Function Characteristics

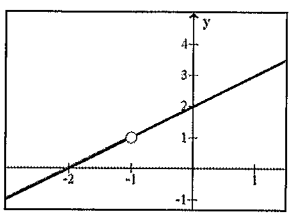
Asymptotes: Horizontal, Vertical, or Slanted lines that a function approaches but does not cross at the extremes.

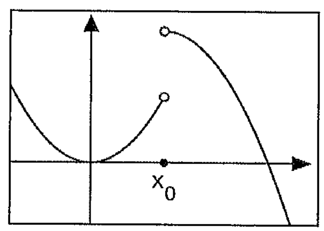
Boundedness: Vertical restrictions on a function.

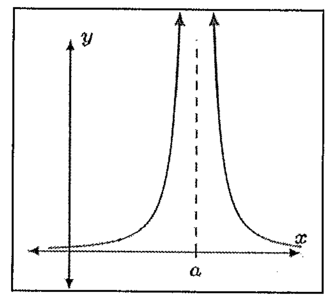
- *Bounded above*: The function has a value that it never climbs above.
- *Bounded below*: The function has a value that it never dips below.
- Or simply *bounded*: The function is both bounded above and bounded below.
- *Unbounded*: A function that is not bound.

Continuity: A function that is not connected.

- *Hole*, a form of *removable discontinuity*: a point at which a graph is not connected but can be made connected by filling in a single point.
- *Jump*, a form of *non-removable discontinuity*: the graph jumps from one piece of the graph to another.
- *Infinite*, a form of *non-removable discontinuity*: the graph has pieces to either side of a vertical asymptote that approach infinity.







Domain: All values of x where the function is defined.

End Behavior: What the function is doing toward the end of the graph.

- Is it increasing without bound (∞)?
- Is it decreasing without bound ($-\infty$)?
- Is it approaching a specific value (like a horizontal asymptote)?
- $as\ x \to \infty, f(x) \to$ _?_ What is the function doing as it goes off the right side of the graph?
- $as\ x \to -\infty, f(x) \to$ _?_ What is the function doing as it goes off the left side of the graph?

Extrema: The maximum and minimum function values that it reaches.

- *Absolute extrema*: The highest (or lowest) value the function ever reaches.
- *Relative extrema*: A high (or low) value the function reaches for a certain interval of x-values.

Intercepts: The point where the graph intersects an axis.

- *x-intercepts*: found where the graph crosses the x-axis, when y = 0. Also known as *roots* or *zeros*.
- *y-intercepts*: found where the graph crosses the y-axis, when x = 0.

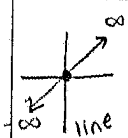
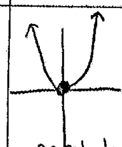
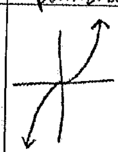
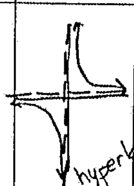
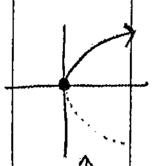
Range: All values of y that a function can be.

Transformations: Changes made to a parent graph to create a different but related function.

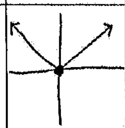
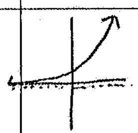
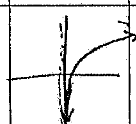
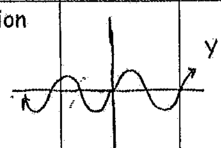
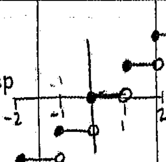
- *Vertical stretch/compress*
- *Horizontal stretch/compress*
- *Vertical shift*
- *Horizontal shift*
- *Vertical Reflection*
- *Horizontal Reflection*

April & May 2021 Units 10 Function Analysis				
19	20	21	22 **9.13 Unit 9 Test**	23 10.01 10 Basic Functions CW: Complete the table
26 **American Lit EOC in AM** 10.01 10 Basic Functions	27 **American Lit EOC in AM** 10.01 Finishing Reviewing Basic Functions	28 **Biology EOC in AM** 10.02 Boundedness, Even/Odd, Asymptotes HW: 10.02 Practice	29 **US History EOC in AM** 10.03 Transformations • Discovery • Chart HW: 10.03 Practice	30 **EOC in AM** 10.04 Extra Practice HW: 10.04 Practice
5/3 **AP Exams** 10.05 Composition of Functions HW: 10.05 Practice	4 **AP Exams** 10.06 Piecewise Functions HW: 10.06 Practice	5 **AP Exams** 10.07 More Piecewise Functions HW: 10.07 Practice	6 **AP Exams (APUSH)** 10.08 Review HW: 10.08 Review	7 **AP Exams** **10.09 Assessment**

