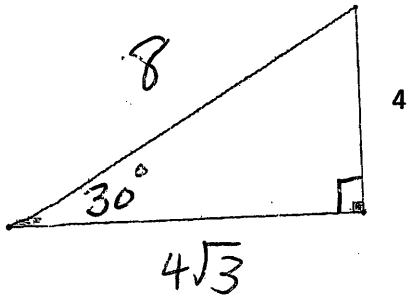
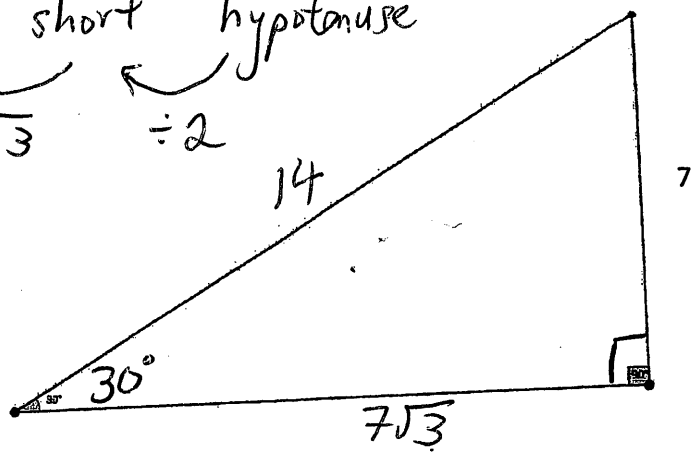
**Geometry** **Trigonometric Ratios** **Name**_____ Key

**Review Similar Triangles**

long $\xrightarrow{\div\sqrt{3}}$ short $\xrightarrow{\times 2}$ hypotenuse

long $\xleftarrow{\times\sqrt{3}}$ short $\xleftarrow{\div 2}$ hypotenuse




Are the triangles similar? Why or why not? yes AA Theorem

What type of triangles are they? Special right triangle 30-60-90

What is the length of the hypotenuse of each triangle?

8 and 14

What is ratios of the given legs to their hypotenuses?

1st Triangle $\frac{\text{short}}{\text{hyp}} = \frac{4}{8} = \boxed{\frac{1}{2}}$ | $\frac{\text{long}}{\text{hyp}} = \frac{4\sqrt{3}}{8} = \boxed{\frac{\sqrt{3}}{2}}$

2nd Triangle $\frac{\text{short}}{\text{hyp}} = \frac{7}{14} = \boxed{\frac{1}{2}}$ | $\frac{\text{long}}{\text{hyp}} = \frac{7\sqrt{3}}{14} = \boxed{\frac{\sqrt{3}}{2}}$

**Trigonometry** is a branch of mathematics that studies relationships involving lengths and angles of triangles.

A **Trigonometric Ratio** is a ratio (relationship between 2 numbers, e.g. fraction) of the lengths of 2 sides of a right triangle of a certain angle. The three basic trigonometric ratio are sine, cosine, and tangent. They are abbreviated as **sin, cos**, and **tan.**

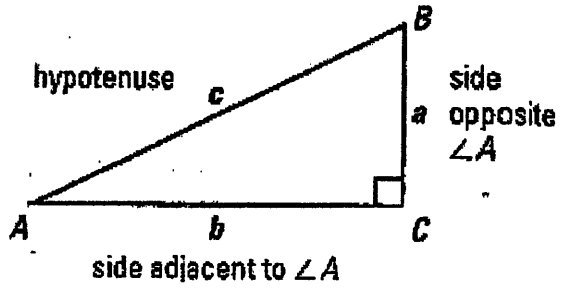
<u>Trigonometric Ratios:</u>

**Let $\triangle ABC$ be a right triangle. The sine, the cosine, and the tangent of the acute angle $\angle A$ are defined as follows.**

