta)}$

5. Find the equation of the tangent line to the curve $y = \tan x$ at $x = \frac{\pi}{6}$

$$y' = \sec^2 x$$

$$y' = [\sec x]^2$$

$$y'(\pi/6) = [\sec(\pi/6)]^2$$

$$= \left[\frac{1}{\frac{\sqrt{3}}{2}}\right]^2 = \left[\frac{2}{\sqrt{3}}\right]^2$$

$$y'(\pi/6) = \frac{4}{3}$$

point: $y(\pi/6) = \tan(\pi/6) = \frac{\sqrt{3}}{3}$

point: $(\pi/6, \sqrt{3}/3)$ slope: $m = 4/3$

$$\boxed{y - \frac{\sqrt{3}}{3} = \frac{4}{3}(x - \pi/6)}$$

6. The position of a particle moving along the x-axis is given by $x(t) = \cot(2t)$ find the acceleration of the particle at $t = \pi/6$

$$x'(t) = -\csc^2(2t) \cdot 2$$

$$v(t) = -2[\csc(2t)]^2$$

$$a(t) = -4[\csc(2t)] \cdot -\csc(2t)\cot(2t) \cdot 2$$

$$a(t) = 8[\csc(2t)]^2 \cot(2t)$$

$$a(\pi/6) = 8[\csc(\pi/3)]^2 \cot(\pi/3)$$

$$a(\pi/6) = 8\left[\frac{2}{\sqrt{3}}\right]^2 \left[\frac{1}{\sqrt{3}}\right]$$

$$= 8\left[\frac{4}{3}\right]\left[\frac{1}{\sqrt{3}}\right]$$

$$\boxed{a(\pi/6) = \frac{32}{3\sqrt{3}} \text{ or } \frac{32\sqrt{3}}{9}}$$

AP Calculus AB No Calculators.

Trig Quiz #2 (Chapter 2) Leave all answers reasonably simplified

Name: Key B **Period:** 30 pts
Each question is worth 5 points.

1. If $y = sin^4(3x^2)$ find $\frac{dy}{dx}$.

$y = [\sin(3x^2)]^4$

$y' = 4[\sin(3x^2)]^3 \cdot \cos(3x^2) \cdot 6x$

$\boxed{y' = 24x \sin^3(3x^2)\cos(3x^2)}$

2. If $y = \frac{\sec(3x)}{\cot(2x)}$ find $\frac{dy}{dx}$.

$y' = \frac{\sec(3x)\tan(3x) \cdot 3\cot(2x) - \sec(3x) \cdot -\csc^2(2x)2}{\cot^2(2x)}$

$y' = \frac{\sec(3x)[3\tan(3x)\cot(2x) + 2\csc^2(2x)]}{\cot^2(2x)}$

$3x \cdot \tan(16x^4)$

3. If $\csc(y) = 3xtan(2x)^4$ find $\frac{dy}{dx}$.

$-\csc y \cot y \left(\frac{dy}{dx}\right) = 3 \cdot \tan(16x^4) + 3x \cdot \sec^2(16x^4) \cdot 64x^3$

$\boxed{\frac{dy}{dx} = \frac{3\tan(16x^4) + 192x^4\sec^2(16x^4)}{-\csc y \cot y}}$

4. $y = \frac{2}{5}\csc(2\theta)$ find $\frac{dy}{d\theta}$.

$\frac{dy}{d\theta} = -\frac{2}{5}\csc(2\theta)\cot(2\theta) \cdot 2$

$\boxed{\frac{dy}{d\theta} = -\frac{4}{5}\csc(2\theta)\cot(2\theta)}$

5. Find the equation of the tangent line to the curve $y = \sec x$ at $x = \frac{\pi}{3}$

$y' = \sec x \tan x$

$y'(\pi/3) = \sec(\pi/3)\tan(\pi/3)$

$= \left(\frac{1}{1/2}\right)\cdot\left(\frac{\sqrt{3}}{1}\right) = 2\sqrt{3}$

$=$

$y(\pi/3) = \sec(\pi/3) = 2$

point: $(\pi/3, 2)$

slope: $m = 2\sqrt{3}$

$\boxed{y - 2 = 2\sqrt{3}(x - \pi/3)}$

6. The position of a particle moving along the x-axis is given by $x(t) = \tan 3t$ find the acceleration of the particle at $t = \frac{\pi}{4}$