Properties of Families of Functions

Name	Graph	Algebraic Equation	Continuity (any breaks or gaps)	Extrema max or min	End Behavior	Symmetry Even, Odd	Asymptotes	Domain and Range
Identity Function Linear	 ∞, $-\infty$, line	$y = x^1$	No breaks continuous	none	As $x \to \infty$ $y \to \infty$ As $x \to -\infty$ $y \to -\infty$	odd can rotate 180° at origin and it aligns w/ itself	none	D: $(-\infty, \infty)$ all x-values R: $(-\infty, \infty)$ all y-values
Quadratic Function	 parabola	$y = x^2$	continuous	no max 1 min at (0,0)	As $x \to \infty$ $y \to \infty$ As $x \to -\infty$ $y \to \infty$	Even can flip over y-axis and it aligns	none	D: $(-\infty, \infty)$ R: $[0, \infty)$
Cubic Function		$y = x^3$	continuous	parent has none *can be 1max + 1min in family	As $x \to \infty$ $y \to \infty$ As $x \to -\infty$ $y \to -\infty$	ODD	none	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
Rational Function Reciprocal Function	 hyperbola	$y = \frac{1}{x}$ $y = 1/x$	infinite discontinuity at $x = 0$ Asymptote	none HA	As $x \to \infty$ $y \to 0$ As $x \to -\infty$ $y \to 0$	ODD	VA: $x = 0$ (y axis) HA: $y = 0$ (x-axis)	D: $(-\infty, 0) \cup (0, \infty)$ R: $(-\infty, 0) \cup (0, \infty)$
Square Root Function	 ½ of parabola	$y = \sqrt{x}$	Continuous	abs min @(0,0)	As $x \to \infty$ $y \to \infty$ as $x \to -\infty$ $y \to$ dne	none	none	D: $[0, \infty)$ R: $[0, \infty)$

