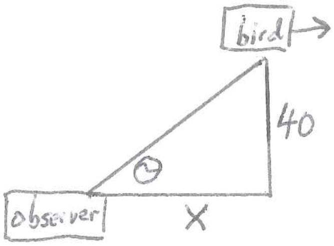
AP Calculus AB Trig Unit Test Review WS #3 Name______________________

1. Suppose a bird is flying horizontally 40 ft above your head at 20 ft/sec. How fast is the angle of elevation changing when your horizontal distance from the bird is 30 ft?



$x = 30\text{ ft}$ $\quad \frac{dx}{dt} = 20\text{ ft/s}$

$y = 40\text{ ft}$

$z = 50\text{ ft}$

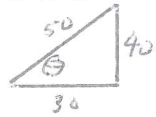
$\tan\theta = \frac{40}{x}$

$\tan\theta = 40x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -40x^{-2}\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt} = \frac{-40}{x^2\sec^2\theta}\cdot\frac{dx}{dt}$

$= \frac{-40}{x^2}\cdot\cos^2\theta\cdot\frac{dx}{dt}$



$\cos\theta = \frac{30}{50} = \frac{3}{5}$

$\frac{d\theta}{dt} = \frac{-40}{30^2}\cdot\left(\frac{3}{5}\right)^2\cdot 20 = \frac{-40\cdot 9\cdot 20}{30\cdot 30\cdot 25}$

$\boxed{\frac{d\theta}{dt} = -8/25\text{ rad/sec}}$

2. Find dy/dx for the equation $xe^{\sin(2y)} + 4x = \ln y$

*product rule

$1e^{\sin(2y)} + xe^{\sin(2y)}\cdot 2\cos(2y)\cdot\frac{dy}{dx} + 4 = \frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}\left[2xe^{\sin(2y)}\cos(2y) - \frac{1}{y}\right] = -4 - e^{\sin 2y}$

$\frac{dy}{dx} = \frac{-4 - e^{\sin 2y}}{2xe^{\sin 2y}\cos 2y - \frac{1}{y}}\cdot\frac{(y)}{(y)}$

$= \boxed{\frac{-4y - ye^{\sin 2y}}{2xye^{\sin 2y} - 1}}$

3. Find the equation of the tangent line to function $f(x) = x\arccos(x^2)$ to the graph at the point $(0, 2\pi)$ in the given domain $[0, \pi]$

*product rule $f'g + fg'$ $\quad$ * $\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

$f'(x) = 1\arccos(x^2) + x\cdot\left(\frac{-2x}{\sqrt{1-x^4}}\right)$

$f'(0) = \arccos(0^2) + 0\left(\frac{-2(0)}{\sqrt{1-0^2}}\right)$

$= \arccos(0)$

$= \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$

point: $(0, 2\pi)$

slope: $m = \pi/2$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2\pi = \frac{\pi}{2}(x - 0)}$

~~Find dy/dx for ln(xy) + e~~

$\tan(\arctan(x+y)) = \tan(y^2 + \pi/4)$

$x + y = \tan(y^2 + \pi/4)$

$1 + \frac{dy}{dx} = \sec^2(y^2 + \pi/4)\cdot 2y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = \sec^2(y^2 + \pi/4)\cdot 2y\left(\frac{dy}{dx}\right) - 1$

$\frac{dy}{dx}_{(1,0)} = \sec^2(\pi/4)\cdot 2(0)\left(\frac{dy}{dx}\right) - 1$

$= 0 - 1 = -1$

4. Find derivative at the point (1,0) for $\arctan(x + y) = y^2 + \frac{\pi}{4}$

find equation of tangent line

point: $(1, 0)$

slope: $m = -1$

$\boxed{y - 0 = -1(x - 1)}$

5. Sketch the graph of function f(x) = sin(2x) on interval [0, π]. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

$f'(x) = 2\cos(2x)$

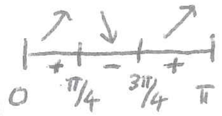
$0 = 2\cos(2x)$

$\cos(2x) = 0$

$(2x) = \cos^{-1}(0)$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \cancel{\frac{5\pi}{4}}, \cancel{\frac{7\pi}{4}}$



