10.01 Exponentials Review Date: ______________

Properties of Exponents:

Product of Powers	$x^m \cdot x^n = x^{m+n}$
Quotient of Powers	$\frac{x^m}{x^n} = x^{m-n}$
Power of a Product	$(xy)^n = x^n \cdot y^n$
Power of a Quotient	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
Power of a Power	$(x^m)^n = x^{mn}$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$
Zero Exponent	$x^0 = 1$
Rational Exponent	$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Rational Radical

$x^m \cdot x^n \neq x^{mn}$

$(x+y)^n \neq x^n + y^n$

$x^{4/3} \rightarrow \sqrt[3]{x^4}$

Simplify. Your answers should only contain positive exponents.

1. $(4a^3b^{-2})^3$
2. $(x^3)^3 2x^{-1}$
3. $\left(\frac{x^{-2}}{y^4}\right)^3$
4. $\frac{a^4b^2}{ab^5}$
5. $\frac{z}{(2z^0)^2}$
6. $\left(\frac{3^{4x}}{3^{2x}}\right)^3$

Evaluate.

7. 2^{-5}
8. $36^{1/2}$
9. $27^{4/3}$
10. $8^{-2/3}$

Solve.

11. $10^{4x+3} = 10^{2x+23}$
12. $3^x = 9^{x-2}$
13. $25^{2x-4} = 125^{x+4}$

1) $(4^1a^3b^{-2})^3 \to 4^3a^9b^{-6} \to \boxed{\dfrac{64a^9}{b^6}}$

2) $(x^3)^3 2x^{-1} \to x^9\cdot 2x^{-1} \to \dfrac{x^9\cdot 2}{x} \to \boxed{2x^8}$

$\hookrightarrow x^9\cdot x^{-1} \to \boxed{2x^8}$

3) $\left(\dfrac{x^{-2}}{y^4}\right)^3 = \dfrac{x^{-6}}{y^{12}} \to \boxed{\dfrac{1}{x^6y^{12}}}$

4) $\dfrac{a^4b^2}{a^1b^5} \to a^3b^{-3} \to \boxed{\dfrac{a^3}{b^3}}$

5) $\dfrac{z}{(2^1z^0)^2} \to \dfrac{z}{2^2z^0} \to \boxed{\dfrac{z}{4}}$

6) $\left(\dfrac{3^{4x}}{3^{2x}}\right)^3 \to \dfrac{3^{12x}}{3^{6x}}$ or $3^{12x}\cdot 3^{-6x} \to \boxed{3^{6x}}$

7) $2^{-5} \rightarrow \frac{1}{2^5} \rightarrow \boxed{\frac{1}{32}}$

8) $36^{1/2} \rightarrow \sqrt{36^1} \rightarrow \boxed{6}$

9) $27^{4/3} \rightarrow \left(\sqrt[3]{27}\right)^4 \rightarrow 3^4 = \boxed{81}$

10) $8^{-2/3} \rightarrow \frac{1}{8^{2/3}} \rightarrow \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$

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11) $\cancel{10}^{4x+3} = \cancel{10}^{2x+23}$ $\quad a^m = a^n \rightarrow \boxed{m=n}$

$4x+3 = 2x+23$

$2x = 20$

$\boxed{x=10}$

12) $3^x = 9^{x-2}$

$3^x = \left(3^2\right)^{x-2}$

$\cancel{3}^x = \cancel{3}^{2(x-2)}$

$x = 2x-4$

$4 = 1x$

$\boxed{x=4}$

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13) $25^{2x-4} = 125^{x+4}$

$\downarrow$

$\cancel{5}^{2(2x-4)} = \cancel{5}^{3(x+4)}$

$2(2x-4) = 3(x+4)$

$4x-8 = 3x+12$

$1x = 20$

$\boxed{x=20}$

10.01 Practice:

Simplify. Your answers should only contain positive exponents.

1. $(x^{-2}x^{-3})^4$

$(x^{-5})^4 = x^{-20}$

$= \boxed{\frac{1}{x^{20}}}$

2. $\frac{\cancel{3}xy^3 2y^3}{\cancel{3}x^2y^{-4}} \rightarrow \frac{2x^1y^6}{x^2y^{-4}}$

$\frac{2x^1y^6y^4}{x^2} \rightarrow \boxed{\frac{2y^{10}}{x}}$

3. $\frac{2a^{-4}}{(2a^{-4})^3}$

$\frac{2a^{-4}}{(2^1a^{-4})^3}$

$\frac{2a^{-4}}{2^3a^{-12}} \rightarrow \frac{2a^{-4}\cdot a^{12}}{8} \rightarrow \boxed{\frac{a^8}{4}}$

Solve. *If $a^m = a^n$, then $m = n$.

4. $27^x = 3^{2x+3}$

$\cancel{3}^{3x} = \cancel{3}^{2x+3}$

$3x = 2x + 3$

$\boxed{x=3}$

5. $8^{2x+2} = 4^{x+15}$

$\cancel{2}^{3(2x+2)} = \cancel{2}^{2(x+15)}$

$6x+6 = 2x+30$

$4x = 24$

$\boxed{x=6}$

6. $3^{x^2} + 5 = 6$

$3^{x^2} = 1$

$3^{x^2} = 3^0$

$x^2 = 0$

$\boxed{x=0}$

7. $16^a \cdot 64^{3-3a} = 64$

$4^{2a} \cdot 4^{3(3-3a)} = 4^3$

$\cancel{4}^{2a+3(3-3a)} = \cancel{4}^3$

$2a+9-9a = 3$

$-7a+9 = 3$

$-7a = -6$

$\boxed{a = 6/7}$

8. $243^{x+2} \cdot 9^{2x-1} = 9$

$3^{5(x+2)} \cdot 3^{2(2x-1)} = 3^2$

$\cancel{3}^{5(x+2)+2(2x-1)} = \cancel{3}^2$

$5x+10+4x-2 = 2$

$9x+8 = 2$

$9x = -6$

$\boxed{x = \frac{-6}{9} = -\frac{2}{3}}$

9. $\frac{125^{-3a}}{25^{3a}} = 125$

$\frac{5^{3(-3a)}}{5^{2(3a)}} = 5^3$

$\cancel{5}^{-9a-6a} = \cancel{5}^3$

$-15a = 3$

$\boxed{a = \frac{-3}{15} = \frac{-1}{5}}$

10. $2^x \cdot \frac{1}{32} = 32$

$2^x \cdot \frac{1}{2^5} = 2^5$

$2^x \cdot 2^{-5} = 2^5$

$\cancel{2}^{x-5} = \cancel{2}^5$

$x-5 = 5$

$\boxed{x=10}$

11. $\frac{4^{3x-1}}{64} = 4^x$

$\frac{4^{3x-1}}{4^3} = 4^x$

*subtract exponents

$4^{3x-1-3} = 4^x$

$\cancel{4}^{3x-4} = \cancel{4}^x$

$3x-4 = 1x$

$2x = 4$

$\boxed{x=2}$

12. $\frac{343^{-2x}}{49^{x-3}} = 5^0$

$\frac{7^{3(-2x)}}{7^{2(x-3)}} = 7^0$

$\cancel{7}^{-6x-2(x-3)} = \cancel{7}^0$

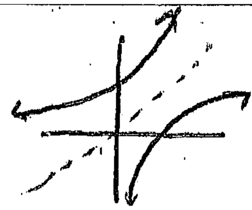
$-6x-2x+6 = 0$

$-8x+6 = 0$

$x = -6/-8$ or $\boxed{x = 3/4}$

$y = b^x \rightarrow x = b^y$

$y = \log_b x$



4

Inverse function:
1) swap x and y
2) Solve for y

10.02 Intro to Log Function

Date: ______________________

A logarithmic function is the *inverse* of an exponential function.

<u>Definition</u>: Let b and y be positive numbers with $b \neq 1$. Then, the ***logarithm of y with base b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \text{ if and only if } b^x = y$$

<u>Examples</u>: Convert from exponential form into logarithmic form.

1. $2^3 = 8$

$\log_2 8 = 3$

2. $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

3. $81^{1/4} = 3$

$\log_{81} 3 = \frac{1}{4}$

<u>Examples</u>: Convert from logarithmic form into exponential form.

4. $\log_5 25 = 2$

$5^2 = 25$

5. $\log_7 \frac{1}{343} = -3$

$7^{-3} = \frac{1}{343}$

6. $\log_{32} 2 = \frac{1}{5}$

$32^{1/5} = 2$

$e \approx 2.72$

- ***Common log*** is log base 10
- Denoted as $\log N$, it is understood to mean $\log_{10} N$
- The LOG button on the calculator evaluates $\log_{10} N$

- ***Natural log*** is log base e
- Denoted as $\ln N$, it is understood to mean $\log_e N$
- The LN button on the calculator evaluates $\log_e N$

<u>Examples</u>: Rewrite in exponential form.

7. $\log 100 = 2$

$\log_{10} 100 = 2$

$\boxed{10^2 = 100}$

8. $\ln 7 = x$

$\log_e 7 = x$

$e^x = 7$

<u>Examples</u>: Rewrite in logarithmic form.

9. $10^{-2} = \frac{1}{100}$

$\log_{10} \frac{1}{100} = -2$

$\boxed{\log \frac{1}{100} = -2}$

10. $e^2 = 7.389$

$\log_e 7.389 = 2$

$\boxed{\ln 7.389 = 2}$

<u>Examples</u>: Evaluate the following logarithms.

11. $\log 1 = x$

$\log_{10} 1 = x$

$10^x = 1$

$\boxed{x = 0}$

12. $\log_{64} 8 = x$

$64^x = 8$

$\boxed{x = \frac{1}{2}}$

13. $\log_5 \frac{1}{625} = x$

$5^x = \frac{1}{625}$

$\boxed{x = -4}$

Inverse function: steps
1) swap x and y 2) Solve for y

$y = b^x$ 4 $x = b^y$

$y = \log_b x$

10.02 Intro to Log Function Date: ____________

A logarithmic function is the *inverse* of an exponential function.

Definition: Let b and y be positive numbers with $b \neq 1$. Then, the ***logarithm of y with base b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \text{ if and only if } b^x = y$$

$\log_b y = x$

Examples: Convert from exponential form into logarithmic form.

1. $2^3 = 8$ 2. $5^{-3} = \frac{1}{125}$ 3. $81^{1/4} = 3$

$\log_2 8 = 3$ $\log_5 \frac{1}{125} = -3$ $\log_{81} 3 = \frac{1}{4}$

Examples: Convert from logarithmic form into exponential form.

4. $\log_5 25 = 2$ 5. $\log_7 \frac{1}{343} = -3$ 6. $\log_{32} 2 = \frac{1}{5}$

$5^2 = 25$ $7^{-3} = \frac{1}{343}$ $32^{1/5} = 2$

$e \approx 2.72$

- ***Common log*** is log base 10
- Denoted as $\log N$, it is understood to mean $\log_{10} N$
- The LOG button on the calculator evaluates $\log_{10} N$

- ***Natural log*** is log base e
- Denoted as $\ln N$, it is understood to mean $\log_e N$
- The LN button on the calculator evaluates $\log_e N$

Examples: Rewrite in exponential form.

7. $\log 100 = 2$ 8. $\ln 7 = x$

$\log_{10} 100 = 2$ $\boxed{10^2 = 100}$

$\log_e 7 = x$ $\boxed{e^x = 7}$

Examples: Rewrite in logarithmic form.

9. $10^{-2} = \frac{1}{100}$ 10. $e^2 = 7.389$

$\log_{10} \frac{1}{100} = -2$ $\boxed{\log \frac{1}{100} = -2}$

$\log_e 7.389 = 2$ $\boxed{\ln 7.389 = 2}$

Examples: Evaluate the following logarithms.

11. $\log 1 = x$ 12. $\log_{64} 8 = x$ 13. $\log_5 \frac{1}{625} = x$

$\log_{10} 1 = x$

$10^x = 1$ $\boxed{x = 0}$

$64^x = 8$ $\boxed{x = 1/2}$

$5^x = \frac{1}{625}$ $\boxed{x = -4}$

10.02 Homework: Evaluate each logarithmic expression.

1. $\log_5 125 = x$

$5^x = 125$

$\boxed{x=3}$

2. $\log_8 1 = x$

$8^x = 1$

$\boxed{x=0}$

3. $\log_6 \frac{1}{36} = x$

$6^x = \frac{1}{36}$ $\boxed{x=-2}$

4. $\log_4 2 = x$

$4^x = 2$

$x = 1/2$

5. $\log_7 -49 = x$

$7^x = -49$

$\boxed{\text{no solution}}$

6. $\log_{10} 10{,}000 = x$

$10^x = 10000$

$\boxed{x=4}$

7. $\ln e^2 = x$

$\log_e e^2 = x$

$e^x = e^2$

$\boxed{x=2}$

8. $\log_{256} 4 = x$

$256^x = 4$

$\boxed{x=1/4}$

9. $\log_{1/5} 25 = x$

$\left(\frac{1}{5}\right)^x = 25$

$\boxed{x=-2}$

10. $\log\sqrt{10} = x$

$\log_{10}\sqrt{10} = x$

$10^x = \sqrt{10}$

$10^x = 10^{1/2}$

$\boxed{x=1/2}$

11. $\log_{1/32} 2 = x$

$\left(\frac{1}{32}\right)^x = 2$

$\boxed{x=-1/5}$

12. $\log_{\sqrt{3}} 27 = x$

$(\sqrt{3})^x = 27$

$\cancel{3}^{\frac{1}{2}x} = \cancel{3}^3$

$\frac{1}{2}x = 3$

$\boxed{x=6}$

13. $\log_2 2^9 = x$

$2^x = 2^9$

$\boxed{x=9}$

14. $3 \cdot \ln e^4 = x$

$3\log_e e^4 = x$

$\log_e e^4 = \frac{x}{3}$

$e^{x/3} = e^4$

$\frac{x}{3} = 4$ $\boxed{x=12}$

15. $\ln -5 = x$

$\log_e -5 = x$

$e^x = -5$

$\boxed{\text{No solution}}$

10.03 Properties of Logarithms Date: ______________

Opener: Simplify the exponential expression.

1. $x^3 * x^7 * x$ $\boxed{x^{11}}$

2. $\frac{m^5n^2}{m^2n^9}$ $\boxed{\frac{m^3}{n^7}}$

3. $(g^4)^{11}$ $\boxed{g^{44}}$

Special properties of exponents and logarithms, where *b* is positive and not 1:

$\log_b 1 =$ 0	Why? $b^0 = 1$
$\log_b b =$ 1	Why? $b^1 = b$
$\log_b b^x =$ x	Why? $b^x = b^x$ ✓
$b^{\log_b x} =$ x	Why? $\log_b x = \log_b x$

Properties of Logarithms:

Argument is a Product			
$\log_b u$	$\log_b v$	$\log_b uv$	General Rule: $\log_b u + \log_b v = \log_b u \cdot v$
$\log_2 4 =$	$\log_2 8 =$	$\log_2 32 =$	

Argument is a Quotient			
$\log_b u$	$\log_b v$	$\log_b \frac{u}{v}$	General Rule: $\log_b u - \log_b v = \log_b\left(\frac{u}{v}\right)$
$\log_5 3125 =$	$\log_5 25 =$	$\log_5 125 =$	

Argument is a Power		
$\log_b u^n$	$n \log_b u$	General Rule: $\log_b u^n = n \log_b u$
$\log_2 4^5 =$	$5 \log_2 4 =$	

1) $\log u + \log v = \log uv$

2) $\log u - \log v = \log\left(\frac{u}{v}\right)$

3) $\log u^n = n \log u$

Examples: Use properties of logarithms to expand each expression. The expanded logarithm expressions should have arguments with no exponent, product, or quotient.

1. $\log_5 2x =$ $\log_5 2 + \log_5 x$

2. $\log_2 8a^2b^5 =$ $\log_2 8 + \log_2 a^2 + \log_2 b^5$

   $\boxed{\log_2 8 + 2\log_2 a + 5\log_2 b}$

3. $\log_7 \frac{g}{h}$ $\log_7 g - \log_7 h$

4. $\log_4 \frac{16w^3}{x^6}$ $\log_4 16 + \log_4 w^3 - \log_4 x^6 \rightarrow \boxed{\log_4 16 + 3\log_4 w - 6\log_4 x}$

5. $\log \sqrt{r}$ $\log_{10} r^{1/2}$ $\boxed{\frac{1}{2}\log_{10} r}$

6. $\ln \frac{a+1}{\sqrt[3]{b-2c}}$ $\log_e\left(\frac{a+1}{(b-2c)^{1/3}}\right) = \log_e(a+1) - \log_e(b-2c)^{1/3}$

   $\boxed{\log_e(a+1) - \frac{1}{3}\log_e(b-2c)}$

Examples: Use properties of logarithms to condense each expression. The condensed logarithm expression should be written as a single logarithm with no coefficient.

7. $3\log 4 - 2\log k$ $\log_{10} 4^3 - \log_{10} k^2$ $\log_{10}\left(\frac{4^3}{k^2}\right) \rightarrow \boxed{\log_{10}\left(\frac{64}{k^2}\right)}$

8. $-5\ \log_2(x+1) + 3\log_2(6x)$ $\log_2(6x)^3 - \log_2(x+1)^5 = \boxed{\log_2 \frac{(6x)^3}{(x+1)^5}}$

9. $\frac{1}{3}\log_4 10 + \frac{1}{3}\log_4 h - 6\log_4 g$ $\log_4 10^{1/3} + \log_4 h^{1/3} - \log_4 g^6 \rightarrow \boxed{\log_4\left(\frac{10^{1/3}h^{1/3}}{g^6}\right)}$

10. $\ln(3m+5) - 4\ln m - \ln(m-1)$ $\log_e(3m+5) - \log_e m^4 - \log_e(m-1) = \log_e\left(\frac{3m+5}{m^4(m-1)}\right)$ or $\ln\left(\frac{3m+5}{m^5-m^4}\right)$

11. $\log 20 + 2\log\frac{1}{2} - \log x + 3\log y$

    $\log 20 + \log\left(\frac{1}{2}\right)^2 - \log x + \log y^3 \rightarrow \log\left(\frac{20\cdot\frac{1}{4}y^3}{x}\right) = \boxed{\log\left(\frac{5y^3}{x}\right)}$

10.03 Practice

Use properties of logarithms to expand each expression. The expanded logarithm expressions should have arguments with no exponent, product, or quotient.