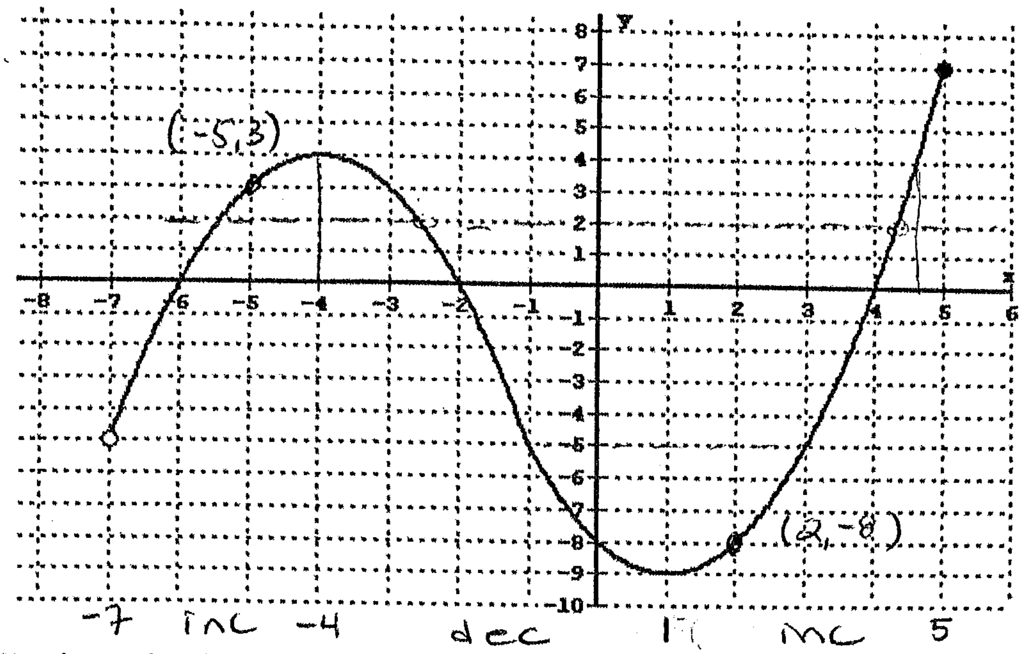
Properties of Families of Functions

Name	Graph	Algebraic Equation	Continuity	Extrema	End Behavior	Symmetry	Asymptotes	Domain and Range
Absolute Value Function		Y = |x|	continuous	abs min at (0,0)	As $x\to\infty$ $y\to\infty$ As $x\to-\infty$ $y\to\infty$	even	none	D: $(-\infty,\infty)$ R: $[0,\infty)$
Exponential Function	 "J-curve"	Y = e^x Or Y = a^x	continuous	none	As $x\to\infty$ $y\to\infty$ As $x\to-\infty$ $y\to0$	none	HA $y=0$	D: $(-\infty,\infty)$ R: $(0,\infty)$
Logarithmic Function		Y = $\log_b x$ Or Y = $\ln x$	continuous	none	As $x\to\infty$ $y\to\infty$ As $x\to-\infty$ y dne	none	VA $x=0$	D: $(0,\infty)$ R: $(-\infty,\infty)$ inverse
Sine Function		Y = sin x	continuous	abs min @ $y=-1$ abs max @ $y=1$	Does not exist oscillates and cannot converge at a value	ODD	none	D: $(-\infty,\infty)$ R: $[-1,1]$
Greatest Integer Function/Step Function		Y = [x]	jump (non-removable) dis. @ x= every integer	none	As $x\to\infty$ $y\to\infty$ As $x\to-\infty$ $y\to-\infty$	none	none	D: $(-\infty,\infty)$ R: all integers $\mathbb{Z}$

The output of [x] is the greatest integer that is less than or equal to x.

[1.2]=1 [1.75]=1 [1.99]=1 [2]=2

Analyze the Graph



Use the graph to find each.

1. x-intercept(s): (-6,0), (-2,0), (4,0)

2. y-intercept(s): (0,-8)

3. Is this a function? yes

4. Domain: (-7,5]

5. Range: [-9,7]

6. Where is $f(x) < 0$? (-7,-6) U (-2,4)
List the x-values, interval notation.

7. Where is $f(x) \geq 0$? [-6,-2] U [4,5]
List the x-values, interval notation.

8. Find $f(2)$. = -8

9. Find $f(-5)$. = 3

10. State the graph's boundedness:
bounded (above and below)

11. How many times does the line $y = 2$ intersect the graph? 3 times

12. Is the graph even, odd, or neither?
neither, no symmetry.

13. Where does $f(x) = 4$? at x = -4 or x = $4\frac{2}{3}$ ish

14. Where does $f(x) = -5$? at x = -1 or x = 3

15. Find $f(-1) - f(2)$. = -5 - -8 = 3

16. Find $3f(1)$. = 3(-9) = -27

17. Absolute Maximum value: (5,7)

18. Absolute Minimum value: (1,-9)

19. Relative Maximum value: (-4,4)

20. Relative Minimum value: none

21. Where is the graph increasing? (-7,-4) U (1,5)
State the x-values, interval notation.

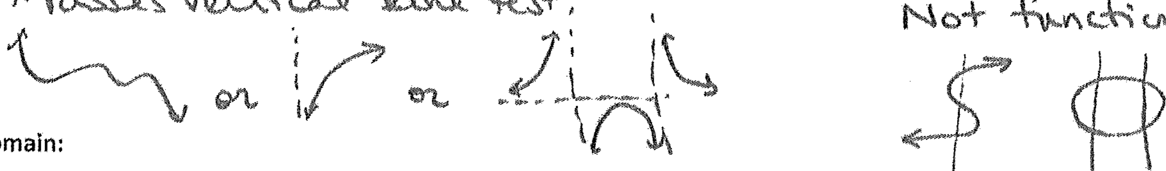
22. Where is the graph decreasing? (-4, 1)
State the x-values, interval notation.

23. Is the graph continuous? If not, what type of discontinuity does it have? yes
(can trace whole graph w/o picking up pencil)

10.02 Function Properties Date: 2019

1. Function versus Not a Function:

Every x-value is paired with exactly 1 y-value.
*Passes vertical line test.



2. Domain:

all possible x-values where the function exists

- All real numbers, ℝ:

- What causes a function to *not exist* for an input value?

① denominator = 0 $\quad \frac{5}{x} \Rightarrow x \neq 0$

② square root of a negative $\quad \sqrt{x-3} \Rightarrow x \geq 3$

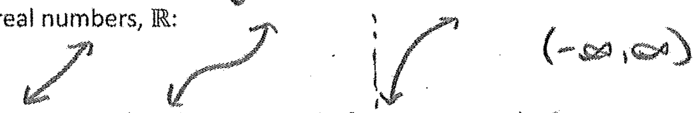
③ log argument is not positive $\quad \log(x+7) \Rightarrow x > -7$

think about excluded values.