$v(t) = \sec^2(3t)\cdot 3 = 3[\sec(3t)]^2$

$a(t) = 6[\sec(3t)]\cdot\sec(3t)\tan(3t)\cdot 3$

$a(t) = 18[\sec(3t)]^2\tan(3t)$

$a(\pi/4) = 18\left[\sec(3\pi/4)\right]^2\tan(3\pi/4)$

$= 18\left[-\frac{2}{\sqrt{2}}\right]^2[-1]$

$= -18\cdot\frac{4}{2} = -18\cdot 2 = -36$

$\boxed{a(\pi/4) = -36}$

AP Calculus AB | Trig Quiz #2 (Chapter 2) | Name: Key C | Period: | 30pts

No Calculators. | Leave all answers reasonably simplified | Each question is worth 5 points.

1. If $y = 3cos^2(3x^3)$, find $\frac{dy}{dx}$

$y = 3\left[\cos(3x^3)\right]^2$

$y' = 6\left[\cos(3x^3)\right] \cdot -\sin(3x^3) \cdot 9x^2$

$y' = -54\cos(3x^3)\sin(3x^3)$

2. If $y = \frac{\tan(2x)}{\csc(3x)}$ find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{\sec^2(2x) \cdot 2 \cdot \csc(3x) - \tan(2x) \cdot -\csc(3x)\cot(3x) \cdot 3}{\csc^2(3x)}$

$\frac{dy}{dx} = \frac{2\sec^2(2x)\csc(3x) + 3\tan(2x)\csc(3x)\cot(3x)}{\csc^2(3x)}$

$\frac{dy}{dx} = \frac{\cancel{\csc(3x)}\left[2\sec^2(2x) + 3\tan(2x)\cot(3x)\right]}{\csc^{\cancel{2}}(3x)}$

$2x \cdot \sin(9x^2)$

3. If $\sec(y) = 2xsin(3x)^2$ find $\frac{dy}{dx}$.

$\sec y \tan y \cdot \frac{dy}{dx} = 2\sin(9x^2) + 2x\cos(9x^2)18x$

$\frac{dy}{dx} = \frac{2\sin(9x^2) + 36x^2\cos(9x^2)}{\sec y \tan y}$

4. $y = \frac{1}{4}\sec(3\theta)$ find $\frac{dy}{d\theta}$.

$\frac{dy}{d\theta} = \frac{1}{4}\sec(3\theta)\tan(3\theta) \cdot 3$

$\frac{dy}{d\theta} = \frac{3}{4}\sec(3\theta)\tan(3\theta)$

5. Find the equation of the tangent line to the curve $y = \cot x$ at $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = -\csc^2 x = -[\csc x]^2$$

$$y'(\pi/4) = -[\csc(\pi/4)]^2$$

$$= -\left[\frac{2}{\sqrt{2}}\right]^2 = \frac{-4}{2} = -2$$

$$y(\pi/4) = \cot(\pi/4) = 1$$

point: $(\pi/4, 1)$

slope: $m = -2$

$$y - 1 = -2(x - \pi/4)$$

6. The position of a particle moving along the x-axis is given by $x(t) = \tan 2t$ find the acceleration of the particle at $t = \frac{\pi}{6}$

$$v(t) = \sec^2(2t) \cdot 2$$

$$v(t) = 2[\sec(2t)]^2$$

$$a(t) = 4[\sec(2t)] \cdot \sec(2t)\tan(2t) \cdot 2$$

$$a(t) = 8[\sec(2t)]^2 \tan(2t)$$

$$a(\pi/6) = 8[\sec \pi/3]^2 \tan(\pi/3)$$

$$a(\pi/6) = 8[2]^2\left[\frac{\sqrt{3}}{1}\right]$$

$$a(\pi/6) = 32\sqrt{3}$$

11/21/13

30 pts.