1. $\ln\frac{4}{5}$

2. $\log_6 3x$

3. $\log\frac{7b}{\sqrt{c}}$

4. $\log_2\frac{m^4}{8n}$

$\log_2 m^4 - \log_2 8 - \log_2 n \rightarrow \boxed{4\log_2 m - \log_2 8 - \log_2 n}$

5. $\ln\sqrt[3]{10g^2}$

$\log_e(10^1 g^2)^{1/3} \rightarrow \log_e 10^{1/3} g^{2/3} \rightarrow \log_e 10^{1/3} + \log_e g^{2/3}$

$\boxed{\frac{1}{3}\log_e 10 + \frac{2}{3}\log_e g}$

6. $\log_3\frac{u-1}{v^5 w^3}$

$\log_3(u-1) - \log_3 v^5 - \log_3 w^3$

$\boxed{\log_3(u-1) - 5\log_3 v - 3\log_3 w}$

7. $\log\frac{a^2 b}{\sqrt[5]{3a-1}}$

$\log_{10}\frac{a^2 b}{(3a-1)^{1/5}} \rightarrow \log_{10} a^2 + \log_{10} b - \log_{10}(3a-1)^{1/5}$

$\boxed{2\log_{10} a + \log_{10} b - \frac{1}{5}\log_{10}(3a-1)}$

Use properties of logarithms to condense each expression. The condensed logarithm expression should be written as a single logarithm with no coefficient.

8. $\log_5 8 - \log_5 12$

9. $3\ln x + 5\ln y$

10. $10\log k - 2\log 3$

11. $\frac{1}{2}\log_5 36 + \log_5 r - 3\log_5 p$

$\log_5 36^{1/2} + \log_5 r - \log_5 p^3$

$\log_5\left(\frac{36^{1/2} r}{p^3}\right) \rightarrow \boxed{\log_5\left(\frac{6r}{p^3}\right)}$

12. $2\log_8 9 - 3\log_8 c - 4\log_8 d$

$\log_8 9^2 - \log_8 c^3 - \log_8 d^4 \rightarrow \boxed{\log_8\left(\frac{81}{c^3 d^4}\right)}$

13. $3\log n - \frac{1}{2}\log(6-n) + \log 7$

$\log_{10} n^3 - \log_{10}(6-n)^{1/2} + \log_{10} 7$

$\boxed{\log_{10}\left(\frac{7n^3}{(6-n)^{1/2}}\right)}$

14. $\frac{2}{5}\ln 32 - \left(3\ln j - \frac{1}{2}\ln 9\right)$

$\log_e 32^{2/5} - \ln j^3 + \ln 9^{1/2}$

$(32^{1/5})^2 \rightarrow (2)^2 = 4$

$\log_e\left(\frac{32^{2/5} \cdot 9^{1/2}}{j^3}\right) \rightarrow \log_e\left(\frac{4 \cdot 3}{j^3}\right) \rightarrow \boxed{\log_e\left(\frac{12}{j^3}\right)}$

10.03 More Practice with Log Properties Date: ______________

Choose "A" or "B" as the correct answer. Then, explain the mistake in the wrong answer.

		Answer A	Answer B
1.	Expand: $\log\left(\frac{j}{kp}\right)$ $\log j - \log k - \log p$	$\log j - \log k + \log p$	$\log j - \log k - \log p$ ✓
2.	Condense: $\frac{\log a}{4}$ $\frac{1}{4}\log a \to \log a^{1/4}$	$\log\left(\frac{a}{4}\right)$	$\log a^{1/4}$ ✓
3.	Expand: $\log cd^3$ $\log_{10} c + \log_{10} d^3$ $\log c + 3\log d$	$\log c + 3\log d$ ✓	$3\log c + 3\log d$
4.	Condense: $\frac{1}{2}\log m - 4\log r + \log u$ $\log m^{1/2} - \log r^4 + \log u$ $\log\left(\frac{m^{1/2}u}{r^4}\right)$	$\log\frac{\sqrt{m}}{r^4 u}$	$\log\frac{u\sqrt{m}}{r^4}$ ✓
5.	Expand: $\ln\sqrt[5]{z^2}$ $\log_e z^{2/5} \to \frac{2}{5}\log_e z = \frac{2}{5}\ln z$	$\frac{2\ln z}{5}$ ✓	$\frac{5\ln z}{2}$
6.	Condense: $\log_2(x+3) + \log_2(x-2)$ $\log_2(x+3)(x-2)$ $\log_2(x^2+3x-2x-6)$	$\log_2(x+1)$	$\log_2(x^2+x-6)$ ✓
7.	Which is equivalent to: $5^x = 100$ $5^x = 100 \to \log_5 100 = x$	$x = \log_5 100$ ✓	$x = \frac{100}{5}$
8.	Which is equivalent to: $e^2 = x$ $\log_e x = 2$ $\boxed{\ln x = 2}$	$\log x = 2$	$\ln x = 2$ ✓
9.	Which is equivalent to: $\log_3 \cancel{3}^{2x}$ $2x$	9^x	$2x$ ✓

$\cancel{\log\left(\frac{a}{4}\right)}$

10.04 Solving Exponential Equations Date: ____________

Recall the **One-to-One Property of Exponential Functions:**
$b^x = b^y$ if and only if $x = y$.

For this property to work, notice that the *bases must be the same.*
Examples: Solve each equation.

1. $32^{x+3} = 4^{2x+10}$

$\cancel{2}^{5(x+3)} = \cancel{2}^{2(2x+10)}$

$5x+15 = 4x+20$

$1x = 5$ $\boxed{x=5}$

2. $\left(\frac{1}{3}\right)^{2x} = 81^{x-3}$

$\cancel{3}^{-2x} = \cancel{3}^{4(x-3)}$

$-2x = 4x-12$

$-6x = -12$

$\frac{-6x}{-6} = \frac{-12}{-6}$

$\boxed{x=2}$

There is a similar property of logarithms:
One-to-One Property of Logarithmic Functions:
$log_b\, x = log_b\, y$ if and only if $x = y$.

Examples: Solve each equation.
3. $\log_4 x = \log_4 3 + \log_4 (x-2)$

$\cancel{\log_4} x = \cancel{\log_4} 3(x-2)$

$x = 3(x-2)$

$x = 3x-6$

$-2x = -6$

$\boxed{x=3}$

This property also works backwards: if x = y, then $\log_b x = \log_b y$.
This method is often called "*taking the log of both sides*" and is helpful to solve exponential equations.
Examples: Solve each equation.

4. $4^x = 1.5$

5. $3.2e^{2x} + 2.5 = 16.9$

6. $6^{2x+4} = 5^{-x+1}$

7. $2^{3x+11} = 9^{2x+1}$

10.04 Solving Exponential Equations

Date: ____________

Recall the **One-to-One Property of Exponential Functions:**
$b^x = b^y$ if and only if $x = y$.

For this property to work, notice that the *bases must be the same*.
Examples: Solve each equation.

1. $32^{x+3} = 4^{2x+10}$

$\cancel{2}^{5(x+3)} = \cancel{2}^{2(2x+10)}$

$5x+15 = 4x+20$

$1x = 5$ $\boxed{x=5}$

2. $\left(\frac{1}{3}\right)^{2x} = 81^{x-3}$

$\cancel{3}^{-2x} = \cancel{3}^{4(x-3)}$

$-2x = 4x - 12$

$-6x = -12$

$\frac{-6x}{-6} = \frac{-12}{-6}$

$\boxed{x=2}$

There is a similar property of logarithms:
One-to-One Property of Logarithmic Functions:
$log_b\, x = log_b\, y$ if and only if $x = y$.

Examples: Solve each equation.
3. $\log_4 x = \log_4 3 + \log_4 (x-2)$

$\cancel{\log_4} x = \cancel{\log_4} 3(x-2)$

$x = 3(x-2)$

$x = 3x - 6$

$-2x = -6$

$\boxed{x=3}$

This property also works backwards: if x = y, then $\log_b x = \log_b y$.
This method is often called "*taking the log of both sides*" and is helpful to solve exponential equations.
Examples: Solve each equation.

4. $4^x = 1.5$

$\log 4^x = \log 1.5$

$\frac{x(\log 4)}{\log 4} = \frac{\log 1.5}{\log 4}$

$\boxed{x = 0.292}$

5. $3.2e^{2x} + 2.5 = 16.9$

$-2.5 \quad -2.5$

$\frac{3.2e^{2x}}{3.2} = \frac{14.4}{3.2}$

$e^{2x} = 4.5$

$\ln e^{2x} = \ln 4.5$

$\frac{2x\, \cancel{\ln e}}{2} = \frac{\ln 4.5}{2}$

$\boxed{x = 0.752}$

6. $6^{2x+4} = 5^{-x+1}$

$\log 6^{2x+4} = \log 5^{-x+1}$

$(2x+4)\log 6 = (-x+1)\log 5$

$2x\log 6 + 4\log 6 = -x\log 5 + \log 5$

$2x\log 6 + x\log 5 = \log 5 - 4\log 6$

$x(2\log 6 + \log 5) = \log 5 - 4\log 6$

$x = \frac{\log 5 - 4\log 6}{2\log 6 + \log 5}$

$\boxed{x = -1.0702}$

7. $2^{3x+11} = 9^{2x+1}$

let $x = e^x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

8. $e^{2x} + 2e^x - 8 = 0$

$(e^x+4)(e^x-2) = 0$

$e^x + 4 = 0$ | $e^x - 2 = 0$

$e^x = -4$ | $e^x = 2$

$\ln e^x = \ln(-4)$ | $\ln e^x = \ln 2$

No solution | $x \cancel{\ln e} = \ln 2$

$\boxed{x = \ln 2}$

9. $4e^{2x} + 8e^x = 5$

$4e^{2x} + 8e^x - 5 = 0$

* factor $4x^2 + 8x - 5 = 0$

-20, 10, -2, 4, 4, 8

$(x + \frac{5}{2})(x - \frac{1}{2}) = 0$

$(2x+5)(2x-1) = 0$

$2e^x + 5 = 0$ | $2e^x - 1 = 0$

$e^x = \frac{-5}{2}$ | $2e^x - 1 = 0$

No solution | $e^x = \frac{1}{2}$

$\ln e^x = \ln \frac{1}{2}$

$x \cancel{\ln e} = \ln \frac{1}{2}$

$\boxed{x = \ln\left(\frac{1}{2}\right)}$

Practice:
Solve.

1. $4^{x+7} = 8^{x+3}$

2. $\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$

3. $1.8^x = 9.6$

4. $8^x - 1 = 3.4$

5. $e^{2x} + 5 = 16$

6. $2.5e^{x+4} = 14$

7. $0.75e^{3.4x} - 0.3 = 80.1$

8. $7^{2x+1} = 3^{x+3}$

9. $9^{x+2} = 2^{5x-4}$

10. $e^{2x} - 15e^x + 56 = 0$

11. $6e^{2x} - 5e^x = 6$

12. $300 = \frac{400}{1+3e^{-2x}}$

pg.13 10.04 Solving Exponential equations classwork $\boxed{\text{Odds}}$

1) $4^{x+7} = 8^{x+3}$

$\cancel{2}^{2(x+7)} = \cancel{2}^{3(x+3)}$

$2(x+7) = 3(x+3)$

$2x+14 = 3x+9$

$-1x = -5$

$\boxed{x=5}$

3) $1.8^{x} = 9.6$

$\log 1.8^{x} = \log 9.6$

$x(\log 1.8) = \log 9.6$

$x = \dfrac{\log 9.6}{\log 1.8}$

$\boxed{x = 3.848}$

5) $e^{2x} + 5 = 16$

$e^{2x} = 11$

$\ln e^{2x} = \ln 11$

$2x \cancel{\ln e} = \ln 11$

$2x = \ln 11$

$x = \dfrac{\ln 11}{2}$

$\boxed{x = 1.199}$

7) $0.75e^{3.4x} - 0.3 = 80.1$

$\dfrac{0.75e^{3.4x}}{0.75} = \dfrac{80.4}{0.75}$

$e^{3.4x} = 107.2$

$\ln e^{3.4x} = \ln 107.2$

$3.4x \cancel{\ln e} = \ln 107.2$

$\dfrac{3.4x}{3.4} = \dfrac{\ln 107.2}{3.4}$

$\boxed{x = 1.375}$

9) $9^{x+2} = 2^{5x-4}$

$\log 9^{x+2} = \log 2^{5x-4}$

$(x+2)\log 9 = (5x-4)\log 2$

$x\log 9 + 2\log 9 = 5x\log 2 - 4\log 2$

$x\log 9 - 5x\log 2 = -4\log 2 - 2\log 9$

$x(\log 9 - 5\log 2) = -4\log 2 - 2\log 9$

$x = \dfrac{-4\log 2 - 2\log 9}{\log 9 - 5\log 2}$

$\boxed{x = 5.650}$

11) $6e^{2x}-5e^{x}=6$ $\quad$ * let $x=e^{x}$

$6e^{2x}-5e^{x}-6=0$

* $6x^2-5x-6=0$ ← factor this

$a\cdot c$

-36

-9	4
6	6

-5

$\frac{-9}{6}\to\frac{-3}{2}$

$\frac{4}{6}\to\frac{2}{3}$

$\left(x-\frac{3}{2}\right)\left(x+\frac{2}{3}\right)=0$

$(2x-3)(3x+2)=0$

↓

$(2e^{x}-3)(3e^{x}+2)=0$

$2e^{x}-3=0$

$2e^{x}=3$

$e^{x}=\frac{3}{2}$

$\ln e^{x}=\ln\frac{3}{2}$

$x\cancel{\ln e}=\ln\left(\frac{3}{2}\right)$

$\boxed{x=\ln\left(\frac{3}{2}\right)}$

$3e^{x}+2=0$

$3e^{x}=-2$

$e^{x}=-\frac{2}{3}$

$\ln e^{x}=\ln\left(\cancel{-2/3}\right)$

$\boxed{\text{no solution}}$

Use the properties of logarithms to expand each expression to match with an equivalent one below. Then decode the answer to: **Why does a moon rock taste better than an Earth rock?**

1. $\log_4(xyz)$

$\log_4 x + \log_4 y + \log_4 z$

$\boxed{A}$

2. $\log_4\left(\frac{x}{yz}\right)$

$\log_4 x - \log_4 y - \log_4 z$

$\boxed{S}$

3. $\log_4 3x^4$

$\log_4 3 + \log_4 x^4$

$\log_4 3 + 4\log_4 x$

$\boxed{R}$

4. $\log_4\left(\frac{x^2y}{z}\right)$

$\log_4 x^2 + \log_4 y - \log_4 z$

$2\log_4 x + \log_4 y - \log_4 z$

$\boxed{M}$

5. $\log_4\left(\frac{3x^5}{y^2z}\right)$

$\log_4 3 + \log_4 x^5 - \log_4 y^2 - \log_4 z$

$\log_4 3 + 5\log_4 x - 2\log_4 y - \log_4 z$

$\boxed{O}$

6. $\log_4\left(\frac{6x^2y^8}{z^3}\right)$

$\log_4 6 + 2\log_4 x + 8\log_4 y - 3\log_4 z$

$\boxed{I}$

7. $\log_4\left(\frac{6y^2z^5}{x^4}\right)$

$\log_4 6 + 2\log_6 y + 5\log_6 z - 4\log_4 x$

$\boxed{T}$

8. $\log_4\left(\frac{3y^2z}{x^7}\right)$

$\log_4 3 + 2\log_4 y + \log_4 z - 7\log_4 x$

$\boxed{E}$

9. $\log_4\left(\frac{xz^6}{\sqrt{y}}\right)$

$\log_4 x + 6\log_4 z - \frac{1}{2}\log_4 y$

$\boxed{L}$

	$\log_4 3 + 5\log_4 x - 2\log_4 y + \log_4 z$ P		$4\log_4 3 + 4\log_4 x$ H
8	$\log_4 3 - 7\log_4 x + 2\log_4 y + \log_4 z$ E	4	$2\log_4 x + \log_4 y - \log_4 z$ M
5	$\log_4 3 + 5\log_4 x - 2\log_4 y - \log_4 z$ O	1	$\log_4 x + \log_4 y + \log_4 z$ A
7	$\log_4 6 - 4\log_4 x + 2\log_4 y + 5\log_4 z$ T	2	$\log_4 x - \log_4 y - \log_4 z$ S
	$5\log_4 3 + 5\log_4 x - 2\log_4 y - \log_4 z$ H		$\log_4 x - \log_4 y + \log_4 z$ N
6	$\log_4 6 + 2\log_4 x + 8\log_4 y - 3\log_4 z$ I	3	$\log_4 3 + 4\log_4 x$ R
	$6\log_4 x - \frac{1}{2}\log_4 y + 6\log_4 z$ C	9	$\log_4 x - \frac{1}{2}\log_4 y + 6\log_4 z$ L

I	T		I	S		A		L	I	T	T	L	E		M	E	T	E	O	R
6	7		6	2		1		9	6	7	7	9	8		4	8	7	8	5	3

10.04 Solving Exponential Equations $\boxed{\text{Evens}}$ Classwork

$\boxed{\text{pg. 13}}$

2) $\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$

$\downarrow$

$\left(\frac{3}{4}\right)^{2(3x-2)} = \left(\frac{3}{4}\right)^{5x+4}$

$2(3x-2) = 5x+4$

$6x-4 = 5x+4$

$1x = 8$

$\boxed{x=8}$

4) $8^x - 1 = 3.4$

$8^x = 4.4$

$\log 8^x = \log 4.4$

$x(\log 8) = \log 4.4$

$x = \frac{\log 4.4}{\log 8}$

$\boxed{x = 0.713}$

6) $\frac{2.5e^{x+4}}{2.5} = \frac{14}{2.5}$

$e^{x+4} = 5.6$

$\ln e^{x+4} = \ln 5.6$

$(x+4)\cancel{\ln e} = \ln 5.6$

$x+4 = \ln 5.6$

$\boxed{x = -2.277}$

8) $7^{2x+1} = 3^{x+3}$

$\log 7^{2x+1} = \log 3^{x+3}$

$(2x+1)\log 7 = (x+3)\log 3$

$2x\log 7 + 1\log 7 = x\log 3 + 3\log 3$

$2x\log 7 - x\log 3 = -\log 7 + 3\log 3$

$x(2\log 7 - \log 3) = -\log 7 + 3\log 3$

$x = \frac{-\log 7 + 3\log 3}{2\log 7 - \log 3}$

$\boxed{x = 0.483}$

$$10)\quad e^{2x} - 15e^{x} + 56 = 0$$

* let $x = e^x$

* factor this

$$* \quad x^2 - 15x + 56 = 0$$

$$(x - 7)(x - 8) = 0$$

$$\downarrow \qquad \downarrow$$

$$(e^x - 7)(e^x - 8) = 0$$

$e^x - 7 = 0$	$e^x - 8 = 0$
$e^x = 7$	$e^x = 8$
$\ln e^x = \ln 7$	$\ln e^x = \ln 8$
$x \cancel{\ln e} = \ln 7$	$x \cancel{\ln e} = \ln 8$
$\boxed{x = \ln 7}$	$\boxed{x = \ln 8}$

$$12)\quad 300 = \frac{400}{1 + 3e^{-2x}}$$

$$\frac{300}{1} = \frac{400}{1 + 3e^{-2x}}$$

$$300(1 + 3e^{-2x}) = 400(1)$$

$$300 + 900e^{-2x} = 400$$

$$\frac{900e^{-2x}}{900} = \frac{100}{900}$$

$$e^{-2x} = \frac{1}{9}$$

$$\ln e^{-2x} = \ln\left(\frac{1}{9}\right)$$

$$-2x \cancel{\ln e} = \ln\left(\frac{1}{9}\right)$$

$$-2x = \ln\left(\frac{1}{9}\right)$$

$$x = \frac{\ln\left(\frac{1}{9}\right)}{-2}$$

$$\boxed{x = 1.099}$$

10.05 Solving Logarithmic Equations   Date: ____________

The opposite of taking the *log of both sides* is to take *exponentiate both sides.* This can be used to cancel a logarithm from one or more sides of an equation. To do this, make each side of the equation the exponent of the value of the base of the logarithm(s):
If $\log_b x = y$, then $b^{\log_b x} = b^y$.