3. Range:

all possible y-values that the function can be.

- All real numbers, ℝ:



- What causes a function to *not exist* for an output value?

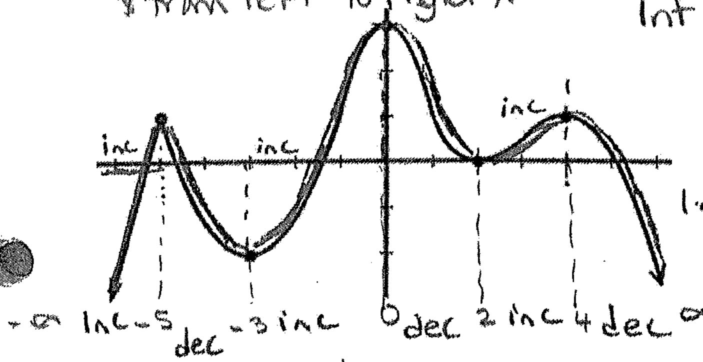
① function turns around and does not go forever up or forever down.

② exponential functions are found to one side of an asymptote

③ Rational functions can also have holes or horizontal asy.

4. Intervals of Increase and Decrease:

from left to right



Int of Inc: A span of x-values for which each output increases.

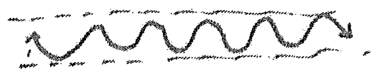
increase $(-\infty, -5) \cup (-3, 0) \cup (2, 4)$

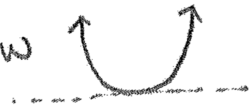
Int of Dec: A span of x-values for which each output decreases

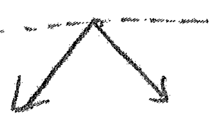
decrease: $(-5, -3) \cup (0, 2)$ ~~(4~~ $(4, \infty)$

5. **Boundedness:** A function having a vertical limit (ceiling or floor)

Not bounded: 

Bounded: 

Bounded below 

Bounded above 

6. **Symmetry:** Mirror images or rotational

Even Function

symmetric over y-axis

EVEN EQUAL

$f(-x) = f(x)$

left is same as right

Odd Function

symmetric about the origin

Odd Opposite

$f(-x) = -f(x)$

left is the opposite of the right

Determine if the function is even, odd, or neither.

*plug in $-x$

a) $f(x) = 3x^4 + 7x^3 - 2x + 1$

$f(-x) = 3(-x)^4 + 7(-x)^3 - 2(-x) + 1$

$= 3x^4 + -7x^3 + 2 + 1$

Not same, not opposite $\boxed{\text{Neither}}$

b) $g(x) = 5x^6 + 3x^2 - 8$

$g(-x) = 5(-x)^6 + 3(-x)^2 - 8$

$= 5x^6 + 3x^2 - 8$

all equal $\Rightarrow$ $\boxed{\text{EVEN}}$

c) $h(x) = 3\sin(x)$

$h(-x) = 3\sin(-x)$

$= 3(-\sin x)$

$= -3\sin x$

opposite $\Rightarrow$ $\boxed{\text{ODD}}$

d) $j(x) = -\frac{4}{x}$

$j(-x) = \frac{-4}{-x} = \frac{4}{x}$

opposite $\Rightarrow$ $\boxed{\text{ODD}}$

7. **Asymptotes and Holes in Rational Functions:**

Holes: occur at (a,b) when $(x-a)$ is a factor of both the numerator and denominator (so it will be cancelled!) and $f(a)=b$ in the simplified function.

