Key

10.5 AP Practice Problems (p.765) – Alternating Series Test & Absolute Convergence

1. Which of the following series converge?

I. $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2}$

II. $\sum_{k=1}^{\infty} (-1)^k \left(\frac{5}{3}\right)^k$

III. $\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$

(A) I only (B) I and II only

(C) I and III only (D) I, II, and III

(Absolute Convergence) i) $\lim_{n\to\infty} \frac{1}{n^2} = 0$ (converge) AST

ii) $\lim_{n\to\infty} \left(\frac{5}{3}\right)^n = \infty$ (diverges)

(conditional Convergence) iii) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ (converges by AST)

2. The alternating series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{10^k}$ converges. What is the maximum error incurred by using the first three nonzero terms to approximate the sum of the series?

(A) −0.083 (B) 0.003 (C) 0.0004 (D) 0.0826

2) Max error is at most the first unused term (a_4)

$a_4 = \frac{(-1)^4 \cdot 4}{10^4} = \boxed{0.0004}$

3. What is the fewest number of terms of the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$ that must be added to approximate the sum so that the error is less than or equal to 0.001?

(A) 7 (B) 9 (C) 10 (D) 11

$\frac{1}{(n+1)^3} \leq \frac{1}{1000}$

$(n+1)^3 \geq 1000$

$n+1 \geq 10$

$n \geq 9$

$n = 9$

Since when $n=10$ (first unused term) the value $a_n = 0.001$, the sum would be the first nine terms to achieve desired accuracy.

4. Which of the following series converge conditionally, but not absolutely?

I. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3}{k}$ (conditional)

II. $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{1}{k}\right)^{4/3}$ (absolute) $\rightarrow \frac{1}{n^{4/3}}, \; p=4/3>1$

III. $\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{3}{4}\right)^{k}$ (absolute convergence)

(A) I only (B) I and II only

(C) I and III only (D) I, II, and III

5. (a) Write out the first five terms of the series $\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}$.

(b) Show the series $\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}$ converges.

58 **(c)** How many terms of the series are necessary to approximate the sum S with an error less than or equal to 0.0001?

a) $\frac{(-1)^0}{0!}+\frac{(-1)^1}{1!}+\frac{(-1)^2}{2!}+\frac{(-1)^3}{3!}+\frac{(-1)^4}{4!}+\frac{(-1)^5}{5!}$

b) $\lim_{n\to\infty}\frac{1}{n!}=0$ (converges by AST)

c) $\frac{1}{10{,}000}\le\frac{1}{(k+1)!}$

$(k+1)!\le 10{,}000$

$8!=40{,}320$

$7!=5{,}040$

Since 8! is the least value to exceed 10,000, then series would go to $n=7$, there 8 terms (since we start at $n=0$ to $n=7$)

6. Determine whether the series $\sum_{k=1}^{\infty}\frac{\cos(2k)}{4^k}$ converges absolutely, converges conditionally, or diverges. Show your work.

$\rightarrow \cos(2k)=1$

$\frac{1}{4^k}$ converges absolutely (Geometric Series Test)

$\left(\frac{1}{4}\right)^k$