This may have the effect of converting the logarithm into its exponential form.

$\cancel{\log_2(-24)}$

Also, the argument of a logarithm ***must be positive***. **Check for extraneous solutions before moving on from each problem. Meaning if the value you get for *x* makes the argument either 0 or a negative, you must exclude that value from the solution set.

Examples: Solve each equation.

1. $-3 \ln x = -24$

$\frac{-3 \ln x}{-3} = \frac{-24}{-3}$

$\ln x = 8$

$\log_e x = 8$

$e^8 = x$

$\boxed{x = e^8}$

2. $4 - 3 \log (5x) = 16$

$-4 \qquad -4$

$\frac{-3 \log(5x)}{-3} = \frac{12}{-3}$

$\log_{10}(5x) = -4$

$10^{-4} = 5x$

$5x = \frac{1}{10^4}$

$x = \frac{1}{10^4} \cdot \frac{1}{5} = \boxed{\frac{1}{50000}}$

3. $\log_3 (x - 1) = -2$

$3^{-2} = x - 1$

$\frac{1}{9} = x - 1$

$\boxed{x = \frac{10}{9}}$

$\cancel{3}^{\cancel{\log_3}(x-1)} = 3^{-2}$

$x - 1 = 3^{-2}$

$\boxed{x = \frac{10}{9}}$

4. $\log_2 (x^2 - 4) = \log_2 21$

$\cancel{2}^{\cancel{\log_2}(x^2-4)} = \cancel{2}^{\cancel{\log_2} 21}$

$x^2 - 4 = 21$

$x^2 = 25$

$\boxed{x = \pm 5}$

You may have to use properties to change the equation to have <u>at most</u> one logarithm on each side of the equation.

5. $3 \log_7 x = \log_7 64$

$\cancel{\log_7} x^3 = \cancel{\log_7} 64$

$x^3 = 64$

$\boxed{x = 4}$

6. $\log_2 5 = \log_2 10 - \log_2 (x - 4)$

$\cancel{\log_2} 5 = \cancel{\log_2}\left(\frac{10}{x-4}\right)$

$\frac{5}{1} = \frac{10}{x-4}$

$5(x-4) = 10$

$5x - 20 = 10$

$5x = 30$

$\boxed{x = 6}$

7. $\log_4 (x - 3) + \log_4 (x + 1) = \log_4 (6x - 18)$

$\cancel{\log_4}(x-3)(x+1) = \cancel{\log_4}(6x-18)$

$x^2 - 3x + 1x - 3 = 6x - 18$

$x^2 - 2x - 3 - 6x + 18 = 0$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5, \ x \neq 3$

$\boxed{x = 5}$

8. $\ln (3x - 4) = 1 + \ln (2x + 3)$

$\ln(3x-4) - \ln(2x+3) = 1$

$\ln\left(\frac{3x-4}{2x+3}\right) = 1$

$\log_e\left(\frac{3x-4}{2x+3}\right) = 1$

$e^1 = \frac{3x-4}{2x+3}$

$\frac{e}{1} = \frac{3x-4}{2x+3}$

$2xe + 3e = 3x - 4$

$2xe - 3x = -3e - 4$

$x(2e - 3) = -3e - 4$

$x = \frac{-3e-4}{2e-3} \rightarrow \cancel{-4.988}$

No solution

Practice:

Solve. Don't forget to check for extraneous solutions!

1. $-8\log x = -64$

$\frac{-8\log x}{-8} = \frac{-64}{-8}$

$\log_{10} x = 8$

$10^8 = x$

$\boxed{x = 10^8}$

2. $2 + 3\log 3d = 5$

$-2 \qquad -2$

$\frac{3\log(3d)}{3} = \frac{3}{3}$

$\log_{10}(3d) = 1$

$10^1 = 3d$

$\boxed{d = \frac{10}{3}}$

3. $14 + 20\ln 7x = 54$

$-14 \qquad -14$

$\frac{20\ln(7x)}{20} = \frac{40}{20}$

$\ln(7x) = 2$

$\log_e(7x) = 2$

$e^2 = 7x$

$\boxed{x = \frac{e^2}{7}}$

4. $7{,}000\ln x = -21{,}000$

5. $log_8(x^2 + 11) = log_8 92$

6. $log_{11} 3x = log_{11}(x + 5) - log_{11} 2$

7. $\ln x + \ln(x + 7) = \ln 18$

8. $\ln(3x + 1) + \ln(2x - 3) = ln10$

9. $\ln(x - 3) + \ln(2x + 3) = \ln(-4x^2)$

10. $log(5x^2 + 4) = 2log3x^2 - log(2x^2 - 1)$

11. $\log(3x + 2) = 1 + log2x$

12. $log_9 9x - 2 = -log_9 x$

10.05 p.15

4) $\frac{7000 \ln x}{7000} = \frac{-21000}{7000}$

$\ln x = -3$

$\log_e x = -3$

$e^{-3} = x$

$\boxed{x = \frac{1}{e^3}}$

5) $\log_8 (x^2+11) = \log_8 92$

$x^2 + 11 = 92$

$x^2 = 81$

$x = \pm 9$

$\boxed{x = 9,\ x = -9}$

6) $\log_{11} 3x = \log_{11}(x+5) - \log_{11}(2)$

$\log_{11} 3x = \log_{11}\left(\frac{x+5}{2}\right)$

$\frac{3x}{1} = \frac{x+5}{2}$

$6x = x + 5$

$5x = 5$

$\boxed{x = 1}$

7) $\ln(x) + \ln(x+7) = \ln 18$

$\ln(x)(x+7) = \ln 18$

$\cancel{\log_e}(x^2+7x) = \cancel{\log_e} 18$

$x^2 + 7x = 18$

$x^2 + 7x - 18 = 0$

$(x-2)(x+9) = 0$

$x = 2,\ x = \cancel{-9}$ extraneous solution

$\boxed{x = 2}$

8) $\ln(3x+1) + \ln(2x-3) = \ln(10)$

$\ln(3x+1)(2x-3) = \ln(10)$

$\ln(6x^2 + 2x - 9x - 3) = \ln 10$

$\ln(6x^2 - 7x - 3) = \ln 10$

$\cancel{\log_e}(6x^2 - 7x - 3) = \cancel{\log_e} 10$

$6x^2 - 7x - 3 = 10$

$6x^2 - 7x - 13 = 0$

$\left(x - \frac{13}{6}\right)\left(x + \frac{6}{6}\right)$

$(6x-13)(x+1) = 0$

$x = 13/6,\ x = \cancel{-1}$ (extraneous)

$\boxed{x = 13/6}$

9) $\ln(x-3) + \ln(2x+3) = \ln(-4x^2)$

$\ln(x-3)(2x+3) = \ln(-4x^2)$

$\ln(2x^2-6x+3x-9) = \ln(-4x^2)$

$\log_e(2x^2-3x-9) = \log_e(-4x^2)$

$2x^2-3x-9 = -4x^2$

$6x^2-3x-9=0$

$3(2x^2-1x-3)=0$

$3\left(x-\frac{3}{2}\right)\left(x+\frac{2}{2}\right)=0$

$3(2x-3)(x+1)=0$

$2x-3=0$ | $x+1=0$

$x=3/2$ | $x=-1$

Both are extraneous solutions. $\boxed{\text{No solution}}$

10) $\log(5x^2+4) = 2\log 3x^2 - \log(2x^2-1)$

$\log(5x^2+4) = \log(3x^2)^2 - \log(2x^2-1)$

$\log(5x^2+4) = \log(9x^4) - \log(2x^2-1)$

$\log(5x^2+4) = \log\left(\frac{9x^4}{2x^2-1}\right)$

$\frac{5x^2+4}{1} = \frac{9x^4}{2x^2-1}$

$(5x^2+4)(2x^2-1) = 9x^4(1)$

$10x^4+8x^2-5x^2-4 = 9x^4$

$1x^4+3x^2-4=0$

$(x^2+4)(x^2-1)=0$

$x^2+4=0$ | $x^2-1=0$

$\sqrt{x^2}=\sqrt{-4}$ | $\sqrt{x^2}=\pm\sqrt{1}$

No solution | $\boxed{x=1,-1}$

11) $\log(3x+2) = 1 + \log(2x)$

$\log(3x+2) - \log(2x) = 1$

$\log\left(\frac{3x+2}{2x}\right) = 1$

$\log_{10}\left(\frac{3x+2}{2x}\right) = 1$

$10^1 = \frac{3x+2}{2x}$

$\frac{10}{1} = \frac{3x+2}{2x}$

$20x = 3x + 2$

$17x = 2$

$\boxed{x = \frac{2}{17}}$

---

12) $\log_9(9x) - 2 = -\log_9(x)$

$\log_9(9x) + \log_9(x) = 2$

$\log_9(9x)(x) = 2$

$\log_9(9x^2) = 2$

$9^2 = 9x^2$

$81 = 9x^2$

$9 = x^2$

$\pm\sqrt{9} = \sqrt{x^2}$

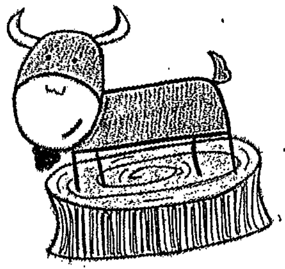
$\pm 3 = x$

$\boxed{x = 3}$, $x \neq -3$

Extraneous solution

$\boxed{10.06}$ p. 16-17

Name: ______________________ Date: ______________ Period: ______



DOODLE-ING MATH

Why do Lumberjacks Make Good Music?

Directions: For each problem, solve the exponential or logarithmic function. Doodle or color on the lumberjack below according to your answer choice.

1. $2^{3x-19} = 4$

$2^{3x-19} = 2^2$

$3x - 19 = 2$
$+19 \quad +19$

$3x = 21$

$\boxed{x = 7}$

If your answer is 7 color his pants grey.

If your answer is -7 color his pants blue.

2. $\log_2 x + \log_2(x+7) = 3$

$\log_2 x(x+7) = 3$

$\log_2 x^2 + 7x = 3$

$2^3 = x^2 + 7x$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$x = \cancel{-8}$ AND $\boxed{1}$

CHECK: ① $\cancel{\log_2 -8}$ + log... ② $\log_2 1 + \log_2(1+7) = 3$; $\log_2 1 + \log_2 8 = 3$; $\log_2 8 = 3$; $3 = 3$ ✓

If your answer is 1 and -8 draw the following hat:

If your answer is 1 draw the following hat:

3. $2e^{2x} - 7e^x + 6 = 0$

12
3 4

$u = e^x$

$2u^2 - 7u + 6 = 0$

$(u - \frac{4}{2})(u - \frac{3}{2}) = 0$

$(u-2)(2u-3) = 0$

$u = 2 \quad 2u = 3$
$u = \frac{3}{2}$

$e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2 \approx \boxed{.69}$

$e^x = \frac{3}{2}$

$\ln e^x = \ln \frac{3}{2}$

$x = \ln \frac{3}{2} \approx \boxed{.41}$

If your answer is .69 and .41 color his hat red.

If your answer is 4.48 and 7.39 color his hat blue.

4. $\log(x+1) - \log(3x-2) = \log\left(\frac{2}{x}\right)$

$\cancel{\log} \frac{x+1}{3x-2} = \cancel{\log} \frac{2}{x}$

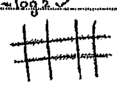
$\frac{x+1}{3x-2} = \frac{2}{x} \Rightarrow x^2 + x = 6x - 4$
$-6x - 4 \quad -6x + 4$

$5x + 4 = 0$

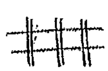
$\boxed{x = 4 \text{ AND } 1}$

CHECK: $\log 5 - \log 10 = \log \frac{1}{2}$; $\log \frac{5}{10} = \log \frac{1}{2}$ ✓; $\log 2 - \log 1 = \log 2$; $\log 2 = \log 2$ ✓

If your answer is 1 draw the following pattern on his hat:



If your answer is 1 and 4 draw the following pattern on his hat:



5. $7^x = 156$

$\ln 7^{x} = \ln 156$

$\frac{x \ln 7}{\ln 7} = \frac{\ln 156}{\ln 7}$

$x = \frac{\ln 156}{\ln 7} \approx 2.595$

If your answer is 22.3 draw a sledge hammer in his hands.

If your answer is 2.595 draw a guitar in his hands.

6. $3^{x^2-7} = 27^{2x}$

$3^{x^2-7} = 3^{3(2x)}$

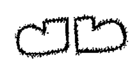
$x^2 - 7 = 6x$
$-6x$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1)$

$\boxed{x = 7 \text{ AND } -1}$

If your answer is 7 and 1 draw the following shoes on the lumberjack:



If your answer is -7 and 1 draw the following shoes on the lumberjack:

7. $\frac{6\ln(2x)}{6} = \frac{12}{6}$

$\ln(2x) = 2$

$e^{\ln(2x)} = e^2$

$\frac{2x}{2} = \frac{e^2}{2} \quad x = \frac{e^2}{2} \approx 3.69$

If your answer is 3.69 color his shoes yellow.

If your answer is 0 color his shoes brown.

8. $\log(x-3) = \log(7x-23) - \log(x+1)$

$\log(x-3) = \log \frac{7x-23}{x+1}$

$x - 3 = \frac{7x-23}{x+1}$

$(x+1)(x-3) = 7x - 23$

$x^2 - 2x - 3 = 7x - 23$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0 \quad \boxed{x = 4 \text{ AND } 5}$

CHECK: $\log 1 = \log 5 - \log 5$; $\log 1 = \log \frac{5}{5}$; $\log 1 = \log 1$ ✓; $\log 2 = \log 12 - \log 6$; $\log 2 = \log \frac{12}{6}$; $\log 2 = \log 2$ ✓

If your answer is -10 and 2 color his jacket red.

If your answer is 4 and 5 color his jacket blue.

9. $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4(x-1) = 1$

$\log_4 \frac{x^2}{x-1} = 1$

$\left(\frac{x^2}{x-1}\right) = 4^1$

$\frac{x^2}{x-1} = 4$

$x^2 = 4x - 4$

$x^2 - 4x + 4 = 0$

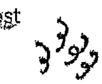
$(x-2)(x-2) = 0$

$\boxed{x = 2}$

CHECK: $\underbrace{\log_4 4}_{=1} - \underbrace{\log_4 1}_{=0} = 1$; $1 + 0 = 1$; $1 = 1$ ✓

If your answer is 1 draw 3 chest hairs on the lumberjack.

If your answer is 2 draw 5 chest hairs on the lumberjack.



10. $\log_2(x+2)-\log_2(x-5)=3$

$\frac{x+2}{x-5}=2^3$

$\frac{x+2}{x-5}=8$

$x+2=8x-40$
$-8x-2 \quad -8x-2$

$-7x=-42$

$\boxed{x=6}$

CHECK:
$\log_2 8-\underbrace{\log_2 1}_{=0}=3$
$\log_2 2^3$
$3-0=3$
$3=3$ ✓

If your answer is 6, draw a pile of 6 logs next to the lumberjack.
If your answer is -4, draw a pile of 4 logs next to the lumberjack.

11. $27^{2x+4}=9^{x-2}$

$3^{3(2x+4)}=3^{2(x-2)}$

$3^{6x+12}=3^{2x-4}$

$6x+12=2x-4$
$-2x-12 \quad -2x-12$

$4x=-16$

$\boxed{x=-4}$

If your answer is -6, draw 3 snowcapped mountains:
If your answer is -4, draw 3 mountains:

12. $\log_4(x-6)=-2$

$x-6=4^{-2}$

$x-6=\frac{1}{16}$
$+6 \quad +6$

$x=\frac{1}{16}+\frac{96}{16}=\boxed{\frac{97}{16}}$

CHECK:
$\log_4\left(\frac{97}{16}-6\right)=-2$
$\log_4\left(\frac{97}{16}-\frac{96}{16}\right)=-2$
$\log_4\left(\frac{1}{16}\right)=-2$
$4^{-2}=\frac{1}{16}$ ✓

If your answer is $\frac{291}{48}$ draw 2 trees near your log pile.
If your answer is $\frac{97}{16}$ draw 1 tree near your log pile.

