Trig Unit Morning Test Review WS 4

1. Practice writing out the 6 trig derivative rules, 6 arc trig derivative rules, rule for $\ln u$ & e^u

2. An object's position is given by $F(t) = 2sin^2(3t)$, continuous and differentiable on $0 \leq t \leq \pi$ seconds. F(t) is given in meters

   a) Find the average velocity (rate of change) from $0 \leq t \leq \frac{\pi}{4}$

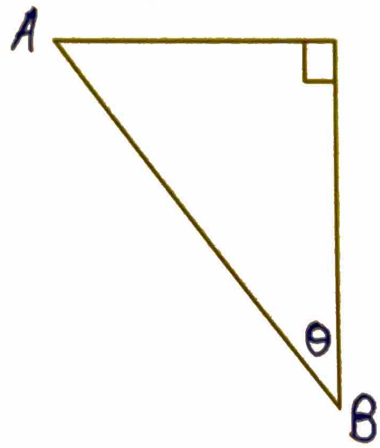
   b) At what point in time does the instantaneous velocity equal the average velocity from part (a)? Set up equation but do not solve.

   c) When on the interval $0 \leq t \leq \pi$, does $F(t) = 1$? (Show work leading to answer)

3)

a) Two cyclists are traveling on perpendicular roads. Cyclist A travels East at 15 mph. Cyclist B travels South at 20 mph. Find the rate at which the distance between the 2 cyclists are changing when x = 8 miles and y = 15 miles.

b) What is the rate of change of θ at that moment?



4) Find $\frac{dy}{dx}$ $x\cot 2y - y = \sqrt{\tan^5(5y^2)} + 3x - 7$

5) Find $\frac{dy}{dx}$ $y = x^3 \cos^{-1}(\pi x) - \sec(3x^2) + 5\pi e^2$

6) Solve for t: $2\cos\left(\frac{5\pi t}{2}\right) = 1 \quad on\ [0, 2]$

7)

Sketch the graph of function $f(x) = \frac{1}{2}x - \cos x$ on interval $[-\pi, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

<u>Include "Because" statements for parts a,b, & c</u>

a) the relative extrema min: ____________ max: ____________

b) the point(s) of inflection: ____________________________

c) concavity intervals up: ____________________________

down: ____________________________

d) Absolute extrema: Absolute min: ____________ Absolute max: ____________

d) Sketch the graph

Key

Trig Unit Morning Test Review WS 4

1. Practice writing out the 6 trig derivative rules, 6 arc trig derivative rules, rule for $\ln u$ & e^u

$\frac{d}{dx}\sin u = \cos u \cdot u'$ $\quad \frac{d}{dx}\cos u = -\sin u \cdot u'$

$\frac{d}{dx}\tan u = \sec^2 u \cdot u'$ $\quad \frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

$\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$ $\quad \frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

$\frac{d}{dx}\ln u = \frac{u'}{u}$ $\quad \frac{d}{dx}e^u = e^u \cdot u'$

$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$ $\quad \frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$ $\quad \frac{d}{dx}\text{arccot}\, u = \frac{-u'}{1+u^2}$

$\frac{d}{dx}\text{arcsec}\, u = \frac{u'}{|u|\sqrt{u^2-1}}$ $\quad \frac{d}{dx}\text{arccsc}\, u = \frac{-u'}{|u|\sqrt{u^2-1}}$

2. An object's position is given by $F(t) = 2\sin^2(3t)$, continuous and differentiable on $0 \le t \le \pi$ seconds. F(t) is given in meters $F(t) = 2[\sin(3t)]^2$

a) Find the average velocity (rate of change) from $0 \le t \le \pi/4$

$F(0) = 2[\sin(0)]^2 = 0$

$F(\pi/4) = 2\left[\sin\left(\frac{3\pi}{4}\right)\right]^2 = 2\left[\frac{\sqrt{2}}{2}\right]^2 = 2 \cdot \frac{1}{2} = 1$

Avg. velocity (Avg. ROC) $= \frac{1-0}{\pi/4 - 0} = 1 \cdot \frac{4}{\pi} = \boxed{\frac{4}{\pi} \text{ m/s}}$

b) At what point in time does the instantaneous velocity equal the average velocity from part (a)? Set up equation but do not solve.