A.P. Calculus AB **Trig Quiz #3** Name Answer Key A

NO CALCULATORS! All answers must be justified!

5 1. Determine whether or not the mean value theorem applies to the function, $f(x) = \cos x + x$, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. If so, find the value(s) of c as defined in the theorem. $c = 0$

$f(x)$ is cont on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ✓

$f'(x) = -\sin x + 1$

$f(x)$ is diff on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ✓

$$\text{S.O.S.} = \frac{0 + \frac{\pi}{2} - \left(0 - \frac{\pi}{2}\right)}{\frac{\pi}{2} - -\frac{\pi}{2}} = \frac{\pi}{\pi} = 1$$

$-\sin x + 1 = 1$

$\sin x = 0$

$x = 0$

5 2. Determine whether or not Rolle's theorem applies to the function $f(x) = \csc x$ on $[0, 2\pi]$. If so, find the value(s) of c as defined in the theorem.

does not apply b/c $f(x)$ is not continuous on $[0, 2\pi]$

3. Given the function $y = 2x + \cot x$ on the interval $(0, \pi)$, find:

4 a) the intervals of direction: increasing: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ b/c $y' > 0$

decreasing: $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ b/c $y' < 0$

$y' = 2 - \csc^2 x$

$\csc^2 x = 2$

$\csc x = \pm\sqrt{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$-$ $+$ $-$

$0 \searrow \frac{\pi}{4} \nearrow \frac{3\pi}{4} \searrow \pi$

4 3. (continued) Given the function $y = 2x + \cot x$ on the interval $(0, \pi)$, find:

b) the relative extrema

min: $\left(\frac{\pi}{4}, \frac{\pi}{2}+1\right)$ b/c y' chgs from $(-)$ to $(+)$

max: $\left(\frac{3\pi}{4}, \frac{3\pi}{2}-1\right)$ b/c y' chgs from $(+)$ to $(-)$

4 c) the point(s) of inflection: $\left(\frac{\pi}{2}, \pi\right)$ b/c y'' chgs sign

$$y'' = -2\csc x(-\csc x \cot x)$$
$$= 2\csc^2 x \cot x$$

$\cos x = 0$ $\quad$ $\sin x = 0$

$x = \frac{\pi}{2}$ $\quad$ $x = 0, \pi$

$+ \quad -$

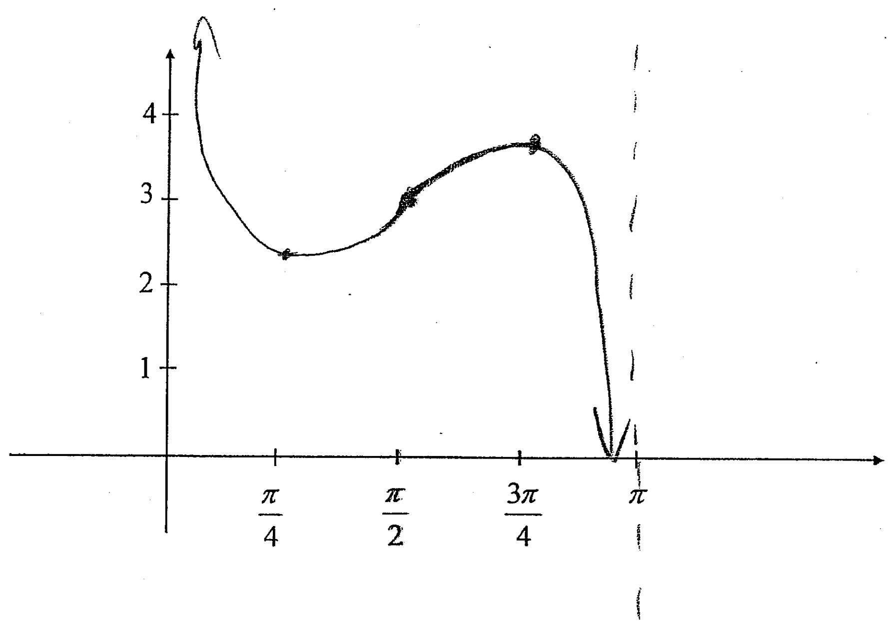
$0 \quad \frac{\pi}{2} \quad \pi$

4 d) concavity intervals

up: $\left(0, \frac{\pi}{2}\right)$ b/c $y'' > 0$

down: $\left(\frac{\pi}{2}, \pi\right)$ b/c $y'' < 0$

4 e) Sketch the graph based on your answers to parts a – d.