13. $\ln\sqrt{2x-4}=0$

e e

$(\sqrt{2x-4})^2=(1)^2$

$2x-4=1$
$+4 \quad +4$

$\frac{2x}{2}=\frac{5}{2}$

$\boxed{x=\frac{5}{2}}$

If your answer is 2, draw the following mustache on the lumberjack:
If your answer is 2.5 draw the following mustache on the lumberjack:

14. $e^{2x}-5e^x+6=0$

$(e^x-3)(e^x-2)=0$

$e^x=3 \qquad e^x=2$

$x=\ln 3 \qquad x=\ln 2$
$\approx .69 \qquad \approx 1.099$

If your answer is .69 and 1.099 color his beard brown and his mustache grey.
If your answer is 1.792 color his beard grey and his mustache brown.

15. $\log_2(3x-1)=3$

2 2

$3x-1=8$
$+1 \quad +1$

$3x=9$

$\boxed{x=3}$

CHECK:
$\underbrace{\log_2 8}=3$
$\log_2 2^3=3$
$3=3$ ✓

If your answer is 3, the answer to the puzzle is: "because they've got natural logarithm."
If your answer is $\frac{10}{3}$ the answer to the puzzle is: "because they've got great logarithm."

Why do Lumberjacks Make Good Music?

15. BECAUSE THEY'VE GOT NATURAL LOGARITHM!

$\boxed{10.06}$ $\boxed{p.16}$

1) $2^{3x-19} = 4$

$2^{3x-19} = 2^2$

$3x - 19 = 2$

$3x = 21$

$\boxed{x = 7}$

---

2) $\log_2 x + \log_2 (x+7) = 3$

$\log_2 x(x+7) = 3$

$\log_2 (x^2 + 7x) = 3$

$2^3 = x^2 + 7x$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$\cancel{x = -8}$, $x = 1$

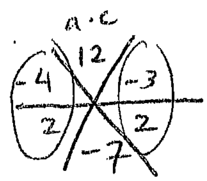
$\boxed{x = 1}$

---

3) $2e^{2x} - 7e^x + 6 = 0$

*let $x = e^x$

* $2x^2 - 7x + 6 = 0$



$(x-2)(x-3/2)$

$(x-2)(2x-3) = 0$

original scenario $(e^x - 2)(2e^x - 3) = 0$

$e^x - 2 = 0$

$e^x = 2$

$\log e^x = \log 2$

$x \log e = \log 2$

$x = \frac{\log 2}{\log e} = \boxed{0.693}$

$2e^x - 3 = 0$

$e^x = \frac{3}{2}$

$\log e^x = \log(3/2)$

$x \log e = \log(3/2)$

$x = \frac{\log(3/2)}{\log e} = \boxed{0.405}$

4) $\log(x+1) - \log(3x-2) = \log\left(\frac{2}{x}\right)$

$\cancel{\log}\left(\frac{x+1}{3x-2}\right) = \cancel{\log}\left(\frac{2}{x}\right)$

$\frac{x+1}{3x-2} = \frac{2}{x}$

$x(x+1) = 2(3x-2)$

$x^2 + x = 6x - 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\boxed{x = 4,\ x = 1}$

5) $7^x = 156$

$\log 7^x = \log 156$

$x \log 7 = \log 156$

$x = \frac{\log 156}{\log 7} = \boxed{2.595}$

6) $3^{x^2-7} = 27^{2x}$

$\cancel{3}^{x^2-7} = \cancel{3}^{3(2x)}$

$x^2 - 7 = 6x$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$\boxed{x = 7,\ x = -1}$

7) $\frac{6\ln(2x)}{6} = \frac{12}{6}$

$\ln(2x) = 2$

$\log_e(2x) = 2$

$e^2 = 2x$

$\frac{e^2}{2} = x$

$\boxed{x = \frac{e^2}{2}} \approx \boxed{3.695}$

8) $\log(x-3) = \log(7x-23) - \log(x+1)$

$\cancel{\log}(x-3) = \cancel{\log}\left(\frac{7x-23}{x+1}\right)$

$\frac{x-3}{1} = \frac{7x-23}{x+1}$

$(x-3)(x+1) = (7x-23)$

$x^2-2x-3 = 7x-23$

$x^2-9x+20=0$

$(x-4)(x-5)=0$

$\boxed{x=4,\ x=5}$

9) $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4(x-1) = 1$

$\log_4\left(\frac{x^2}{x-1}\right) = 1$

$\frac{4^1}{1} = \frac{x^2}{x-1}$

$x^2 = 4x-4$

$x^2-4x+4=0$

$(x-2)(x-2)=0$

$\boxed{x=2}$

10) $\log_2(x+2) - \log_2(x-5) = 3$

$\log_2\left(\frac{x+2}{x-5}\right) = 3$

$2^3 = \frac{x+2}{x-5}$

$\frac{8}{1} = \frac{x+2}{x-5}$

$8(x-5) = x+2$

$8x-40 = x+2$

$7x = 42$

$\boxed{x=6}$

11) $27^{2x+4} = 9^{x-2}$

$\cancel{3}^{3(2x+4)} = \cancel{3}^{2(x-2)}$

$3(2x+4) = 2(x-2)$

$6x+12 = 2x-4$

$4x = -16$

$\boxed{x = -4}$

12) $\log_4(x-6) = -2$

$4^{-2} = x-6$

$\frac{1}{4^2} = x-6$

$\frac{1}{16} + 6 = x$

$x = \frac{1}{16} + \frac{96}{16} = \boxed{\frac{97}{16}}$

13) $\ln\sqrt{2x-4} = 0$

$\log_e \sqrt{2x-4} = 0$

$e^0 = \sqrt{2x-4}$

$1 = \sqrt{2x-4}$

$(1)^2 = \left(\sqrt{2x-4}\right)^2$

$1 = 2x-4$

$5 = 2x$

$\boxed{5/2 = x}$

14) $e^{2x}-5e^{x}+6=0$ $\quad$ *let $x=e^{x}$

* $x^2-5x+6=0$

$(x-3)(x-2)=0$

Bring e^x back → $(e^x-3)(e^x-2)=0$

$e^x-3=0$	$e^x-2=0$
$e^x=3$	$e^x=2$
$\log e^x=\log 3$	$\ln e^x=\ln 2$
$x\log e=\log 3$	$x\cancel{\ln e}=\ln 2$
$x=\frac{\log 3}{\log e}=\boxed{0.691}$	$x=\ln 2$
	$\boxed{x=1.099}$

15) $\log_2(3x-1)=3$

$2^3=3x-1$	$3x=9$
$8=3x-1$	$\boxed{x=3}$ ✓
$9=3x$	

10.07 Solving Exponential and Log Equations Solve the exponential/log equations

1.

$$5^{2x-1} = 5^4$$

$2x-1=4$

$2x=5$

$\boxed{x=5/2}$

2.

$$\frac{3\log_2(x+2)}{3} = \frac{6}{3}$$

$\log_2(x+2)=2$

$2^2=x+2$

$4-2=x$

$\boxed{x=2}$ ✓

3.

$$\ln x + \ln 4 = 2$$

$\ln(4x)=2$

$\log_e(4x)=2$

$e^2=4x$

$\frac{e^2}{4}=x$

$\boxed{x=\frac{e^2}{4}}$ ✓

4.

$$2\log x - \log 4 = 2$$

$\log x^2-\log 4=2$

$\log\left(\frac{x^2}{4}\right)=2$

$\log_{10}\left(\frac{x^2}{4}\right)=2$

$10^2=\frac{x^2}{4}$

$10^2\cdot 4=x^2$

$400=x^2$

$x=\pm\sqrt{400}$

$\boxed{x=20}$ $\cancel{x=-20}$ Extraneous solution

5.

$$e^{2x} = 25$$

$\ln e^{2x}=\ln 25$

$2x\cancel{\ln e}=\ln 25$

$\boxed{x=\frac{\ln 25}{2}}$

6.

$$\log(3x-5) = \log(2x-1)$$

$3x-5=2x-1$

$1x=4$

$\boxed{x=4}$ ✓

7.

$$\frac{1}{25} = 5^{3x+2}$$

$5^{-2}=5^{3x+2}$

$3x+2=-2$

$3x=-4$

$\boxed{x=\frac{-4}{3}}$

8.

$$3^{2x-7} = 27^x$$

$3^{2x-7}=3^{3x}$

$3x=2x-7$

$\boxed{x=-7}$

9.

$$4^{x-1} = 4^3$$

$x - 1 = 3$

$\boxed{x = 4}$

10.

$$5^{2x+3} = 125^x$$

$5^{2x+3} = 5^{3x}$

$2x + 3 = 3x$

$3 = x$

$\boxed{x = 3}$

11.

$$\log(6x) = \log(4x+5)$$

$6x = 4x + 5$

$2x = 5$

$\boxed{x = 5/2}$ ✓

12.

$$\frac{2\ln e^x}{2} = \frac{9}{2}$$

$\ln e^x = \frac{9}{2}$

$x\cancel{\ln e} = 9/2$

$\boxed{x = 9/2}$

13.

$$2\log(3x) - \log 9 = 1$$

$\log(3x)^2 - \log 9 = 1$

$\log\left(\frac{9x^2}{9}\right) = 1$

$\log(x^2) = 1$

$\log_{10}(x^2) = 1$

$10^1 = x^2$

$\boxed{x = \sqrt{10}}$, $-\sqrt{10}$ extraneous solution

14.

$$2\ln x + \ln x^2 = 4$$

$\ln x^2 + \ln x^2 = 4$

$\ln x^4 = 4$

$\frac{4\ln x}{4} = \frac{4}{4}$

$\ln x = 1$

$\log_e x = 1$

$e^1 = x$

$\boxed{x = e}$ ✓

15.

$$\frac{1}{16} = 4^{3x-1}$$

$4^{-2} = 4^{3x-1}$

$3x - 1 = -2$

$3x = -1$

$\boxed{x = -1/3}$

16.

$$\frac{3\ln e^{2x}}{3} = \frac{30}{3}$$

$\ln e^{2x} = 10$

$2x\cancel{\ln e} = 10$

$2x = 10$

$x = \frac{10}{2}$

$\boxed{x = 5}$ ✓

17.

$\ln e^{x+5} = 17$

$(x+5)\cancel{\ln e} = 17$

$x+5 = 17$

$\boxed{x = 12}$ ✓

18.

$\log x - \log 4 = 3$

$\log\left(\frac{x}{4}\right) = 3$

$\log_{10}\left(\frac{x}{4}\right) = 3$

$10^3 = \frac{x}{4}$

$1000 \cdot 4 = x$

$\boxed{x = 4000}$ ✓

19.

$\log(8-3x) = \log(7-5x)$

$8 - 3x = 7 - 5x$

$1 = -2x$

$\boxed{-\frac{1}{2} = x}$ ✓

20.

$\frac{5\ln(3x-2)}{5} = \frac{15}{5}$

$\ln(3x-2) = 3$

$\log_e(3x-2) = 3$

$e^3 = 3x - 2$

$\frac{e^3 + 2}{3} = \frac{3x}{3}$

$\boxed{x = \frac{e^3+2}{3}}$ ✓

21.

$\log(3x-2) = 3$

$\log_{10}(3x-2) = 3$

$10^3 = 3x - 2$

$1000 = 3x - 2$

$1002 = 3x$

$\frac{1002}{3} = x$

✓ $\boxed{x = 334}$

22.

$4^{x-1} = 64^x$

$4^{x-1} = 4^{3(x)}$

$x - 1 = 3x$

$-1 = 2x$

$\boxed{-\frac{1}{2} = x}$

23.

$\frac{4\ln x}{4} = \frac{-2}{4}$

$\ln x = -\frac{1}{2}$

$\log_e x = -\frac{1}{2}$

$e^{-\frac{1}{2}} = x \rightarrow \boxed{x = \frac{1}{e^{1/2}}}$ ✓

$\ln x^4 = -2$

$\log_e x^4 = -2$

$e^{-2} = x^4$

$\frac{1}{e^2} = x^4$

$\sqrt[4]{\frac{1}{e^2}} = x$

$x = \left(\frac{1}{e^2}\right)^{1/4} = \boxed{\frac{1}{e^{1/2}}}$

24.

$e^{x-4} = 2$

$\ln e^{(x-4)} = \ln 2$

$(x-4)\cancel{\ln e} = \ln 2$

$x - 4 = \ln 2$

$\boxed{x = 4 + \ln 2}$

25.

$\ln x + \ln 4x = 16$

$\ln x(4x) = 16$

$\ln(4x^2) = 16$

$\log_e(4x^2) = 16$

$e^{16} = 4x^2$

$\frac{e^{16}}{4} = x^2$

$\pm\sqrt{\frac{e^{16}}{4}} \rightarrow \left(\frac{e^{16}}{4}\right)^{1/2} \rightarrow \boxed{x = \frac{e^8}{2}}, \cancel{-\frac{e^8}{2}}$ extraneous solution

26.

$\frac{4\log x}{4} = \frac{-4}{4}$

$\log_{10} x = -1$

$10^{-1} = x$

$\boxed{x = \frac{1}{10}}$ ✓

27.

$\frac{1}{4} = 2^{2x-3}$

$\cancel{2}^{-2} = \cancel{2}^{2x-3}$

$2x - 3 = -2$

$2x = 1$

$\boxed{x = 1/2}$

28.

$\log 5x = \log(9 + 8x)$

$5x = 9 + 8x$

$-3x = 9$

$\cancel{x = -3}$ Extraneous solution $\boxed{\text{No solution}}$

29.

$\ln(x-1) - \ln 2 = 3$

$\ln\left(\frac{x-1}{2}\right) = 3$

$\log_e\left(\frac{x-1}{2}\right) = 3$

$e^3 = \frac{x-1}{2}$

$\cancel{\frac{e^3}{1} = \frac{x-1}{2}}$

$x - 1 = 2e^3$

$\boxed{x = 2e^3 + 1}$ ✓

30.

$\ln x = -1$

$\log_e x = -1$

$e^{-1} = x$

$\boxed{x = \frac{1}{e}}$

31.

$e^{\frac{x}{5}} = 32$

$\ln e^{\frac{x}{5}} = \ln 32$

$\frac{x}{5}\cancel{\ln e} = \ln 32$

$\boxed{x = 5\ln 32}$

32.

$\frac{-2\log_3 6x}{-2} = \frac{2}{-2}$

$\log_3(6x) = -1$

$3^{-1} = 6x$

$\frac{1}{3} = 6x$

$\frac{1}{3} \cdot \frac{1}{6} = x$

$\boxed{x = \frac{1}{18}}$ ✓

Find Inverse:
1) swap x and y
2) solve for y

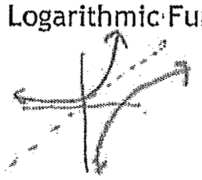
10.08 Intro to Graphing Logarithmic Functions

Date: ____________

Graph $f(x) = 2^x$

$2^{-1} = \frac{1}{2}$ | $2^1 = 2$

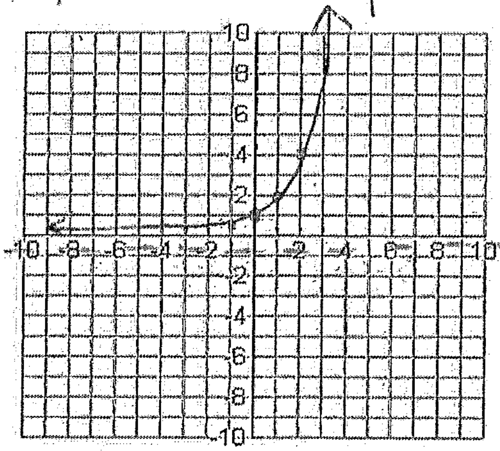
$2^0 = 1$ | $2^2 = 4$



$y = 2^x$

$x = 2^y$

x	y
−1	1/2
0	1
1	2
2	4



Find the inverse of $f(x) = 2^x$ then graph it

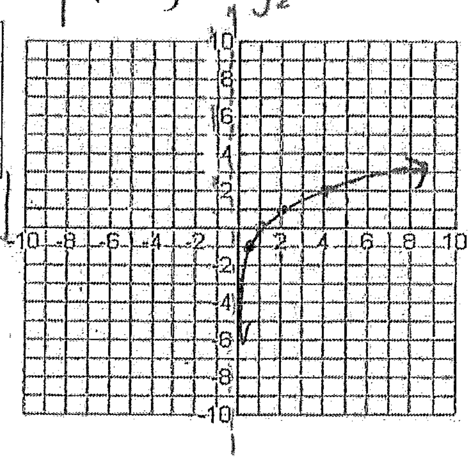
$2^y = x$

$\log_2 2^y = \log_2 x$

$y \cancel{\log_2 2} = \log_2 x$

$f(x) =$ $f(x) = \log_2 x$

x	y
1/2	−1
1	0
2	1
4	2



ℝ

Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Asymptote: $y = 0$ x-intercept: none

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-intercept: $(1, 0)$

What do you notice about the two graphs? Ordered pairs are switched b/t function and inverse