*MVT $F'(c) = \frac{F(b)-F(a)}{b-a}$

$F(t) = 2[\sin(3t)]^2$

out: $2[\ ]^2$

in: $\sin(3t)$

$F'(t) = 4[\sin(3t)] \cdot \cos(3t) \cdot 3$

$\boxed{12\sin(3t)\cos(3t) = \frac{4}{\pi}}$

Instantaneous ROC

Avg. ROC

c) When on the interval $0 \le t \le \pi$, does F(t) = 1 ? (Show work leading to answer)

$2[\sin(3t)]^2 = 1$

$\sqrt{[\sin(3t)]^2} = \sqrt{\frac{1}{2}}$

$\sin(3t) = \pm\frac{\sqrt{2}}{2}$

$3t = \sin^{-1}\left(\pm\frac{\sqrt{2}}{2}\right)$

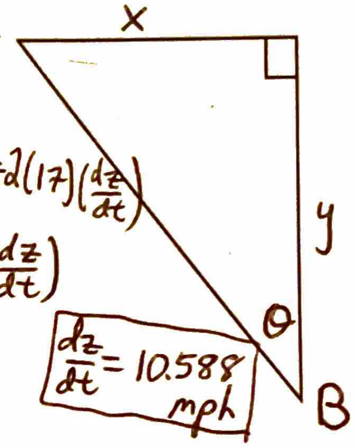
$\frac{1}{3}\left[3t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}\right]$

$\boxed{t = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12},}$ $\cancel{\frac{15\pi}{12}}$

3)

a) Two cyclists are traveling on perpendicular roads. Cyclist A travels East at 15 mph. Cyclist B travels South at 20 mph. Find the rate at which the distance between the 2 cyclists are changing when x = 8 miles and y = 15 miles.

b) What is the rate of change of θ at that moment?



a) $x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

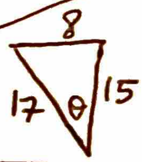
$x=8$ $\quad dx/dt=-15$

$y=15$ $\quad dy/dt=20$

$z=$ ___ $\quad dz/dt=$ ___

$z^2=8^2+15^2$

$z=17$



$\sec\theta=\frac{17}{15}$

$2(8)(-15)+2(15)(20)=2(17)\left(\frac{dz}{dt}\right)$

$-240+600=34\left(\frac{dz}{dt}\right)$

$360=34\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=10.588 \text{ mph}}$

b) $\tan\theta=\frac{x}{y}$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right)=\frac{\overbrace{(dx/dt)}^{f'}\overbrace{(y)}^{g}-\overbrace{x}^{f}\cdot\overbrace{(dy/dt)}^{g'}}{\underbrace{y^2}_{g^2}}$$

$$\left(\frac{17}{15}\right)^2\frac{d\theta}{dt}=\frac{(-15)(15)-8(20)}{15^2}$$

$$\left(\frac{17}{15}\right)^2\left(\frac{d\theta}{dt}\right)=\frac{-385}{225}$$

$$\boxed{\frac{d\theta}{dt}=\frac{-385}{225}\cdot\frac{15^2}{17^2}=\frac{-385}{289} \text{ or } -1.332 \text{ rad/hr}}$$

4) Find $\frac{dy}{dx}$ $\quad x\cot 2y-y=\sqrt{\tan^5(5y^2)}+3x-7$

$$\underbrace{x}_{f}\underbrace{\cot y}_{g}-y=[\tan(5y^2)]^{5/2}+3x-7$$

*chain Rule

out: $[\quad]^{5/2}$

in: $\tan(5y^2)$

$$\overbrace{1}^{f'}\cdot\overbrace{\cot(y)}^{g}+\overbrace{x}^{f}\cdot\overbrace{-\csc^2y\left(\frac{dy}{dx}\right)}^{g'}-1\left(\frac{dy}{dx}\right)=\frac{5}{2}[\tan(5y^2)]^{3/2}\cdot\sec^2(5y^2)\cdot10y\left(\frac{dy}{dx}\right)+3-0$$

$$-x\csc^2y\left(\frac{dy}{dx}\right)-1\left(\frac{dy}{dx}\right)-25y[\tan(5y^2)]^{3/2}\sec^2(5y^2)\left(\frac{dy}{dx}\right)=-\cot(y)+3$$

$$\frac{dy}{dx}\left(-x\csc^2y-1-25y[\tan(5y^2)]^{3/2}\sec^2(5y^2)\right)=-\cot(y)+3$$

$$\boxed{\frac{dy}{dx}=\frac{-\cot(y)+3}{-x\csc^2y-1-25y[\tan(5y^2)]^{3/2}\sec^2(5y^2)}}$$