A.P. Calculus AB **Trig Quiz #3** Name: Answer Key B

NO CALCULATORS! All answers must be justified!

5 1. Determine whether or not the mean value theorem applies to the function, $f(x) = \sin x + x$, on $[0, \pi]$. If so, find the value(s) of c as defined in the theorem. $c = \frac{\pi}{2}$

$f(x)$ is cont on $[0, \pi]$ ✓

$f'(x) = \cos x + 1$

$f(x)$ is diff on $(0, \pi)$ ✓

$\text{S.O.S.} = \dfrac{0 + \pi - 0}{\pi} = 1$

$\cos x + 1 = 1$

$\cos x = 0$

$x = \frac{\pi}{2}$

5 2. Determine whether or not Rolle's theorem applies to the function $f(x) = \sec x$ on $[0, \pi]$. If so, find the value(s) of c as defined in the theorem.

does not apply b/c $f(x)$ is not cont. on $[0, \pi]$

3. Given the function $y = 2(x + 1) - \tan x$ on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find:

4 a) the intervals of direction: increasing: $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ b/c $y' > 0$

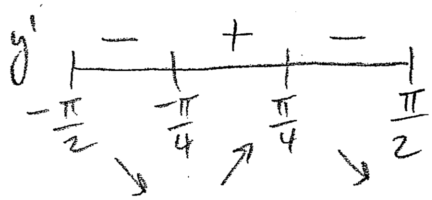
decreasing: $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ b/c $y' < 0$

$y' = 2 - \sec^2 x$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$x = -\frac{\pi}{4}, \frac{\pi}{4}$



3. (continued) Given the function $y = 2(x+1) - \tan x$ on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find:

4 b) the relative extrema

min: $\left(\frac{-\pi}{4}, \frac{-\pi}{2}+3\right)$ b/c y' chgs from $(-)$ to $(+)$

max: $\left(\frac{\pi}{4}, \frac{\pi}{2}+1\right)$ b/c y' chgs from $(+)$ to $(-)$

4 c) the point(s) of inflection: $(0, 2)$ b/c y'' chgs sign

$y'' = -2\sec x(\sec x \tan x)$

$= -2\sec^2 x \tan x$

$\sin x = 0 \quad \cos x = 0$

$x = 0 \quad x = -\frac{\pi}{2}, \frac{\pi}{2}$

$+ \quad -$

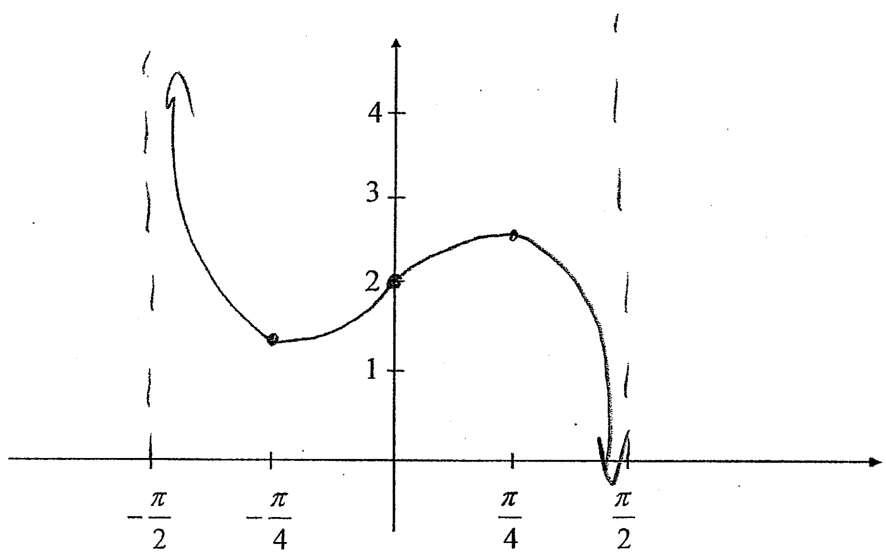
$-\frac{\pi}{2} \quad 0 \quad \frac{\pi}{2}$