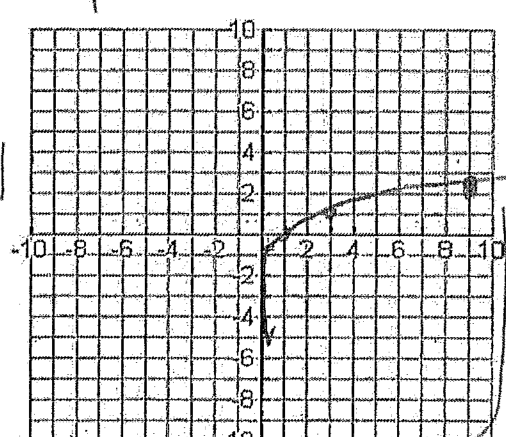
Graph $f(x) = \log_3 x$

$y = \log_3 x$

$3^y = x$

x	y
$1/\sqrt{3}$	$-1/2$
1	0
3	1
9	2

$3^{-1/2} = \frac{1}{\sqrt{3}}$



x	y
1	0
3	1
3^2	2
3^3	3

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-intercept: $(1, 0)$

$f(x) = \ln x$

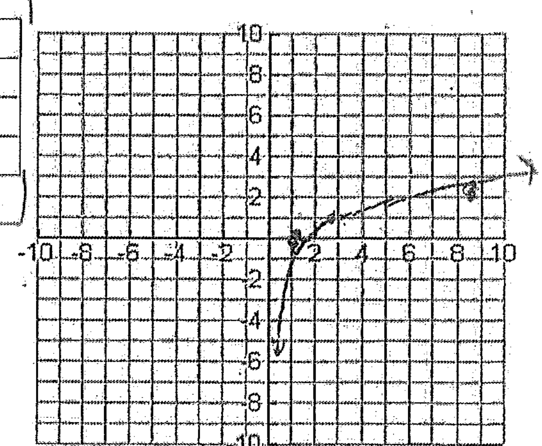
$y = \log_e x$

$e^y = x$

x	y
$1/\sqrt{e}$	$-1/2$
1	0
e	1
e^2	2

$e^{-1/2} = \frac{1}{\sqrt{e}}$

$e \approx 2.7$



x	y
e	1
e^2	2
e^3	3
1	0

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-intercept: $(1, 0)$

10.09 Graphing Logarithmic Functions

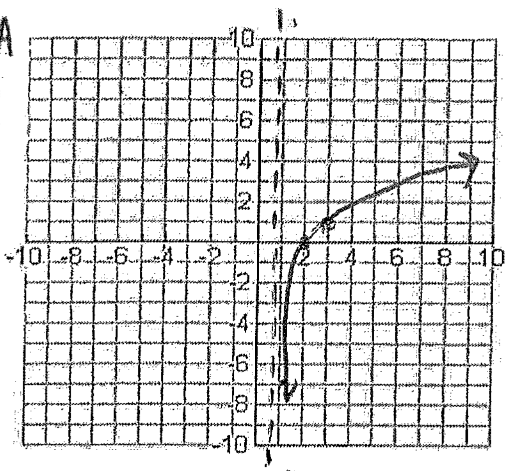
Helpful Log Characteristics

1) $\log_b(x)$ set argument = 0 to find V.A.
2) $\log_b(1) = 0$
3) $\log_b(b) = 1$
4) $\log_b\left(\frac{1}{b}\right) = -1$

Graph the following functions. $x - 1 = 0$

1. $f(x) = \log_2(x-1)$ $x = 1$

x	y
1	VA
2	0
3	1

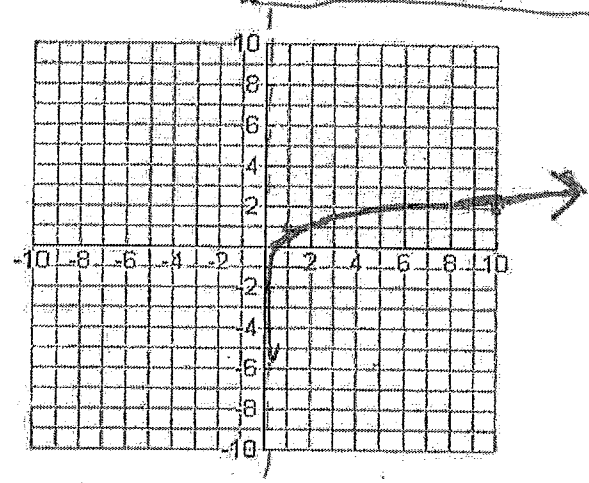


Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 1$ x-intercept: $(2, 0)$

2. $f(x) = \log_{10}(x) + 1$

x	y
0	VA
1	1
10	2
$\frac{1}{10}$	0



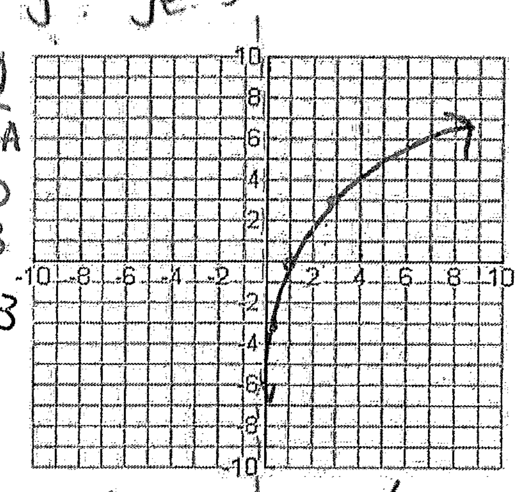
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-intercept: $\left(\frac{1}{10}, 0\right)$

3. $f(x) = 3\ln x$ VA $x = 0$

$y = 3\log_e(x)$

x	y
0	VA
1	0
e	3
$\frac{1}{e}$	-3

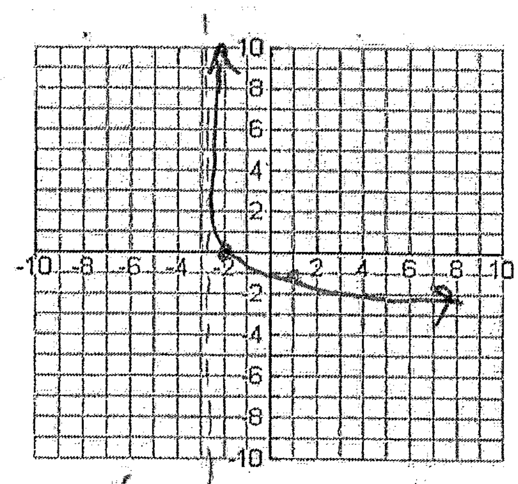


Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-intercept: $(1, 0)$

4. $f(x) = -\log_4(x+3)$ $x + 3 = 0$ $x = -3$ (VA)

x	y
-3	VA
-2	0
1	-1



Domain: $(-3, \infty)$ Range: $(-\infty, \infty)$

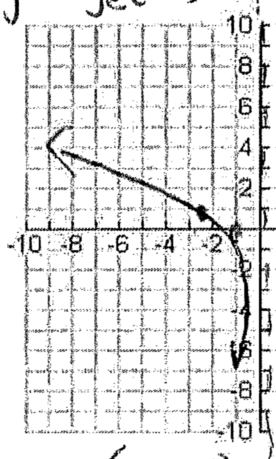
Asymptote: $x = -3$ x-intercept: $(-2, 0)$

5. $f(x) = \ln(-x)$

$y = \log_e(-x)$

VA: $x=0$

x	y
-1	0
-e	1



Domain: $(-\infty, 0)$ Range: $(-\infty, \infty)$

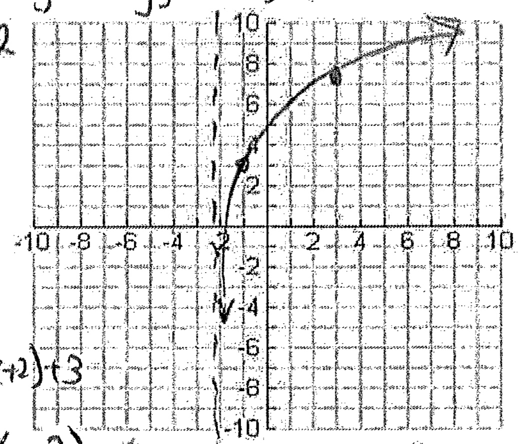
Asymptote: $x=0$ x-intercept: $(-1, 0)$

6. $f(x) = 4\log_5(x+2)+3$

$y = 4\log_5(x+2)+3$

VA: $x=-2$

x	y
-1	3
3	7



$0 = 4\log_5(x+2)+3$

$\frac{-3}{4} = \log_5(x+2)$

$5^{-3/4} = x+2 \rightarrow \boxed{x = 5^{-3/4}-2}$

Domain: $(-2, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=-2$ x-intercept: $(5^{-3/4}-2, 0)$

Practice:

Sketch and analyze the graph of the function. Describe the domain, range, intercepts, asymptote, end behavior, and where the function is increasing or decreasing.

1. $f(x) = log_{\frac{1}{4}}x$

$y = \log_{\frac{1}{4}}(x)$

VA: $x=0$

As $x \to \infty$, $f(x) \to -\infty$

As $x \to 0$, $f(x) \to +\infty$

x	y
1	0
1/4	1

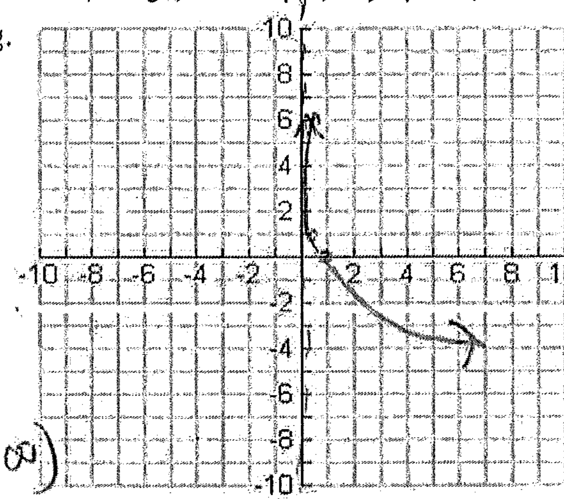
D: $(0, \infty)$

R: $(-\infty, \infty)$

VA: $x=0$

X-int: $(1, 0)$

* $f(x)$ is decreasing $(0, \infty)$



Use the graph of f(x) to describe the transformation that results in g(x) then sketch both graphs.

2. $f(x) = logx$; $g(x) = -\log(x-2)$ parent

* Transformations:

$y = \ominus a\log(x-b)+c$

Reflection X-axis; vertical stretch or compress; shift left(+) right(-); shift up(+) down(-)

g(x) transformations

i) Reflection x-axis

ii) shift Right 2 units.

f(x)

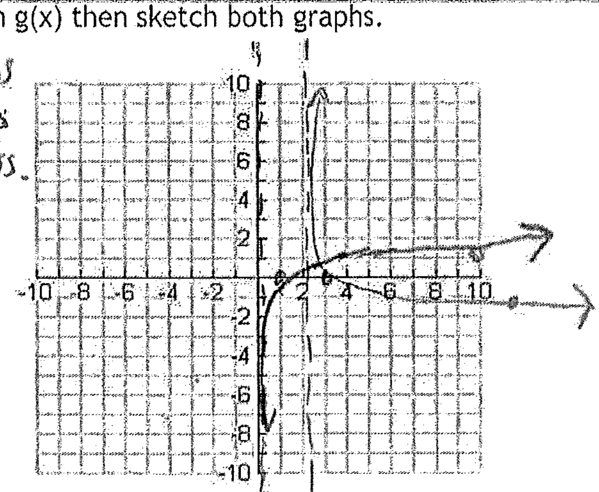
VA: $x=0$

x	y
1	0
10	1

g(x)

VA: $x=2$

x	y
3	0
12	-1



3. $f(x) = \ln x;\ g(x) = 3\ln(x) + 1$

Transformations:

$g(x)$ vertical stretch by 3

$g(x)$ vertical shift up 1

$y = \ln x$

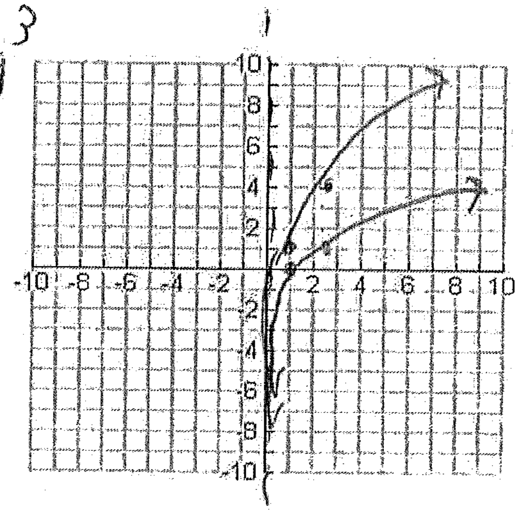
VA: $x = 0$

x	y
1	0
e	1

$y = 3\log_e x + 1$

VA: $x = 0$

x	y
1	1
e	4

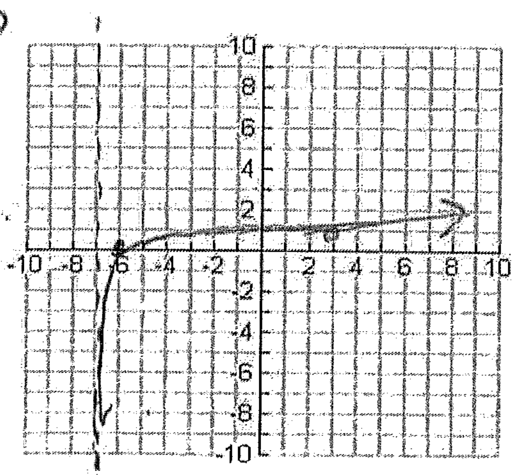


Determine the domain, range, x-intercept, and asymptote.

4. $y = \log(x + 7)$

VA: $x = -7$

x	y
-6	0
3	1



D: $(-7, \infty)$

R: $(-\infty, \infty)$

x-int: $(-6, 0)$

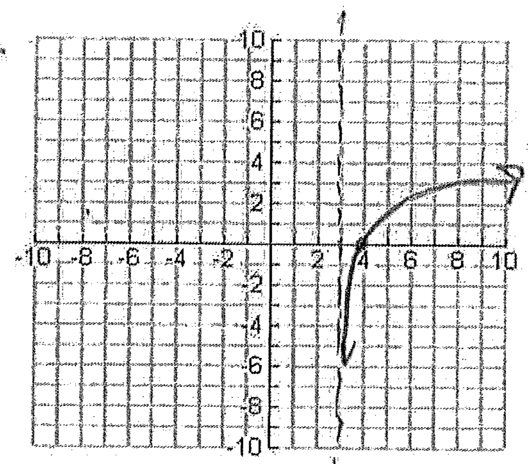
VA: $x = -7$

5. $y = \ln(x - 3)$

VA: $x = 3$

$y = \log_e(x - 3)$

x	y
4	0



D: $(3, \infty)$

R: $(-\infty, \infty)$

x-int: $(4, 0)$

VA: $x = 3$

8.10 Log Test Review

Date: _______________

Complete each problem without using a calculator.

Write each logarithm in exponential form.

Write each exponential in logarithmic form.

1. $log_{49} 7 = \frac{1}{2}$

$49^{1/2} = 7$

2. $log_{10} x = 4.5$

$10^{4.5} = x$

3. $3^{-4} = \frac{1}{81}$

$\log_3\left(\frac{1}{81}\right) = -4$

4. $e^5 = 148.413$

$\log_e 148.413 = 5$

$\boxed{\ln 148.413 = 5}$

Evaluate.

5. $log_5 \sqrt[3]{25} = x$

$5^x = \sqrt[3]{25} = \sqrt[3]{5^2}$

$5^x = 5^{2/3}$

$\boxed{x = 2/3}$

6. $log_2 \frac{1}{16} = x$

$2^x = \frac{1}{16} = 2^{-4}$

$\boxed{x = -4}$

7. $log_6(-36) = x$

$6^{-x} = -36$

$\boxed{\text{No solution}}$

8. $log_{10} 0.01 = x$

$10^x = 0.01 = \frac{1}{100}$

$10^x = 10^{-2}$

$\boxed{x = -2}$

9. $ln\, e^4 = x$

$\log_e e^4 = x$

$4\log_e e = x$

$\boxed{x = 4}$

10. $log_9 \sqrt[4]{3} = x$

$9^x = \sqrt[4]{3}$

$3^{2x} = 3^{1/4}$

$2x = \frac{1}{4}$

$x = \frac{1}{2} \cdot \frac{1}{4}$

$\boxed{x = \frac{1}{8}}$

11. $log_3 27 + 2\, log_5 25$

$\log_3 3^3 + 2\log_5 5^2$

$3\cancel{\log_3 3} + 2 \cdot 2\cancel{\log_5 5}$

$3 + 2(2) = \boxed{7}$

12. $8\, log_2 \sqrt{32}$

$8\log_2 \sqrt{2^5}$

$8\log_2 2^{5/2}$

$8 \cdot \frac{5}{2}\cancel{\log_2 2}$

$8\left(\frac{5}{2}\right) = \boxed{20}$

Use properties of logs to expand. Simplify, if possible.

13. $log_9 \frac{3x^4}{y}$

$\log_9 3x^4 - \log_9 y$

$\boxed{\log_9 3 + 4\log_9 x - \log_9 y}$

14. $log_3 \sqrt[5]{x^2y^3z^4}$

$\log_3 (x^2y^3z^4)^{1/5}$

$\frac{1}{5}\log_3(x^2y^3z^4) \to \frac{1}{5} \cdot 2\log_3 x + \frac{1}{5} \cdot 3\log_3 y + \frac{1}{5} \cdot 4\log_3 z$

$\to \boxed{\frac{2}{5}\log_3 x + \frac{3}{5}\log_3 y + \frac{4}{5}\log_3 z}$

15. $ln \sqrt[5]{x^3(x+1)}$

$\ln\left[x^3(x+1)\right]^{1/5} \to \frac{1}{5}\ln x^3 + \frac{1}{5}\ln(x+1)$

$\boxed{\frac{3}{5}\ln x + \frac{1}{5}\ln(x+1)}$

Use properties of logs to condense. Simplify, if possible.

16. $5log_4 a + 6log_4 b - \frac{1}{3}log_4 7c$

$\log_4 a^5 + \log_4 b^6 - \log_4 (7c)^{1/3} \rightarrow \boxed{\log_4\left(\frac{a^5b^6}{\sqrt[3]{7c}}\right)}$

17. $2\log(x+1) - \log(x^2-1)$

$\rightarrow \log(x+1)^2 - \log(x^2-1)$

$\rightarrow \log\left(\frac{(x+1)^2}{x^2-1}\right) \rightarrow \log\left(\frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)}\right) \rightarrow \boxed{\log\left(\frac{x+1}{x-1}\right)}$

18. $\frac{5}{2}lnx + \frac{1}{2}\ln(y+8) - 3lny - \ln(10-x)$

$\ln x^{5/2} + \ln(y+8)^{1/2} - \ln y^3 - \ln(10-x) \rightarrow \boxed{\ln\left(\frac{x^{5/2}(y+8)^{1/2}}{y^3(10-x)}\right)}$

Solve. Write answers in simplest form.