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$

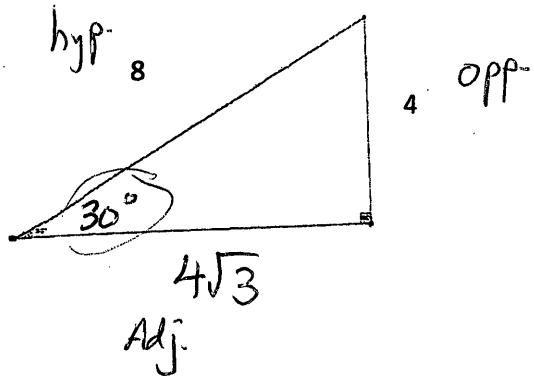


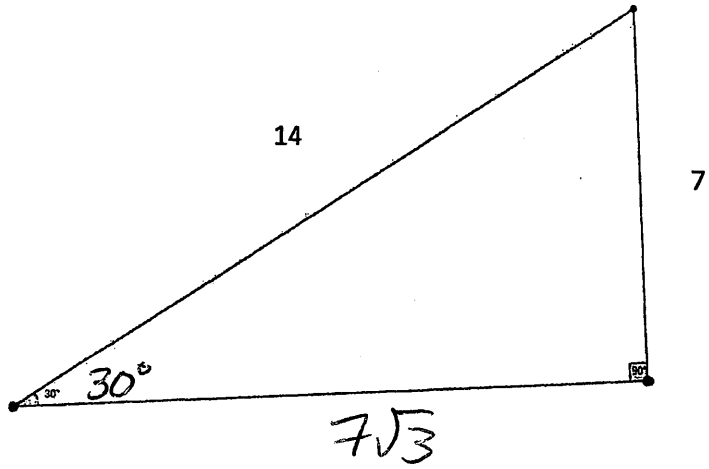
SOH – CAH – TOA

$Sin\ \measuredangle = \frac{Opp}{Hyp}$ $Cos\ \measuredangle = \frac{Adj}{Hyp}$ $Tan\ \measuredangle = \frac{Opp}{Adj}$

**Some Old Horse Caught Another Horse Taking Oats Away**

**Revisit the first triangles**



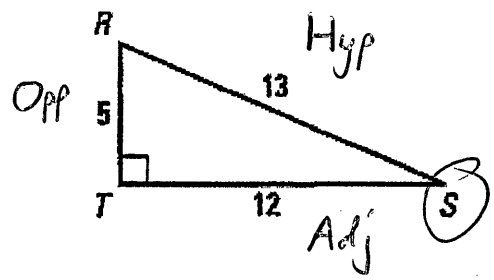




$\sin 30° = \frac{O}{H} = \frac{4}{8} = \boxed{\frac{1}{2}}$

$\sin 30° = \frac{O}{H} = \frac{7}{14} = \boxed{\frac{1}{2}}$

Example 1

Find the sine, the cosine, and the tangent of the indicated angle.




**a.** $\angle S$

$\sin S = \frac{O}{H} = \frac{5}{13}$

$\cos S = \frac{A}{H} = \frac{12}{13}$

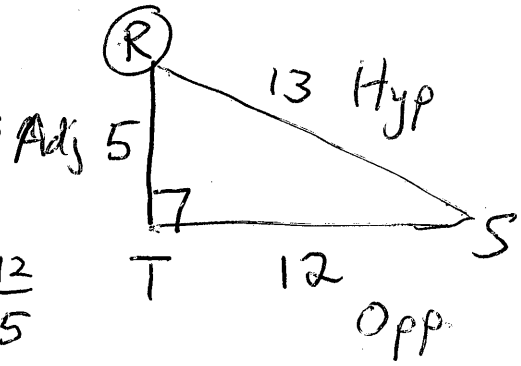
$\tan S = \frac{O}{A} = \frac{5}{12}$

**b.** $\angle R$

$\sin R = \frac{O}{H} = \frac{12}{13}$

$\cos R = \frac{A}{H} = \frac{5}{13}$

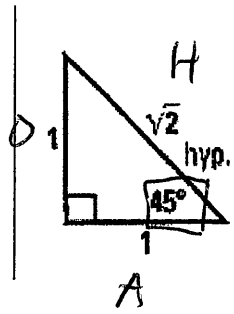
$\tan R = \frac{O}{A} = \frac{12}{5}$




$Sin\ \measuredangle = \frac{Opp}{Hyp}$ $Cos\ \measuredangle = \frac{Adj}{Hyp}$ $Tan\ \measuredangle = \frac{Opp}{Adj}$

**Example 2**

Find the sine, the cosine, and the tangent of 45°.