4 d) concavity intervals

up: $\left(-\frac{\pi}{2}, 0\right)$ b/c $y'' > 0$

down: $\left(0, \frac{\pi}{2}\right)$ b/c $y'' < 0$

4 e) Sketch the graph based on your answers to parts a – d.



A.P. Calculus AB **Trig Quiz #3** Name: Answer Key C

NO CALCULATORS! All answers must be justified!

5 1. Determine whether or not the mean value theorem applies to the function, $f(x) = \sec x$, on $[0, \pi]$. If so, find the value(s) of c as defined in the theorem.

does not apply b/c $f(x)$ is not cont on $[0, \pi]$

5 2. Determine whether or not Rolle's theorem applies to the function $f(x) = \sin x$ on $[0, \pi]$. If so, find the value(s) of c as defined in the theorem.

$c = \frac{\pi}{2}$

$f(x)$ is cont on $[0, \pi]$ ✓

$f'(x) = \cos x$

$f(x)$ is diff on $(0, \pi)$ ✓

$\cos x = 0$

$x = \frac{\pi}{2}$

3. Given the function $y = \tan x - 2x$ on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find:

4 a) the intervals of direction: increasing: $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ b/c $y' > 0$

decreasing: $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ b/c $y' < 0$

$y' = \sec^2 x - 2$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$x = -\frac{\pi}{4}, \frac{\pi}{4}$

y': + on $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$, − on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$, + on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$-\frac{\pi}{2}$ $\quad -\frac{\pi}{4}$ $\quad \frac{\pi}{4}$ $\quad \frac{\pi}{2}$

3. (continued) Given the function $y = \tan x - 2x$ on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, find:

4 b) the relative extrema

min: $\left(\frac{\pi}{4}, 1-\frac{\pi}{2}\right)$ b/c y' chgs from $(-)$ to $(+)$

max: $\left(\frac{-\pi}{4}, -1+\frac{\pi}{2}\right)$ b/c y' chgs from $(+)$ to $(-)$

4 c) the point(s) of inflection: $(0,0)$ b/c y'' chgs sign

$$y'' = 2\sec x(\sec x \tan x)$$
$$= 2\sec^2 x \tan x$$

$\sin x = 0$ $\quad \cos x = 0$

$x = 0$ $\quad x = \frac{-\pi}{2}, \frac{\pi}{2}$

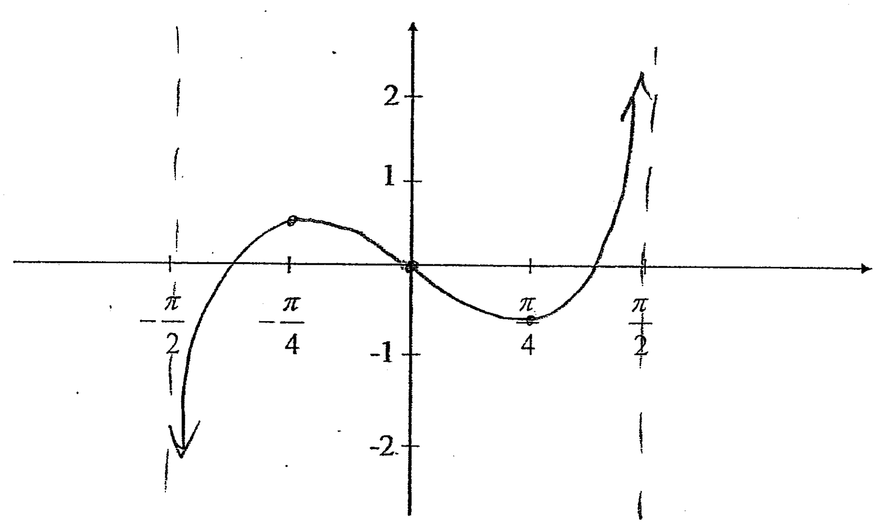
y'': $-\frac{\pi}{2}$ $\;-\;$ 0 $\;+\;$ $\frac{\pi}{2}$