13. $9^{3x+1} = 81$

$3^{2(3x+1)} = 3^4$

$2(3x+1) = 4$

$6x+2 = 4$

$6x = 2$ $\boxed{x = 1/3}$

14. $125^{x-2} = 25^{2x+1}$

$5^{3(x-2)} = 5^{2(2x+1)}$

$3x - 6 = 4x + 2$

$-8 = 1x$

$\boxed{x = -8}$

15. $4e^{2x} - 13 = 5$

$4e^{2x} = 18$

$e^{2x} = \frac{18}{4} = 4.5$

$\ln e^{2x} = \ln 4.5$

$2x \cancel{\ln e} = \ln 4.5$

$2x = \ln 4.5$

$\boxed{x = \frac{\ln 4.5}{2}}$

16. $4^{x-1} = 12$

$\log 4^{x-1} = \log 12$

$(x-1)\log 4 = \log 12$

$x - 1 = \frac{\log 12}{\log 4}$

$\boxed{x = \frac{\log 12}{\log 4} + 1}$

17. $e^{2x} - 3e^x = 10$ $x = e^x$

$e^{2x} - 3e^x - 10 = 0$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$(e^x - 5)(e^x + 2) = 0$

$e^x - 5 = 0$

$e^x = 5$

$\ln e^x = \ln 5$

$\boxed{x = \ln 5}$

$e^x + 2 = 0$

$e^x = -2$

$\ln e^x = \ln(-2)$

$\boxed{\text{No solution}}$

18. $3e^{4x} - 5e^{2x} - 2 = 0$

$x = e^x$

$3x^4 - 5x^2 - 2 = 0$

-6, -6, 1, 3, -5, 3

$(x^2 - 2)(x^2 + \frac{1}{3})$

$(x^2 - 2)(3x^2 + 1)$

$(e^{2x} - 2)(3e^{2x} + 1) = 0$

$e^{2x} - 2 = 0$

$e^{2x} = 2$

$\ln e^{2x} = \ln 2$

$2x \cancel{\ln e} = \ln 2$

$\boxed{x = \frac{\ln 2}{2}}$

$3e^{2x} + 1 = 0$

$3e^{2x} = -1$

$e^{2x} = -1/3$

$\ln e^{2x} = \ln(-1/3)$

$\boxed{\text{No solution}}$

19. $4^{2x+3} = 11^{2-x}$

$\log 4^{2x+3} = \log 11^{2-x}$

$(2x+3)\log 4 = (2-x)\log 11$

$2x\log 4 + 3\log 4 = 2\log 11 - x\log 11$

$2x\log 4 + x\log 11 = 2\log 11 - 3\log 4$

$x(2\log 4 + \log 11) = 2\log 11 - 3\log 4$

$\boxed{x = \frac{2\log 11 - 3\log 4}{2\log 4 + \log 11}}$

Solve.

20. $log_8(3x+1)=2$

$8^2=3x+1$

$64=3x+1$

$\frac{63}{3}=\frac{3x}{3}$

$\boxed{21=x}$ ✓

21. $ln(3x)+5=5$

$ln(3x)=0$

$log_e(3x)=0$

$e^0=3x$

$1=3x$

$\boxed{\frac{1}{3}=x}$ ✓

22. $log_4 x+log_4(x+6)=2$

$log_4 x(x+6)=2$

$log_4(x^2+6x)=2$

$4^2=x^2+6x$

$0=x^2+6x-16$

$0=(x+8)(x-2)$

$x=\cancel{-8}$, $\boxed{x=2}$

extraneous solution.

23. $ln(4x^2)=2\,ln\,(x+4)$

$ln(4x^2)=ln(x+4)^2$

$4x^2=(x+4)^2$

$4x^2=(x+4)(x+4)$

$4x^2=x^2+8x+16$

$3x^2-8x-16=0$

$(3x+4)(x-4)=0$

$3x+4=0$ | $x-4=0$

$\boxed{x=\frac{-4}{3}}$ | $\boxed{x=4}$

24. $log_7(x+6)-log_7(2x)=log_7(x+1)$

$log_7\left(\frac{x+6}{2x}\right)=log_7(x+1)$

$\frac{x+6}{2x}=\frac{x+1}{1}$

$2x(x+1)=x+6$

$2x^2+2x=x+6$

$2x^2+1x-6=0$

$(2x-3)(x+2)=0$

$2x-3=0$ | $x+2=0$

$\boxed{x=\frac{3}{2}}$ | $x=-2$

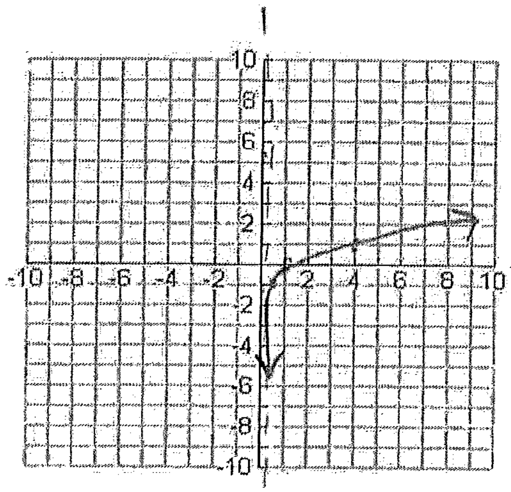
(extraneous solution)

$*\log_b \frac{1}{b} = -1$ $\quad *\log_b b = 1$ $\quad *\log_b(0) \rightarrow$ Vertical Asymptote

$*\log_b 1 = 0$

25. $f(x) = log_4 x$ Graph the parent function, *f(x)*. State its asymptote, domain, range, and x-intercept

$y = \log_4 x$



x	y
0	VA: $x=0$
$\frac{1}{4}$	-1
1	0
4	1

Asymptote: VA: $X = 0$

Domain: $(0, \infty)$

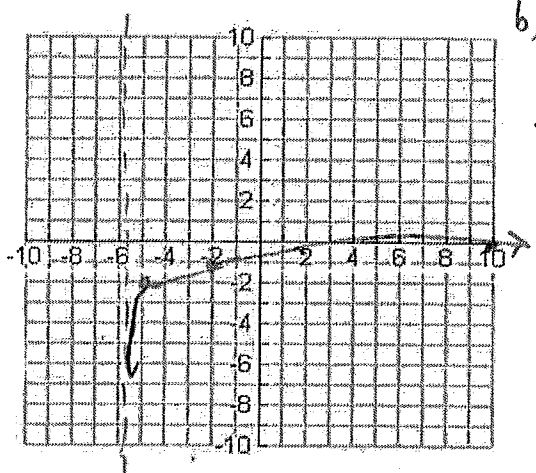
Range: $(-\infty, \infty)$

X-Intercept: $(1, 0)$

Next, analyze the other functions as transformations of *f(x) from #25 above*. Graph each. Then state its asymptote, domain and range.

26. $g(x) = log_4(x+6) - 2$

Transformations to map *f(x)* onto *g(x)*: a) translated (shift) left 6 units
b) shift down 2 units



x	y
-6	VA
-5	-2
-2	-1

Asymptote: $X = -6$

Domain: $(-6, \infty)$

Range: $(-\infty, \infty)$

X-int: $(10, 0)$

VA: $x+6=0$

$x=-6$

$\boxed{\text{* to find x-int, set } y=0}$

$0 = \log_4(x+6) - 2$

$2 = \log_4(x+6)$

$4^2 = x+6$

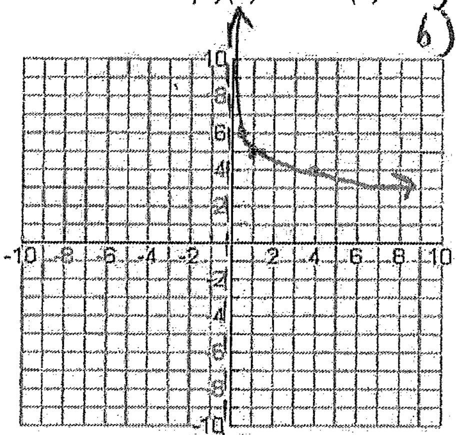
$16 = x+6$

$\boxed{10 = x}$

27. $h(x) = -log_4(x) + 5$

Transformations to map $f(x)$ onto $h(x)$: a) reflection over x-axis
b) vertical translation (shift up) 5 units

x	y
0	V.A.
1	5
4	4
$\frac{1}{4}$	6



Asymptote: VA: $x=0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

X-int: $(4^5, 0)$

x-int: (set $y=0$)

$0 = -\log_4(x) + 5$

$-5 = -\log_4(x)$

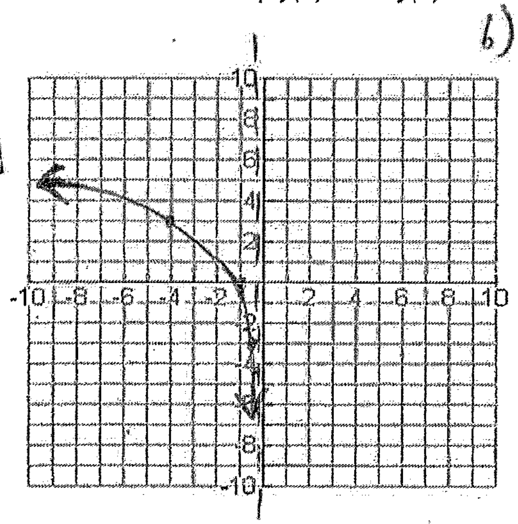
$5 = \log_4 x$

$\boxed{4^5 = x}$

28. $j(x) = 3log_4(-x)$

Transformations to map $f(x)$ onto $j(x)$: a) Vertical stretch, factor of 3
b) reflection over y-axis

x	y
0	VA
-1	0
-4	3
$-\frac{1}{4}$	-3



Asymptote: $x=0$

Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

X-int: $(-1, 0)$

Log Properties:

1) Product Property: $\log uv = \log u + \log v$

2) Quotient Property: $\log\left(\frac{u}{v}\right) = \log u - \log v$

3) Power Property: $\log u^n = n \cdot \log u$

4) $\log\left(\frac{ab}{cde}\right) = \log a + \log b - \log c - \log d - \log e$

5) $\log(u+v) \neq \log u + \log v$

6) $\log_e e^x = x \rightarrow \ln e^x = x$

7) $e^{\log_e x} = x$

Graphing Log Functions:

Helpful Log Characteristics

1) $\log_b(x)$ ← *set the log argument $=0$ to find the Vertical Asymptote.

2) $\log_b(1) = 0$

3) $\log_b(b) = 1$

*4) $\log_b\left(\frac{1}{b}\right) = -1$

---

* $\log_b(b^x) = x$

* $b^{\log_b(x)} = x$

Exponentials and Logs Test Review WS #2

Solve each of the following exponential equations. Round to three decimals when necessary.

1. $2^x = 7$

2. $4^{x+1} = 3$

3. $7 \cdot e^{x-3} = 57$

4. $8e^{2x} = 20$

5. $e^{3-2x} = 4$

6) $5^{2x-1} = 7^{1-x}$

7. $4^x - 5 = 3$

8. $4 - 2e^x = -23$

9. $3^{x+1} = 3^2$

Solve the following logarithmic equations. Round to three decimals when necessary.Check your answer

10. $\ln x = 8$

11. $\log_2(x+2) = 5$

12. $\log_7(25-x) = 3$

13. $4 + 3\log(2x) = 16$

14. $\log(x+2) + \log(x-1) = 1$

15. $5\ln(3-x) = 4$

16. $\log_2(x+2) = \log_2 x^2$

17. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

18. $-5 + 2\ln 3x = 5$

19. $\log_5(-4r-8) = \log_5(r+7)$

Condense each expression to a single logarithm.

24. $2\log_7 x - 4\log_7 y$

25 $5\log_9 a + 15\log_9 b$

26. $3\log_2 x - 4\log_2(x+3)$

Expand each logarithm.

27. $\log_2(x^2y)$

28. $\log_6\left(\frac{a^4}{b}\right)$

29. $\log_2(\frac{8x^4}{5})$

Rewrite each into logarithmic form:

33. $3^x = 12$

34. $2^{-1} = \frac{1}{2}$

35. $e^x = 15$

Rewrite each into exponential form:

36. $\log_{49} 7 = \frac{1}{2}$

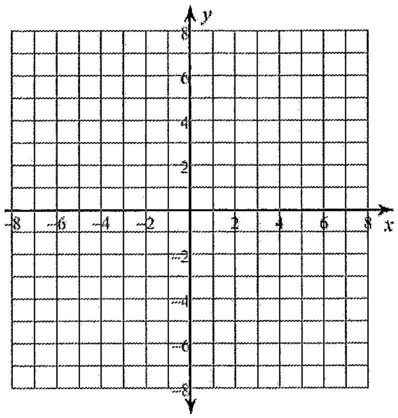
37. $\ln 14 = x$

38. $\log_2 \frac{1}{4} = -2$

Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

39)

$y = \log_3(x-1) - 3$



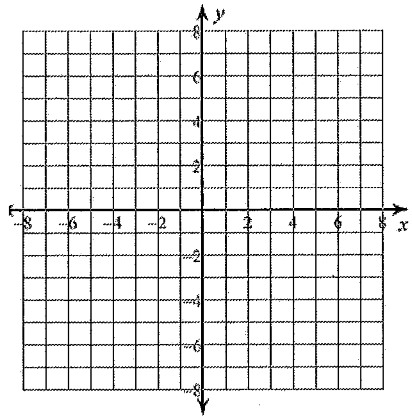
x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

40)

$y = \log_{\frac{1}{3}}(x+4)$

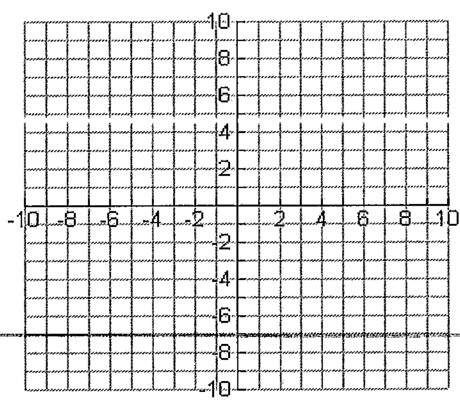


x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

41) $g(x) = -2\ln x + 3$

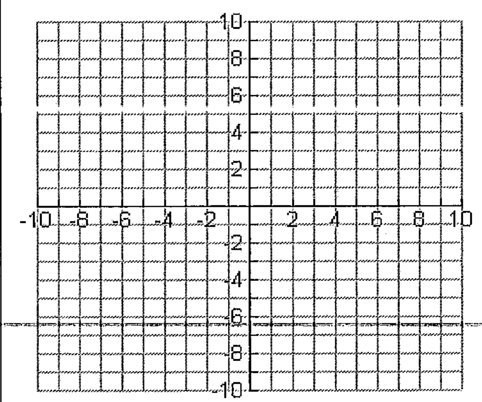


x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

42) $f(x) = \log_2(x+6) - 1$



x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

Key

Exponentials and Logs Test Review WS #2

Solve each of the following exponential equations. Round to three decimals when necessary.

1. $2^x = 7$

$\log 2^x = \log 7$

$x \log 2 = \log 7$

$x = \frac{\log 7}{\log 2} = \boxed{2.807}$

2. $4^{x+1} = 3$

$\log 4^{x+1} = \log 3$

$(x+1)\log 4 = \log 3$

$(x+1) = \frac{\log 3}{\log 4}$

$x+1 = 0.792$

$\boxed{x = -0.208}$

3. $7 \cdot e^{x-3} = 57$

$e^{x-3} = \frac{57}{7} = 8.143$

$e^{x-3} = 8.143$

$\ln e^{x-3} = \ln 8.143$

$(x-3)\cancel{\ln e} = \ln 8.143$

$x = 3 + 2.097$

$\boxed{x = 5.097}$

4. $\frac{8e^{2x}}{8} = \frac{20}{8}$

$e^{2x} = 2.5$

$\ln e^{2x} = \ln 2.5$

$2x \cancel{\ln e} = \ln 2.5$

$x = \frac{\ln 2.5}{2} =$

$\boxed{x = 0.458}$

5. $e^{3-2x} = 4$

$\ln e^{3-2x} = \ln 4$

$(3-2x)\cancel{\ln e} = \ln 4$

$3 - 2x = 1.386$

$-2x = -3 - 1.386$

$\frac{-2x}{-2} = \frac{-1.6137}{-2}$

$\boxed{x = 0.807}$

6) $5^{2x-1} = 7^{1-x}$

$\log 5^{2x-1} = \log 7^{1-x}$

$(2x-1)\log 5 = (1-x)\log 7$

$2x\log 5 - \log 5 = \log 7 - x\log 7$

$2x\log 5 + x\log 7 = \log 7 + \log 5$

$x(2\log 5 + \log 7) = \log 7 + \log 5$

$x = \frac{\log 7 + \log 5}{2\log 5 + \log 7} \approx \boxed{0.688}$

7. $4^x - 5 = 3$

Method 1

$4^x = 8$

$\log 4^x = \log 8$

$x \log 4 = \log 8$

$x = \frac{\log 8}{\log 4} \approx \boxed{\frac{3}{2} = 1.5}$

#7 Method 2

$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$\boxed{x = 3/2}$

8. $4 - 2e^x = -23$

$-4 \quad -4$

$\frac{-2e^x}{-2} = \frac{-27}{-2}$

$e^x = 13.5$

$\ln e^x = \ln 13.5$

$x \cancel{\ln e} = \ln 13.5$

$\boxed{x = 2.603}$

9. $3^{x+1} = 3^2$

$x + 1 = 2$

$\boxed{x = 1}$

Solve the following logarithmic equations. Round to three decimals when necessary. Check your answer

10. $\ln x = 8$

$\log_e x = 8$

$e^8 = x$

$\boxed{x = e^8}$ ✓

11. $\log_2(x+2) = 5$

$2^5 = x + 2$

$32 = x + 2$

$\boxed{30 = x}$ ✓

12. $\log_7(25 - x) = 3$

$7^3 = 25 - x$

$x = 25 - 7^3$

$\boxed{x = -318}$

13. $4 + 3\log(2x) = 16$

$-4 \qquad -4$

$\frac{3\log_{10}(2x)}{3} = \frac{12}{3}$

$\log_{10}(2x) = 4$

$10^4 = 2x$

$\frac{10^4}{2} = x$

$\boxed{x = 5000}$ ✓

14. $\log(x+2) + \log(x-1) = 1$

$\log(x+2)(x-1) = 1$

$\log_{10}(x^2 + x - 2) = 1$

$10^1 = x^2 + x - 2$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$\cancel{x = -4}$ $\boxed{x = 3}$ ✓

extraneous solution

15. $5\ln(3 - x) = 4$

$\log_e(3-x) = \frac{4}{5}$

$\log_e(3-x) = 0.8$

$e^{0.8} = 3 - x$

$x = 3 - e^{0.8}$

$\boxed{x = 0.774}$ ✓

16. $\log_2(x+2) = \log_2 x^2$

$x + 2 = x^2$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

✓ $\boxed{x = 2}$ $\boxed{x = -1}$ ✓

17. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$

$\frac{x+5}{1} = \frac{x-1}{x+1}$

$x - 1 = (x+5)(x+1)$

$x - 1 = x^2 + 5x + 1x + 5$

$0 = x^2 + 5x + 6$

$0 = (x+3)(x+2)$

$\cancel{x = -3}, \cancel{x = -2}$

$\boxed{\text{No solution}}$

18. $-5 + 2\ln 3x = 5$

$\frac{2\ln 3x}{2} = \frac{10}{2}$

$\ln(3x) = 5$

$\log_e(3x) = 5$

$e^5 = 3x$

$\frac{e^5}{3} = x$ $\boxed{x = \frac{e^5}{3}}$ ✓

19. $\log_5(-4r - 8) = \log_5(r + 7)$

$-4r - 8 = r + 7$

$-15 = 5r$

$\frac{-15}{5} = r$

$\boxed{r = -3}$ ✓

Condense each expression to a single logarithm.