$f(\pi/4) = \sin(\pi/2) = 1$

$f(3\pi/4) = \sin(3\pi/2) = -1$

$f''(x) = -4\sin(2x)$

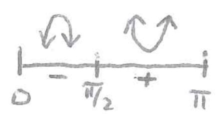
$0 = -4\sin(2x)$

$\sin(2x) = 0$

$(2x) = \sin^{-1}(0)$

$(2x) = 0, \pi, 2\pi, 3\pi$

$x = 0, \pi/2, \pi, \cancel{3\pi/2}$



$f(\pi/2) = \sin(2(\pi/2)) = \sin(\pi) = 0$

a) the relative extrema min: $(3\pi/4, -1)$ b/c $f'(x)$ changes from $-$ to $+$ max: $(\pi/4, 1)$ b/c $f'(x)$ changes from $+$ to $-$

b) the point(s) of inflection: $(\pi/2, 0)$ b/c $f''(x)$ change signs

c) concavity intervals up: $(\pi/2, \pi)$ b/c $f''(x) > 0$

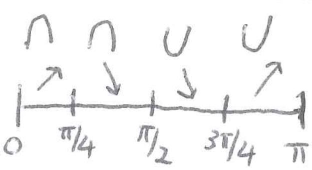
down: $(0, \pi/2)$ b/c $f''(x) < 0$

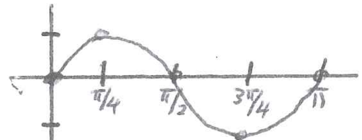
d) Absolute extrema: Absolute min: -1 at $x = 3\pi/4$ Absolute max: 1 at $x = \pi/4$

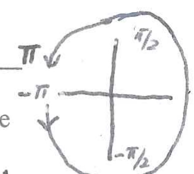
d) Sketch the graph

$f(0) = 0$

$f(\pi) = 0$







6. Sketch the graph of function f(x) = $\frac{1}{2}x - \cos x$ on interval [$-\pi$, π]. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

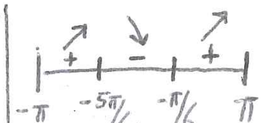
$f'(x) = \frac{1}{2} + \sin x$

$0 = \frac{1}{2} + \sin x$

$\sin x = -1/2$

$x = 7\pi/6, 11\pi/6$

$\boxed{-\pi/6, -5\pi/6}$



$f(-\frac{5\pi}{6}) = \frac{1}{2}(\frac{-5\pi}{6}) - \cos(\frac{-5\pi}{6})$

$= \frac{-5\pi}{12} - (\frac{-\sqrt{3}}{2}) \approx -1.25 + 0.75$

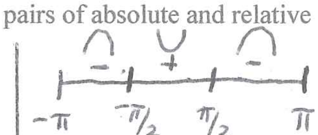
≈ -0.5

$f(-\pi/6) = \frac{1}{2}(-\pi/6) - \cos(-\pi/6)$

$= -\frac{\pi}{12} - \frac{\sqrt{3}}{2} \approx \frac{-1}{4} - \frac{3}{4} \approx -1$

$f''(x) = \cos x$ $\quad \cos x = 0$

$x = \pi/2, -\pi/2$



$f(-\pi/2) = -\pi/4 - \cos(-\pi/2) = -\pi/4$

$f(\pi/2) = \pi/4 - \cos(\pi/2) = \pi/4$

$f(-\pi) = \frac{-\pi}{2} - \cos(-\pi) = \frac{-\pi}{2} + 1$

$f(\pi) = \pi/2 - \cos(\pi) = \pi/2 + 1$

a) the relative extrema min: $(-\pi/6, -\frac{\pi}{12} - \frac{\sqrt{3}}{2})$ max: $(-5\pi/6, -5\pi/12 + \sqrt{3}/2)$

b) the point(s) of inflection: $(-\pi/2, -\pi/4)$ and $(\pi/2, \pi/4)$ b/c $f''(x)$ change signs

c) concavity intervals up: $(-\pi/2, \pi/2)$ b/c $f''(x) > 0$

down: $(-\pi, -\pi/2) \cup (\pi/2, \pi)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: $-\pi/12 - \sqrt{3}/2$ at $x = -\pi/6$ Absolute max: $\pi/2 + 1$ at $x = \pi$

d) Sketch the graph