4 d) concavity intervals

up: $\left(0, \frac{\pi}{2}\right)$ b/c $y'' > 0$

down: $\left(-\frac{\pi}{2}, 0\right)$ b/c $y'' < 0$

4 e) Sketch the graph based on your answers to parts a – d.



A.P. Calculus AB **Trig Quiz #3**

NO CALCULATORS! All answers must be justified!

Name: Answer Key D

5 1. Determine whether or not the mean value theorem applies to the function, $f(x) = \tan x$, on $[0, \pi]$. If so, find the value(s) of c as defined in the theorem.

does not apply b/c $f(x)$ is not cont on $[0, \pi]$

5 2. Determine whether or not Rolle's theorem applies to the function $f(x) = \cos x$ on $[0, 2\pi]$. If so, find the value(s) of c as defined in the theorem.

$c = \pi$

$f(x)$ is cont on $[0, 2\pi]$ ✓

$f'(x) = -\sin x$

$f(x)$ is diff on $(0, 2\pi)$ ✓

$-\sin x = 0$

$x = 0, \pi, 2\pi$

3. Given the function $y = \sin x + \cos x$ on the interval $(-\pi, \pi)$, find:

4 a) the intervals of direction: increasing: $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ b/c $y' > 0$

decreasing: $\left(-\pi, -\frac{3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right)$ b/c $y' < 0$

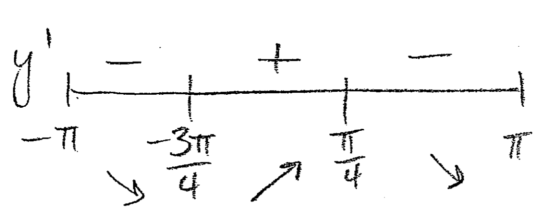
$y' = \cos x - \sin x$

$\cos x - \sin x = 0$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, -\frac{3\pi}{4}$



3. (continued) Given the function $y = \sin x + \cos x$ on the interval $(-\pi, \pi)$, find:

4 b) the relative extrema

min: $\left(-\frac{3\pi}{4}, -\sqrt{2}\right)$ b/c y' chgs from (−) to (+)

max: $\left(\frac{\pi}{4}, \sqrt{2}\right)$ b/c y' chgs from (+) to (−)

4 c) the point(s) of inflection: $\left(-\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, 0\right)$ b/c y'' chgs sign