5) Find $\frac{dy}{dx}$ $y = \underbrace{x^3}_{f}\underbrace{\cos^{-1}(\pi x)}_{g} - \sec(3x^2) + 5\pi e^2$

$$\frac{dy}{dx} = \overbrace{3x^2}^{f'} \cdot \overbrace{\cos^{-1}(\pi x)}^{g} + \overbrace{x^3}^{f} \cdot \overbrace{\frac{-\pi}{\sqrt{1-(\pi x)^2}}}^{g'} - \sec(3x^2)\tan(3x^2)\cdot 6x + 0$$

$$\boxed{\frac{dy}{dx} = 3x^2\cos^{-1}(\pi x) - \frac{\pi x^3}{\sqrt{1-(\pi x)^2}} - 6x\sec(3x^2)\tan(3x^2)}$$

$$2\cos\left(\frac{5\pi t}{2}\right) = 1$$

6) Solve for t: $2\cos\left(\frac{5\pi t}{2}\right) = 1$ *on* $[0, 2]$

$$\cos\left(\frac{5\pi t}{2}\right) = \frac{1}{2}$$

$$\frac{5\pi t}{2} = \cos^{-1}\left(\frac{1}{2}\right)$$

Added $6\pi/3$

$$\frac{2}{5\pi}\left[\frac{5\pi t}{2} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}\right]$$

$$\boxed{t = \frac{2}{15}, \frac{2}{3}, \frac{14}{15}, \frac{22}{15}, \frac{26}{15}}, \cancel{\frac{34}{15}}$$

7)

Sketch the graph of function $f(x) = \frac{1}{2}x - \cos x$ on interval $[-\pi, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

$f'(x) = \frac{1}{2} - (-\sin x)$

$f'(x) = \frac{1}{2} + \sin x$

$0 = \frac{1}{2} + \sin x$

$\sin x = -1/2$

$x = \frac{7\pi}{6}, 11\pi/6$

$\frac{7\pi}{6} - \frac{12\pi}{6} = -5\pi/6$

$\frac{11\pi}{6} - \frac{12\pi}{6} = -\pi/6$

$f'(x)$: $-\pi$ ... $+$ ↗ ... $-\frac{5\pi}{6}$ ... $-$ ↘ ... $-\pi/6$ ... $+$ ↗ ... π ; 0

$f(-\frac{5\pi}{6}) = \frac{1}{2}(-\frac{5\pi}{6}) - \cos(-\frac{5\pi}{6})$

$= -\frac{5\pi}{12} - (-\frac{\sqrt{3}}{2}) = \boxed{-\frac{5\pi}{12} + \frac{\sqrt{3}}{2}}$

$f(-\pi/6) = \frac{1}{2}(-\pi/6) - \cos(-\pi/6)$

$= \boxed{-\frac{\pi}{12} - \frac{\sqrt{3}}{2}}$

$f''(x) = \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{3\pi}{2} - \frac{4\pi}{2} = \frac{-\pi}{2}$

$x = \pi/2, -\pi/2$

$f''(x)$: $-\pi$ ... $\cap$ $-$ ... $-\frac{\pi}{2}$ ... $\cup$ $+$ ... $\pi/2$ ... $\cap$ $-$ ... π ; 0

$f(-\pi/2) = \frac{1}{2}(-\frac{\pi}{2}) - \cos(-\frac{\pi}{2}) = -\frac{\pi}{4} - 0 = \boxed{-\frac{\pi}{4}}$

$f(\pi/2) = \frac{1}{2}(\pi/2) - \cos(\pi/2) = \frac{\pi}{4} - 0 = \boxed{\pi/4}$

Include "Because" statements for parts a,b, & c

a) the relative extrema min: $(-\pi/6, \frac{-\pi}{12} - \sqrt{3}/2)$ b/c $f'(x)$ changes from $-$ to $+$ max: $(-\frac{5\pi}{6}, -\frac{5\pi}{12} + \frac{\sqrt{3}}{2})$ b/c $f'(x)$ changes from $+$ to $-$

b) the point(s) of inflection: $(-\pi/2, -\pi/4)$ and $(\pi/2, \pi/4)$ b/c $f''(x)$ change signs

c) concavity intervals up: $(-\pi/2, \pi/2)$ b/c $f''(x) > 0$

down: $(-\pi, -\pi/2), (\pi/2, \pi)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: $-\frac{\pi}{12} - \frac{\sqrt{3}}{2}$ at $x = \pi/6$ Absolute max: $\frac{\pi}{2} + 1$ at $x = \pi$

d) Sketch the graph

endpoints

$f(-\pi) = \frac{1}{2}(-\pi) - \cos(-\pi) = -\frac{\pi}{2} + 1$

$f(\pi) = \frac{1}{2}(\pi) - \cos(\pi) = \frac{\pi}{2} + 1$