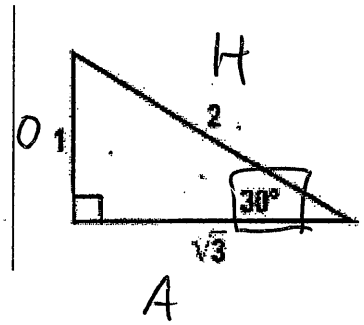

$$\sin 45 = \frac{O}{H} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cos 45 = \frac{A}{H} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\tan 45 = \frac{O}{A} = \frac{1}{1} = \boxed{1}$$

**Example 3**

Find the sine, the cosine, and the tangent of 30°.




$$\sin 30 = \frac{O}{H} = \boxed{\frac{1}{2}}$$

$$\cos 30 = \frac{A}{H} = \boxed{\frac{\sqrt{3}}{2}}$$

$$\tan 30 = \frac{O}{A} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

Name_____

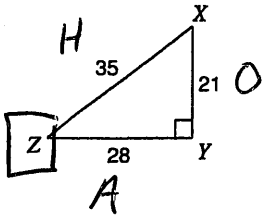
# Trigonometric Ratios

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

Date_____ Period____

**Find the value of each trigonometric ratio.**

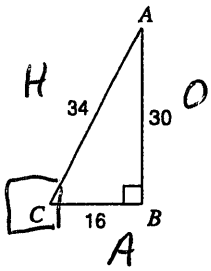
1) tan $Z$ $\quad \tan Z = \frac{O}{A}$




$\tan Z = \frac{21}{28} = \boxed{\frac{3}{4}}$

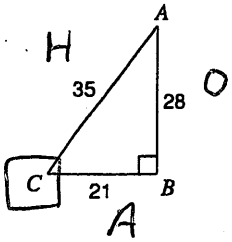
$\boxed{\tan Z = 3/4}$

2) cos $C$ $= \frac{A}{H}$ $\quad \cos C = \frac{16}{34} = \boxed{\frac{1}{2}}$



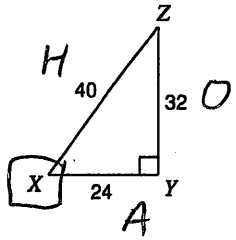

$\boxed{\cos C = 1/2}$

3) sin $C$ $= \frac{O}{H} = \frac{28}{35} = 4/5$



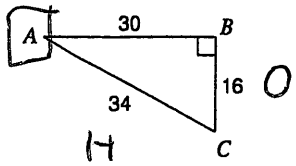

$\boxed{\sin C = 4/5}$

4) tan $X$ $= \frac{O}{A} = \frac{32}{24} = 4/3$



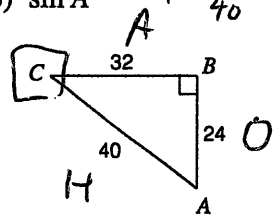

$\boxed{\tan X = 4/3}$

5) cos $A$ $= \frac{A}{H} = \frac{30}{34} = 15/17$



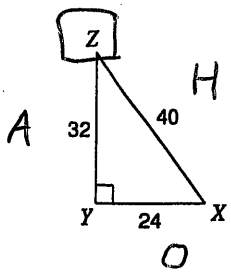

$\boxed{\cos A = \frac{15}{17}}$

6) sin $A$ $= O/H = \frac{24}{40} = 3/5$



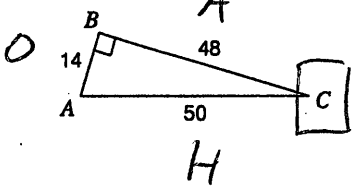

$\boxed{\sin A = 3/5}$

7) sin $Z$ $= O/H = \frac{24}{40} = \boxed{3/5}$



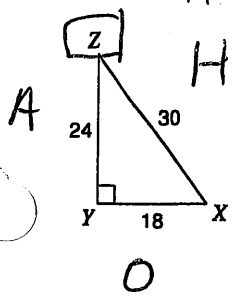

$\boxed{\sin Z = 3/5}$

8) sin $C$ $= \frac{O}{H} = \frac{14}{50}$



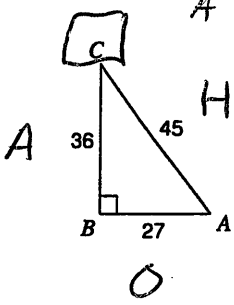

$\boxed{\sin C = 7/25}$

