Key

10.2 AP Practice Problems (p.734) – Infinite Series

1. Find the sum of the series $\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$, if it exists.

(A) 1 (B) 2 $\boxed{\text{(C)}}$ $\frac{1}{2}$ (D) The series diverges.

$A=1$

$\frac{1}{(k+1)(k+2)} = \frac{A}{k+1} + \frac{B}{k+2}$ $B=-1$

$(k=-1)$ $(k=-2)$

$\frac{1}{k+1} - \frac{1}{k+2}$

$= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\ldots = \boxed{\frac{1}{2}}$

2. $\sum_{k=1}^{\infty} \frac{7}{3^{k-1}}$

(A) converges and equals $\frac{7}{4}$.

(B) converges and equals $\frac{14}{3}$.

(C) converges and equals $\frac{21}{2}$.

(D) diverges.

$\sum_{k=1}^{\infty} \frac{7}{3^k \cdot 3^{-1}} = \frac{21}{3^k} = 21\left(\frac{1}{3}\right)^k$ Geometric series

$r = 1/3$

$Sum = \frac{a_1}{1-r} \rightarrow \frac{7}{1-1/3} = \frac{7}{2/3} = 7 \cdot \frac{3}{2} = \boxed{\frac{21}{2}}$

3. The repeating decimal 0.1212... can be expressed as the fraction

(A) $\frac{4}{33}$ (B) $\frac{3}{25}$ (C) $\frac{4}{99}$ (D) $\frac{303}{250}$

$0.12 + 0.0012 + 0.000012 + \ldots$

$12\left[0.01 + (0.01)^2 + (0.01)^3\right]$

$S = \frac{a_1}{1-r}$

$S = \frac{0.12}{1-0.01}$

$S = \frac{0.12}{0.99} = \frac{12}{99} = \boxed{\frac{4}{33}}$

4. Which of the following series diverge?

I. $\sum_{k=1}^{\infty} (\sqrt{2})^{k-1}$ II. $\sum_{k=1}^{\infty} -\frac{3}{4^k}$ III. $\sum_{k=1}^{\infty} \frac{1}{k}$

(A) I and II only $\boxed{\text{(B)}}$ I and III only

(C) II and III only (D) I, II, and III

i) $\frac{(\sqrt{2})^k}{\sqrt{2}}$ $r = \sqrt{2} > 1$

ii) $-3\left(\frac{1}{4}\right)^k$

$r = 1/4 < 1$

harmonic series diverge.

5. Determine whether the series $\sum_{k=1}^{\infty} \frac{7^{k-2}}{8^{k+1}}$ converges or diverges. If it converges, find its sum.

(A) converges and equals $\frac{1}{64}$

(B) converges and equals $\frac{1}{56}$

(C) converges and equals $\frac{1}{8}$

(D) The series diverges.

$a_1 = \frac{7^{-1}}{8^2} = \frac{1}{7(64)}$

$S = \frac{a_1}{1-r} \rightarrow \frac{\frac{1}{7(64)}}{1 - 7/8} = \frac{\frac{1}{7(64)}}{1/8}$

$\frac{1}{7 \cdot 64} \cdot \frac{8}{1} = \boxed{\frac{1}{56}}$

$\sum \frac{7^k \cdot 7^{-2}}{8^k \cdot 8^1} \rightarrow \frac{1}{8(49)} \cdot \left(\frac{7}{8}\right)^k$

$r = 7/8 < 1$

6. An object hanging on a spring is pulled downward a distance of 100 cm from its equilibrium position (the origin) and released. It recoils upward past the origin to a height 90 cm above the origin. It continues to oscillate up and down about the origin, but with each oscillation the spring travels only $\frac{9}{10}$ of the last distance traveled.

(a) Write an infinite series that models the object's movement.

(b) Find the sum of the infinite series.

(c) Interpret the sum of the infinite series in the context of the problem.

a) $100 + 2\sum_{k=1}^{\infty} 90\left(\frac{9}{10}\right)^{k-1}$

b) $100 + 2 \cdot \frac{90}{1 - 9/10} = 1900\text{cm}$

c) 1900cm is the total distance that the object travels above and below the origin.