$y'' = -\sin x - \cos x$

$-\sin x - \cos x = 0$

$\sin x = -\cos x$

$\tan x = -1$

$x = -\frac{\pi}{4}, \frac{3\pi}{4}$

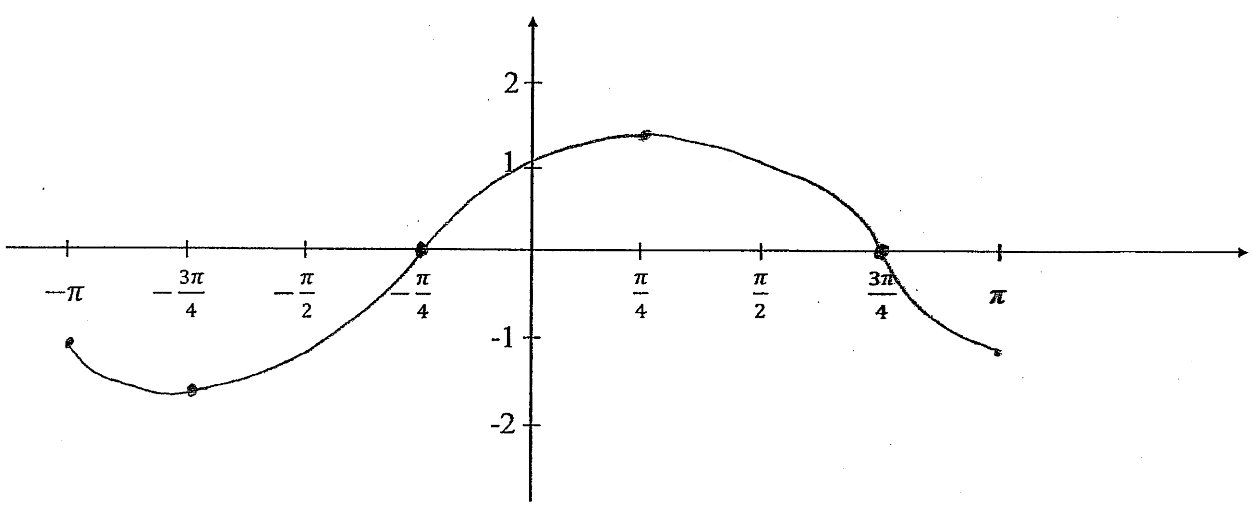
y'': + on $(-\pi, -\frac{\pi}{4})$, − on $(-\frac{\pi}{4}, \frac{3\pi}{4})$, + on $(\frac{3\pi}{4}, \pi)$

4 d) concavity intervals

up: $\left(-\pi, -\frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ b/c $y'' > 0$

down: $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ b/c $y'' < 0$

4 e) Sketch the graph based on your answers to parts a – d.



A.P. Calculus AB **Trig Quiz #3** Answer Key E
NO CALCULATORS! All answers must be justified! Name

5 1. Determine whether or not the mean value theorem applies to the function, $f(x) = \cot x$, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. If so, find the value(s) of c as defined in the theorem.

does not apply b/c f(x) is not cont on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

5 2. Determine whether or not Rolle's theorem applies to the function $f(x) = \cos x$ on $[0, 2\pi]$. If so, find the value(s) of c as defined in the theorem.

$c = \pi$

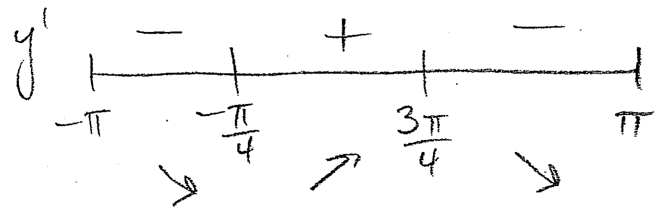
f(x) is cont on $[0, 2\pi]$ ✓
$f'(x) = -\sin x$
f(x) is diff on $(0, 2\pi)$ ✓
$-\sin x = 0$
$x = 0, \pi, 2\pi$

3. Given the function $y = \sin x - \cos x$ on the interval $(-\pi, \pi)$, find:

4 a) the intervals of direction: increasing: $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ b/c $y' > 0$

decreasing: $\left(-\pi, -\frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ b/c $y' < 0$

$y' = \cos x + \sin x$
$\cos x = -\sin x$
$\tan x = -1$
$x = -\frac{\pi}{4}, \frac{3\pi}{4}$



3. (continued) Given the function $y = \sin x - \cos x$ on the interval $(-\pi, \pi)$, find:

4 b) the relative extrema

min: $\left(-\frac{\pi}{4}, -\sqrt{2}\right)$ b/c y' chgs from (−) to (+)

max: $\left(\frac{3\pi}{4}, \sqrt{2}\right)$ b/c y' chgs from (+) to (−)

4 c) the point(s) of inflection: $\left(-\frac{3\pi}{4}, 0\right), \left(\frac{\pi}{4}, 0\right)$ b/c y'' chgs sign

$y'' = -\sin x + \cos x$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, -\frac{3\pi}{4}$

$-\pi$	−	$-\frac{3\pi}{4}$	+	$\frac{\pi}{4}$	−	π

4 d) concavity intervals

up: $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ b/c $y'' > 0$

down: $\left(-\pi, -\frac{3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right)$ b/c $y'' < 0$

4 e) Sketch the graph based on your answers to parts a – d.