24. $2\log_7 x - 4\log_7 y$

$\log_7 x^2 - \log_7 y^4$

$\boxed{\log_7\left(\frac{x^2}{y^4}\right)}$

25 $5\log_9 a + 15\log_9 b$

$\log_9 a^5 + \log_9 b^{15}$

$\boxed{\log_9 a^5 b^{15}}$

26. $3\log_2 x - 4\log_2(x+3)$

$\log_2 x^3 - \log_2(x+3)^4$

$\boxed{\log_2\left(\frac{x^3}{(x+3)^4}\right)}$

Expand each logarithm.

27. $\log_2(x^2y)$

$\boxed{2\log_2 x + \log_2 y}$

28. $\log_6\left(\frac{a^4}{b}\right)$

$\boxed{4\log_6 a - \log_6 b}$

29. $\log_2\left(\frac{8x^4}{5}\right)$

$\log_2 8 + \log_2 x^4 - \log_2 5$

$\boxed{\log_2 8 + 4\log_2 x - \log_2 5}$

Rewrite each into logarithmic form:

33. $3^x = 12 \rightarrow \boxed{\log_3 12 = x}$

$\cancel{\log 3^x = \log 12}$

$\cancel{x\log 3 = \log 12}$

$\cancel{x = \frac{\log 12}{\log 3} = \boxed{2.262}}$

No need to solve

34. $2^{-1} = \frac{1}{2}$

$\boxed{\log_2\left(\frac{1}{2}\right) = -1}$

35. $e^x = 15$

$\log_e 15 = x$

$\boxed{\ln 15 = x}$

OR:

$\ln e^x = \ln 15$

$x\cancel{\ln e} = \ln 15$

$\boxed{x = \ln 15}$

Rewrite each into exponential form:

36. $\log_{49} 7 = \frac{1}{2}$

$\boxed{49^{1/2} = 7}$

37. $\ln 14 = x$

$\log_e 14 = x$

$\boxed{e^x = 14}$

38. $\log_2 \frac{1}{4} = -2$

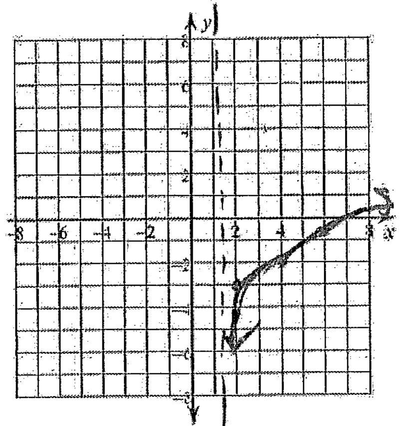
$\boxed{2^{-2} = \frac{1}{4}}$

Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

39) $y = \log_3 (x-1) - 3$

$x - 1 = 0$
$x = 1$ (VA)

Transformations
i) Right 1
ii) Down 3



x	y
1	VA
2	-3
4	-2

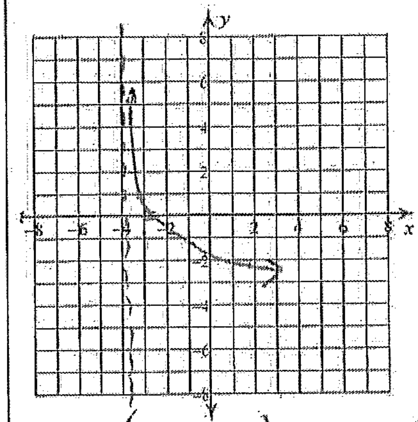
Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 1$ x-int: $(28, 0)$

set $y = 0$
$0 = \log_3(x-1) - 3$
$3 = \log_3(x-1)$
$3^3 = x - 1$
$27 = x - 1$
$28 = x$

40) $y = \log_{\frac{1}{3}} (x+4)$

$x + 4 = 0$
$x = -4$ (VA)



x	y
-4	VA
-3	0
$-11/3$	1

Domain: $(-4, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = -4$ x-int: $(-3, 0)$

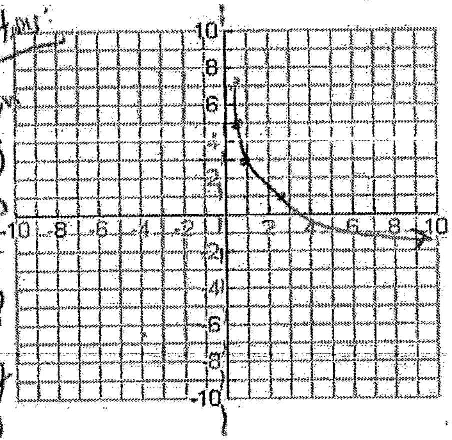
$x + 4 = 1/3$
$x = 1/3 - 4$
$x = -11/3$

Transformations:
i) left 4 units

41) $g(x) = -2 \ln x + 3$

$y = -2\log_e(x) + 3$

Transformations:
i) Reflection (x-axis)
ii) shift up 3 units
iii) vertical stretch by factor of 2



x	y
0	VA
1	3
e	1
$\frac{1}{e}$	5

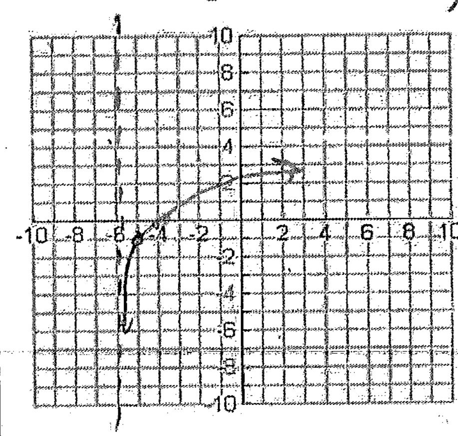
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = 0$ x-int: $(e^{3/2}, 0)$

$0 = -2\log_e(x) + 3$
$-3 = -2\log_e x$
$\frac{3}{2} = \log_e x$
$e^{1.5} = x$
$(e^{1.5}, 0)$

42) $f(x) = \log_2(x+6) - 1$

$x + 6 = 0$
$x = -6$ (VA)



x	y
-6	VA
-5	-1
-4	0

Domain: $(-6, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x = -6$ x-int: $(-4, 0)$

Transformations:
i) shift left 6 units
ii) shift down 1 unit.

Exponentials and Logs Test Review WS #3

Write the equation in logarithmic form.

1) $3^6 = 729$

2) $8^{\frac{2}{3}} = 4$

Write the equation in exponential form.

3) $\log_5 \frac{1}{25} = -2$

4) $\log_{x+y} z = 3$

Evaluate the logarithm.

5) $\log_6 \frac{1}{216}$

6) $\log_9 729$

7) $\log 0.1$

Condense the expression as a single logarithm.

8) $4\log_2 x - 6\log_2 y$

9) $2\log x + \log(x+2)$

Expand the logarithmic expression.

10) $\log_3(12b^4)$

11) $\log_2\left(\frac{c^3}{d}\right)$

Solve the exponential equation.

12) $\frac{1}{25} = 5^{x+2}$

13) $4^{5x-1} = 256$

Solve the logarithmic equation.

14) $\log(x+3) - \log x = 1$

15) $3\log_2 2 + \log_2 x = 6$

Solve for x:

16) $5^{3x-1} \cdot 5^{2x-5} = 5^{x+6}$

17) $5^{x-2} = 10^{2x+1}$

Solve the natural logarithmic equation. Round to nearest hundredth.

18) $\ln(3x+2) = 5$

19) $\ln x - \ln 3 = 0$

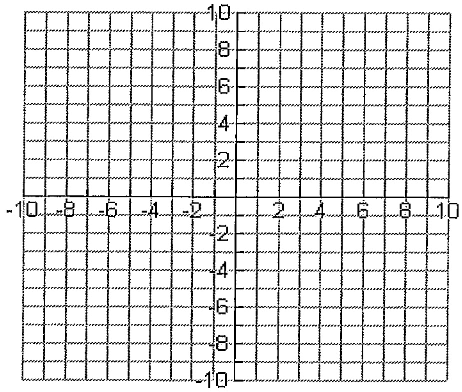
Use natural logarithms to solve the equations. Round to the nearest hundredth.

20) $e^x = \frac{5}{7}$

21) $3e^{-x} + 1 = 7$

Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

22) $f(x) = \log_2(x+8) - 4$

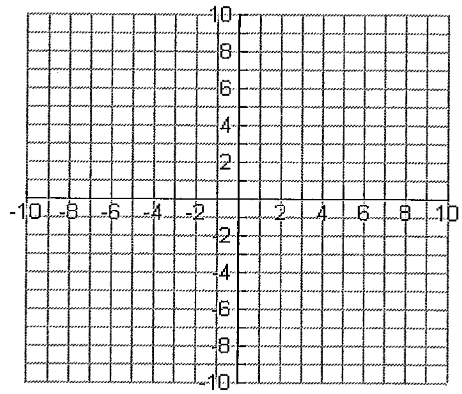


x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

23) $f(x) = -3log_3(x-1)+1$

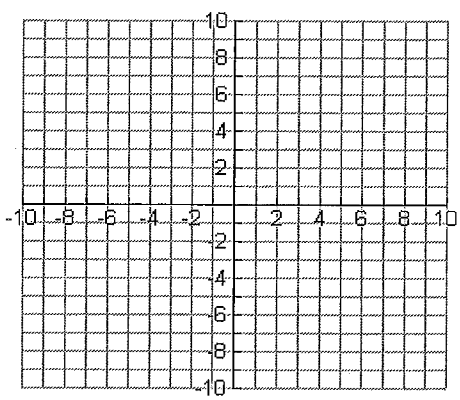


x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

24) $g(x) = 2\ln x - 1$

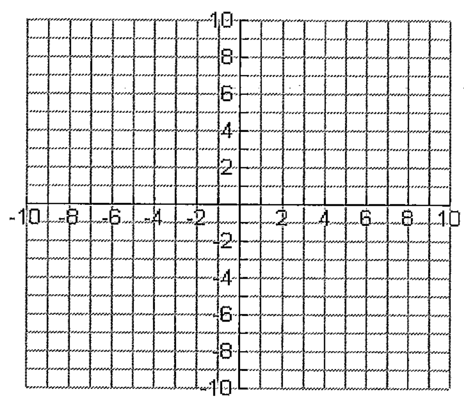


x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

25) $f(x) = \log_{\frac{1}{3}}(x-4)$



x	y

Domain: ______________ Range: ____________

Asymptote: _______ x-int: _______________

Key

Exponentials and Logs Test Review WS #3

Write the equation in logarithmic form.

1) $3^6 = 729$

$\boxed{\log_3 729 = 6}$

2) $8^{\frac{2}{3}} = 4$

$\boxed{\log_8 4 = \frac{2}{3}}$

Write the equation in exponential form.

3) $\log_5 \frac{1}{25} = -2$

$\boxed{5^{-2} = \frac{1}{25}}$

4) $\log_{x+y} z = 3$

$\boxed{(x+y)^3 = z}$

Evaluate the logarithm.

5) $\log_6 \frac{1}{216} = x$

$6^x = \frac{1}{216} \rightarrow 6^x = 6^{-3}$

$\boxed{x = -3}$

6) $\log_9 729 = x$

$9^x = 729$

$\boxed{x = 3} \leftarrow 9^x = 9^3$

7) $\log 0.1 = x$

$\log_{10}\left(\frac{1}{10}\right) = x$

$10^x = \frac{1}{10} \rightarrow 10^x = 10^{-1}$

$\boxed{x = -1}$

Condense the expression as a single logarithm.

8) $4\log_2 x - 6\log_2 y$

$\log_2 x^4 - \log_2 y^6$

$\boxed{\log_2\left(\frac{x^4}{y^6}\right)}$

9) $2\log x + \log(x+2)$

$\log_{10} x^2 + \log_{10}(x+2)$

$\log_{10} x^2(x+2) \rightarrow \boxed{\log(x^3 + 2x^2)}$

Expand the logarithmic expression.

10) $\log_3(12b^4)$

$\log_3 12 + \log_3 b^4$

$\boxed{\log_3 12 + 4\log_3 b}$

11) $\log_2\left(\frac{c^3}{d}\right)$

$\boxed{3\log_2 c - \log_2 d}$

Solve the exponential equation.

12) $\frac{1}{25} = 5^{x+2}$

$5^{-2} = 5^{x+2}$

$-2 = x+2$

$\boxed{-4 = x}$

13) $4^{5x-1} = 256$

$4^{5x-1} = 4^4$

$5x-1 = 4$

$5x = 5$

$\boxed{x=1}$

Solve the logarithmic equation.

14) $\log(x+3) - \log x = 1$

$\log\left(\frac{x+3}{x}\right) = 1$

$\log_{10}\left(\frac{x+3}{x}\right) = 1$

$10^1 = \frac{x+3}{x}$

$\frac{10}{1} = \frac{x+3}{x}$

$10x = x+3$

$9x = 3$

$\boxed{x = \frac{1}{3}}$ ✓

15) $3\log_2 2 + \log_2 x = 6$

$\log_2 2^3 + \log_2 x = 6$

$\log_2 8x = 6$

$2^6 = 8x$

$\frac{2^6}{8} = x$

$\boxed{x = 8}$ ✓

Solve for x:

16) $5^{3x-1} \cdot 5^{2x-5} = 5^{x+6}$

$5^{3x-1+2x-5} = 5^{x+6}$

$5x-6 = x+6$

$4x = 12$

$\boxed{x=3}$

17) $5^{x-2} = 10^{2x+1}$

$\log 5^{x-2} = \log 10^{2x+1}$

$(x-2)\log 5 = (2x+1)\cancel{\log 10} \rightarrow 1$

$x\log 5 - 2\log 5 = 2x+1$

$x\log 5 - 2x = 2\log 5 + 1$

$x(\log 5 - 2) = 2\log 5 + 1$

$\boxed{x = \frac{2\log 5 + 1}{\log 5 - 2}}$

Solve the natural logarithmic equation. Round to nearest hundredth.

18) $\ln(3x+2) = 5$

$\log_e(3x+2) = 5$

$e^5 = 3x+2$

$e^5 - 2 = 3x$

$\boxed{\frac{e^5-2}{3} = x}$ ✓

19) $\ln x - \ln 3 = 0$

$\log_e\left(\frac{x}{3}\right) = 0$

$e^0 = \frac{x}{3}$

$1 = \frac{x}{3}$

$\boxed{x=3}$ ✓

Use natural logarithms to solve the equations. Round to the nearest hundredth.