9) cos $Z$ $= \frac{A}{H} = \frac{24}{30} = 4/5$




$\boxed{\cos Z = 4/5}$

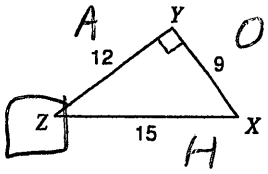
10) tan $C$ $= \frac{O}{A} = \frac{27}{36} = 3/4$




$\boxed{\tan C = 3/4}$

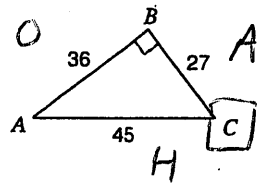
**Find the value of each trigonometric ratio to the nearest ten-thousandth.**

11) cos Z $= \frac{A}{H} = \frac{12}{15} = 4/5$



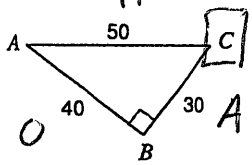

$\boxed{\cos Z = 4/5}$

12) cos C $= \frac{A}{H} = \frac{27}{45} = 3/5$



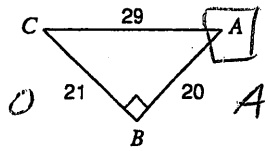

$\boxed{\cos C = 3/5}$

13) tan C $= \frac{O}{A} = \frac{40}{30} = 4/3$



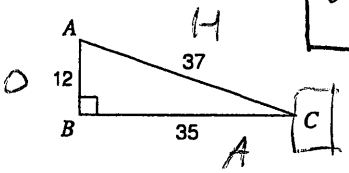

$\boxed{\tan C = 4/3}$

14) tan A $= \frac{O}{A} = \frac{21}{20}$

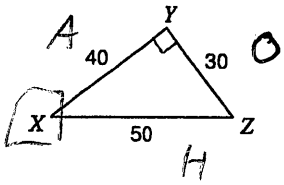



$\boxed{\tan A = \frac{21}{20}}$

15) tan C $= \frac{O}{A} = \frac{12}{35} =$ $\boxed{\tan C = \frac{12}{35}}$

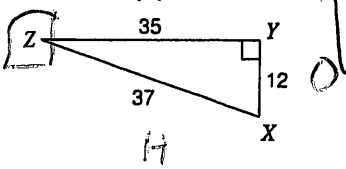



16) tan X $= \frac{O}{A} = \frac{30}{40}$



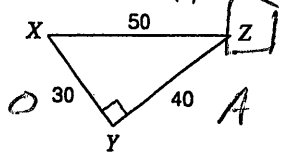

$\boxed{\tan X = 3/4}$

17) sin Z $= \frac{O}{H} = \frac{12}{37}$




$\boxed{\sin Z = \frac{12}{37}}$

18) sin Z $= \frac{O}{H} = \frac{30}{50}$




$\boxed{\sin Z = \frac{3}{5}}$

19) sin 48° $\boxed{0.743}$

20) sin 38° $\boxed{0.616}$

21) cos 61° $\boxed{0.485}$

22) cos 51° $\boxed{0.629}$

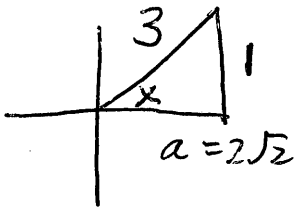
**Critical thinking questions:**

23) Can the sine of an angle ever equal 2? Why or why not?

No, since $\sin\theta = \frac{O}{H}$ and hypotenuse is always larger than opposite side, so the fraction will be less than 1.

24) $\sin x = \frac{1}{3}$

Find cos $x$.




$a^2 + 1^2 = 3^2$

$a^2 = 9 - 1 = 8$

$a = \sqrt{8} = 2\sqrt{2}$

$\cos x = \frac{A}{H}$

$\boxed{\cos x = \frac{2\sqrt{2}}{3}}$