20) $e^x = \frac{5}{7}$

$\ln e^x = \ln\left(\frac{5}{7}\right)$

$x\cancel{\ln e} = \ln(5/7)$

$\boxed{x = \ln\left(\frac{5}{7}\right)}$

21) $3e^{-x} + 1 = 7$

$\frac{3e^{-x}}{3} = \frac{6}{3}$

$e^{-x} = 2$

$\ln e^{-x} = \ln 2$

$-x\cancel{\ln e} = \ln 2$

$\boxed{x = -\ln 2}$

Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

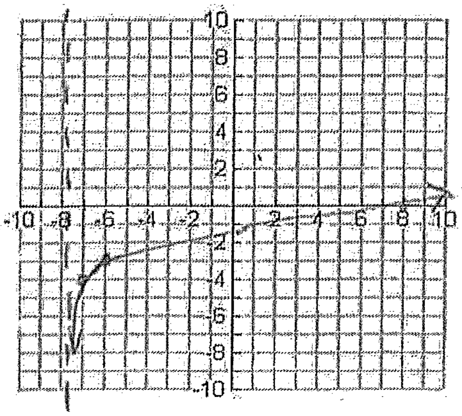
22) $f(x) = \log_2(x+8) - 4$

$x+8=0$

$x=-8$ (VA)

*shifts left 8 units

*shift down 4 units



x	y
-8	VA
-7	-4
-6	-3

Domain: $(-8, 0)$ Range: $(-\infty, \infty)$

Asymptote: $x=-8$ x-int: $(8, 0)$

$0 = \log_2(x+8) - 4$

$4 = \log_2(x+8)$

$2^4 = x+8$

$16-8 = x$

$8 = x$

$(8, 0)$

23) $f(x) = -3\log_3(x-1) + 1$

$x-1=0$

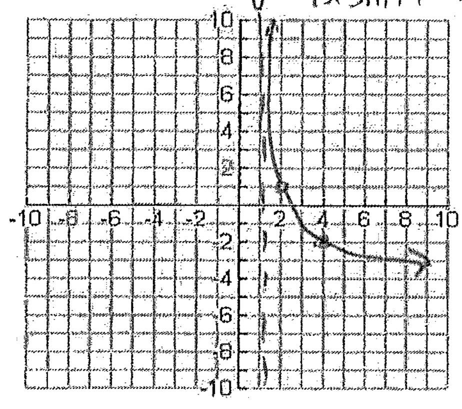
$x=1$ (VA)

*Reflection (x-axis)

*vertical stretch by 3

*shift down 1

*shift up 1



x	y
1	VA
2	1
4	-2

Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=1$ x-int: $(3^{1/3}+1, 0)$

$0 = -3\log_3(x-1) + 1$

$-1 = -3\log_3(x-1)$

$\frac{1}{3} = \log_3(x-1)$

$3^{1/3} = x-1$

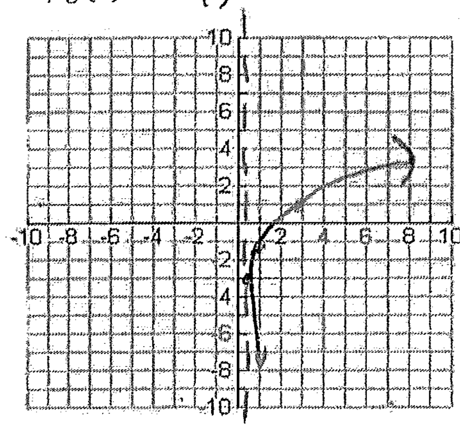
$3^{1/3}+1 = x$

24) $g(x) = 2\ln(x) - 1$

$x=0$ (VA)

*stretch by factor of 2

*shift down 1



x	y
0	VA
1	-1
e	1
$\frac{1}{e}$	-3

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=0$ x-int: $(e^{0.5}, 0)$

$0 = 2\log_e(x) - 1$

$1 = 2\log_e(x)$

$\frac{1}{2} = \log_e(x)$

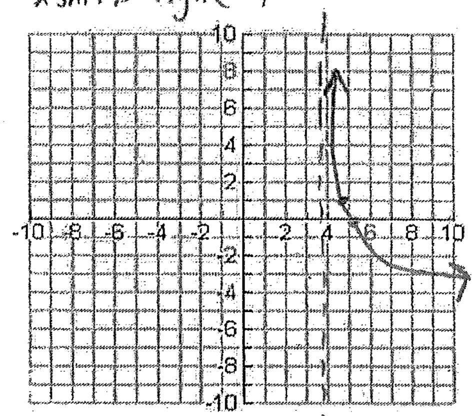
$e^{\frac{1}{2}} = x$

25) $f(x) = \log_{\frac{1}{3}}(x-4)$

$x-4=0$

$x=4$ (VA)

*shifts right 4



x	y
4	VA
5	0
$13/3$	1

Domain: $(4, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=4$ x-int: $(5, 0)$

$x-4 = 1/3$

$x = 4 + 1/3$

$x = \frac{13}{3} \approx 4.33$

Log Properties:

1) Product Property: $\log uv = \log u + \log v$

2) Quotient Property: $\log\left(\frac{u}{v}\right) = \log u - \log v$

3) Power Property: $\log u^n = n \cdot \log u$

4) $\log\left(\frac{ab}{cde}\right) = \log a + \log b - \log c - \log d - \log e$

5) $\log(u+v) \neq \log u + \log v$

6) $\log_e e^x = x \rightarrow \ln e^x = x$

7) $e^{\log_e x} = x$

Graphing Log Functions:

Helpful Log Characteristics

1) $\log_b(x)$ $\leftarrow$ * set the log argument $= 0$ to find the Vertical Asymptote

2) $\log_b(1) = 0$

3) $\log_b(b) = 1$

*4) $\log_b\left(\frac{1}{b}\right) = -1$

---

* $\log_b(b^x) = x$

* $b^{\log_b(x)} = x$

Logs and Exponentials Test Review WS #4

Graph each function and find the following characteristics.

1) $f(x) = log_3(x+4) - 1$

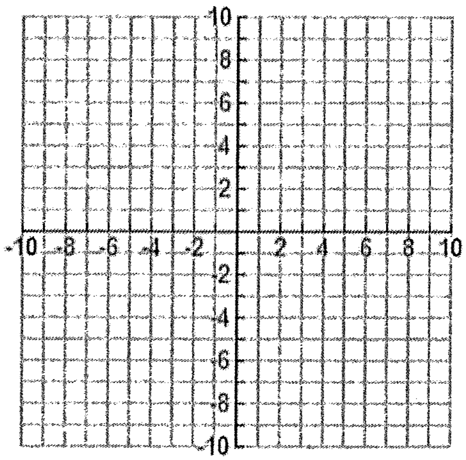
Parent function:

Transformations:

Domain: Range:

Asymptote: Intercepts:

$\lim_{x\to\infty} f(x) =$ $\lim_{x\to 2^+} f(x) =$



2) $g(x) = -3log_2(x-2)$

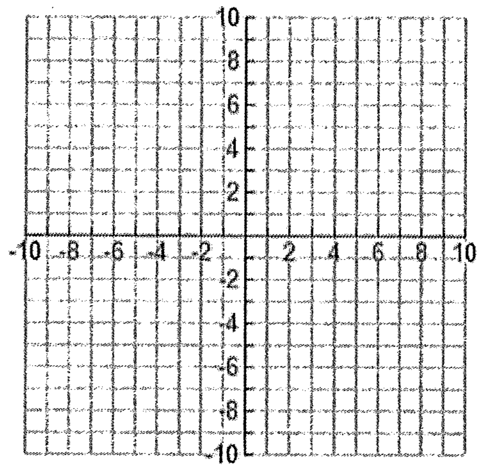
Parent function:

Transformations:

Domain: Range:

Asymptote: Intercepts:

$\lim_{x\to\infty} f(x) =$ $\lim_{x\to 2^+} f(x) =$



Rewrite each equation in logarithmic form.

3) $3^4 = 81$

4) $4^{-3} = \frac{1}{64}$

Rewrite each equation in exponential form.

5) $\log_5 125 = 3$

6) $\ln 10 \approx 2.303$

Evaluate.

7) $\log_3 81$

8) $\log_4 \frac{1}{64}$

9) $\ln e^5$

10) $\log 100{,}000$

11) $\log_{16} 8$

12) $\log_9 \frac{1}{27}$

13) $5^{\log_5 2}$

14) $\log_6 6^{17}$

15) $\log_8 16$

16) $\log_{\frac{1}{27}} 9$

17) $\log 0.0001$

18) $\log_{49} 7$

Expand fully.

19) $\ln\frac{\sqrt{e}}{y^2}$

20) $\log(200a^2b^3)^4$

21) $\log_2\frac{48m}{25n}$

Condense into a single logarithmic expression.

22) $\log_3 x - 3$

23) $\ln b + 3\ln c - 2\ln a$

24) $\frac{1}{2} - 2(3\log m + 4\log n)$

Solve each equation.

25) $\log x = 4$

26) $\log_x 3 = \frac{1}{2}$

27) $4^x = 3$

28) $\ln x = 1.8$

29) $\left(\frac{1}{2}\right)^x = 32^{x-1}$

30) $4(2^{3x-1}) - 3 = 0$

31) $9 = 4 + \log_2(x+5)$

32) $\log x + \log(x-2) = \log 8$

33) $7^{x-1} \cdot 7^{x+3} = 14$

34) $\log_2(x+1) - \log_2(x-5) = 3$

35) $6^{2x} = 30$

36) $\log_3(16x+1) = 4$

37) $\log_7(3x^2+8) - \log_7 8 = 4$

38) $\frac{5^{6x+7}}{5^{3x+5}} = 6$

Key

Logs and Exponentials Test Review WS #4

Graph each function and find the following characteristics.

1) $f(x) = \log_3(x+4) - 1$ $\quad x+4=0$, $x=-4$ (VA)

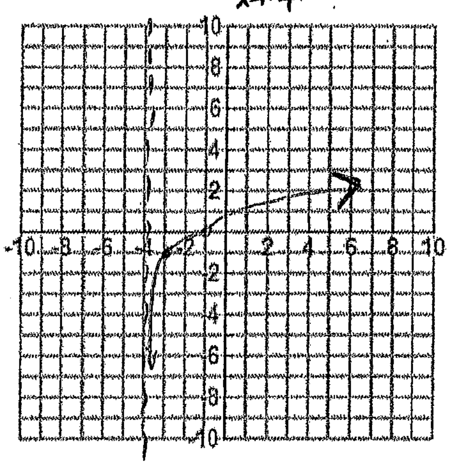
Parent function: $\log_3(x)$

Transformations: translated 4 units left, 1 down

Domain: $(-4, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=-4$ Intercepts: $(-1, 0)$

$\lim_{x \to \infty} f(x) = \infty$ $\quad \lim_{x \to -4^+} f(x) = -\infty$



x	y
-4	VA
-3	-1
-1	0

2) $g(x) = -3\log_2(x-2)$ $\quad x-2=0$, $x=2$ (VA)

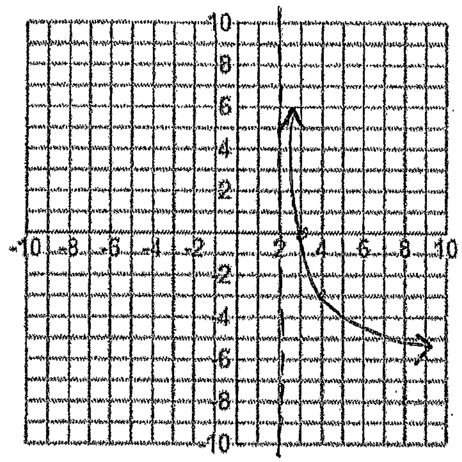
Parent function: $\log_2(x)$

Transformations: reflection (x-axis), vertical stretch by 3, right 2 units

Domain: $(2, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=2$ Intercepts: $(3, 0)$

$\lim_{x \to \infty} f(x) = -\infty$ $\quad \lim_{x \to 2^+} f(x) = +\infty$



x	y
2	VA
3	0
4	-3

Rewrite each equation in logarithmic form.

3) $3^4 = 81$

$\boxed{\log_3 81 = 4}$

4) $4^{-3} = \frac{1}{64}$

$\boxed{\log_4\left(\frac{1}{64}\right) = -3}$

Rewrite each equation in exponential form.

5) $\log_5 125 = 3$

$\boxed{5^3 = 125}$

6) $\ln 10 \approx 2.303$

$\log_e 10 = 2.303$

$\boxed{e^{2.303} = 10}$

Evaluate.

7) $\log_3 81 = x$

$3^x = 81 \to 3^x = 3^4$

$\boxed{x=4}$

8) $\log_4 \frac{1}{64} = x$

$4^x = \frac{1}{64}$

$4^x = 4^{-3}$ $\boxed{x=-3}$

9) $\ln e^5$

$\boxed{=5}$

10) $\log 100{,}000$

$\log_{10} 100{,}000 = x$

$10^x = 100{,}000$

$\boxed{x=5}$

11) $\log_{16} 8 = x$

$16^x = 8$

$2^{4x} = 2^3$

$4x = 3$

$\boxed{x = 3/4}$

12) $\log_9 \frac{1}{27} = x$

$9^x = \frac{1}{27}$

$3^{2x} = 3^{-3}$

$2x = -3$

$\boxed{x = -3/2}$

13) $5^{\log_5 2}$

$\boxed{=2}$

14) $\log_6 6^{17}$

$\boxed{=17}$

15) $\log_8 16 = x$

$8^x = 16$

$2^{3x} = 2^4$

$3x = 4$

$\boxed{x = 4/3}$

16) $\log_{\frac{1}{27}} 9 = x$

$\left(\frac{1}{27}\right)^x = 9$

$3^{-3x} = 3^2$

$-3x = 2$

$\boxed{x = -2/3}$

17) $\log 0.0001 = x$

$10^x = \frac{1}{10000}$

$10^x = 10^{-4}$

$\boxed{x=-4}$

18) $\log_{49} 7 = x$

$49^x = 7$

$7^{2x} = 7^1$

$2x = 1$

$\boxed{x = 1/2}$

Expand fully.

19) $\ln \frac{\sqrt{e}}{y^2}$

$\ln e^{1/2} - \ln y^2$

$\frac{1}{2}\cancel{\ln e} - 2\ln y$

$\boxed{\frac{1}{2} - 2\ln y}$

20) $\log(200a^2b^3)^4$

20) $4\log(200a^2b^3)$

$4\log 200 + 4\log a^2 + 4\log b^3$

$\boxed{4\log 200 + 8\log a + 12\log b}$

21) $\log_2 \frac{48m}{25n}$

$\boxed{\log_2 48 + \log_2 m - \log_2 25 - \log_2 n}$

Condense into a single logarithmic expression.

22) $\log_3 x - 3$

$\log_3 x - \log_3 3^3$

$\log_3 x - \log_3 27$

$\boxed{\log_3\left(\frac{x}{27}\right)}$

23) $\ln b + 3\ln c - 2\ln a$

$\ln b + \ln c^3 - \ln a^2$

$\boxed{\ln\left(\frac{bc^3}{a^2}\right)}$

24) $\frac{1}{2} - 2(3\log m + 4\log n)$

$\log_{10} 10^{\frac{1}{2}} - 6\log m - 8\log n$

$\log_{10} 10^{1/2} - \log m^6 - \log n^8$

$\boxed{\log\left(\frac{10^{1/2}}{m^6 n^8}\right)}$

Solve each equation.

25) $\log x = 4$

$\log_{10} x = 4$

$10^4 = x$

$\boxed{x = 10000}$

26) $\log_x 3 = \frac{1}{2}$

$x^{1/2} = 3$

$(x^{1/2})^2 = (3)^2$

$\boxed{x = 9}$

27) $4^x = 3$

$\log 4^x = \log 3$

$x\log 4 = \log 3$

$x = \frac{\log 3}{\log 4}$

$\boxed{x = 0.792}$

28) $\ln x = 1.8$

$\log_e x = 1.8$

$e^{1.8} = x$

$\boxed{x = e^{1.8}}$

29) $\left(\frac{1}{2}\right)^x = 32^{x-1}$

$2^{-x} = 2^{5(x-1)}$

$-x = 5x - 5$

$-6x = -5$

$x = \frac{-5}{-6}$

$\boxed{x = 5/6}$

30) $4(2^{3x-1}) - 3 = 0$

$4(2^{3x-1}) = 3$

$2^{3x-1} = \frac{3}{4}$

$2^{3x-1} = 0.75$

$\log 2^{(3x-1)} = \log 0.75$

$(3x-1)\log 2 = \log 0.75$

$3x\log 2 - \log 2 = \log 0.75$

$3x\log 2 = \log 2 + \log 0.75$

$x = \frac{\log 2 + \log .75}{3\log 2} = \boxed{0.195}$

* check for extraneous solutions

31) $9 = 4 + \log_2(x+5)$

$5 = \log_2(x+5)$

$2^5 = x+5$

$32 = x+5$

$\boxed{x=27}$ ✓

32) $\log x + \log(x-2) = \log 8$

$\log_{10}(x)(x-2) = \log_{10} 8$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4,\ \cancel{x = -2}$ Extraneous solution

$\boxed{x=4}$

33) $7^{x-1} \cdot 7^{x+3} = 14$

$7^{x-1+x+3} = 14$

$7^{2x+2} = 14$

$\log 7^{2x+2} = \log 14$

$(2x+2)\log 7 = \log 14$

$2x\log 7 + 2\log 7 = \log 14$

$2x\log 7 = \log 14 - 2\log 7$

$x = \dfrac{\log 14 - 2\log 7}{2\log 7}$

$\boxed{x \approx -0.322}$

34) $\log_2(x+1) - \log_2(x-5) = 3$

$\log_2\left(\dfrac{x+1}{x-5}\right) = 3$

$2^3 = \dfrac{x+1}{x-5}$

$\dfrac{8}{1} = \dfrac{x+1}{x-5}$

$8x - 40 = x + 1$

$7x = 41$

$\boxed{x = \dfrac{41}{7}}$ ✓

35) $6^{2x} = 30$

$\log 6^{2x} = \log 30$

$2x \log 6 = \log 30$

$x = \dfrac{\log 30}{2\log 6} \approx \boxed{0.949}$

36) $\log_3(16x+1) = 4$

$3^4 = 16x + 1$

$81 = 16x + 1$

$80 = 16x$

$\boxed{x=5}$ ✓

37) $\log_7(3x^2+8) - \log_7 8 = 4$

$\log_7\left(\dfrac{3x^2+8}{8}\right) = 4$

$7^4 = \dfrac{3x^2+8}{8}$

$3x^2 + 8 = 7^4 \cdot 8$

$3x^2 + 8 = 19208$

$3x^2 = 19200$

$x^2 = 6400$

$x = \pm 80$

$\boxed{x = 80,\ x = -80}$

38) $\dfrac{5^{6x+7}}{5^{3x+5}} = 6$

$5^{6x+7-(3x+5)} = 6$

$5^{3x+2} = 6$

$\log 5^{3x+2} = \log 6$

$(3x+2)\log 5 = \log 6$

$3x\log 5 + 2\log 5 = \log 6$

$3x\log 5 = \log 6 - 2\log 5$

$x = \dfrac{\log 6 - 2\log 5}{3\log 5}$

$\boxed{x \approx -0.296}$

Log Properties:

1) Product Property: $\log uv = \log u + \log v$

2) Quotient Property: $\log\left(\frac{u}{v}\right) = \log u - \log v$

3) Power Property: $\log u^n = n \cdot \log u$

4) $\log\left(\frac{ab}{cde}\right) = \log a + \log b - \log c - \log d - \log e$

5) $\log(u+v) \neq \log u + \log v$

6) $\log_e e^x = x \rightarrow \ln e^x = x$

7) $e^{\log_e x} = x$

Graphing Log Functions:

Helpful Log Characteristics

1) $\log_b(x)$ *set the log argument = 0 to find the Vertical Asymptote

2) $\log_b(1) = 0$

3) $\log_b(b) = 1$

*4) $\log_b\left(\frac{1}{b}\right) = -1$

* $\log_b(b^x) = x$

* $b^{\log_b(x)} = x$