Example: $f(x) = \frac{x+4}{x^2+16} = \frac{\cancel{x+4}}{(\cancel{x+4})(x-4)} = \frac{1}{x-4}$

$x+4$ is a common factor

$x=-4$ has a $\boxed{\text{hole @ } (-4, -\frac{1}{8})}$

$y = \frac{1}{-4-4} = \frac{1}{-8}$

$\boxed{\text{VA @ } x=4}$

Vertical Asymptotes: occur at $x=c$ where $(x-c)$ is a factor of only the denominator

Example: $g(x) = \frac{2x^2-9x-5}{x^2-8x+15} = \frac{(2x+1)\cancel{(x-5)}}{\cancel{(x-5)}(x-3)}$

$\boxed{\text{VA @ } x-3=0,\ x=3}$

hole @ $x-5=0$

$x=5$

$y = \frac{2(5)+1}{5-3} = \frac{11}{2}$

$(5, 5.5)$

Horizontal Asymptotes:

- $y = 0$ if degree of numerator is less than degree of denominator

  Example: $f(x) = \dfrac{x+4}{x^2-16}$ ← deg=1 (numerator), ← deg=2 (denominator) $\boxed{y=0 \text{ is HA}}$

- $y = \dfrac{\text{leading coefficient}}{\text{leading coeffcient}}$ if degree of numerator is the same as degree of denominator

  Example: $g(x) = \dfrac{2x^2-9x-5}{x^2-8x+15}$ ← both deg=2

  $\boxed{\text{HA @ } y = \frac{2}{1} = 2}$

- Does not exist if degree of numerator is more than degree of denominator

  Example: $h(x) = \dfrac{x^3-x^2+3x-3}{x^2+1}$ ← deg=3 (numerator), ← deg=2 (denominator)

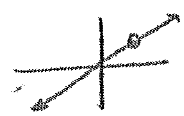
  $\boxed{\text{HA does not exist}}$

  * there is a slant asymp *

8. Continuity versus Discontinuity:

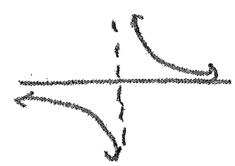
At x-values

Removable Discontinuity : a.k.a. holes

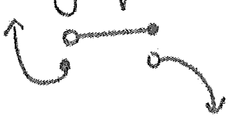


Nonremovable Discontinuity

vertical asymptotes (infinite)



jumps



9. End Behavior:

The value the function is converging on (heading towards) at the ends of a graph

As $x \to \infty$, $f(x) \to$ ___?___

or $\lim\limits_{x \to \infty} f(x) \to$ ___?___

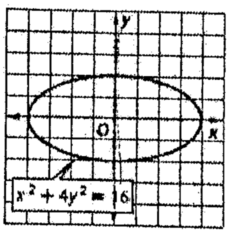
10.02 Practice

Date: Key

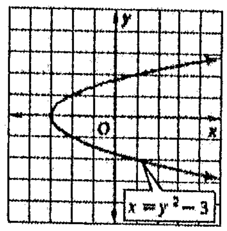
For the equations graphed in #24 – 31:

A) State if the graph shows a function or not a function
B) State if bounded above, bounded below, bounded, or not bounded
C) State if even, odd, or neither
D) State the intervals where the function is increasing, decreasing, or constant. (Use interval notation.)

24.



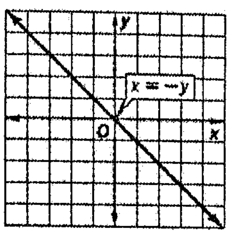
25.



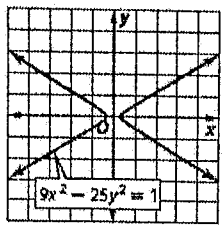
24 a) not a function b) bounded
c) both even & odd d) —

25. a) not a function. b) not bounded
c) neither d) —

26.



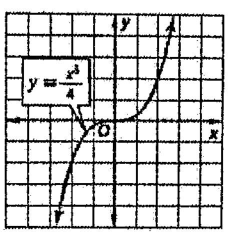
27.



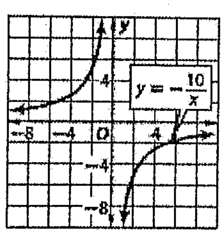
26. a) function b) not bounded
c) odd d) Dec $(-\infty, \infty)$

27. a) not a function b) not bounded
c) even + odd d) —

28.



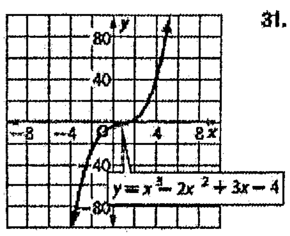
29.



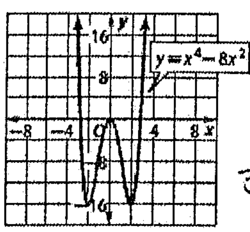
28. a) function b) not bounded
c) odd d) Inc $(-\infty, \infty)$

29. a) function b) not bounded
c) odd d) Inc
$(-\infty, 0) \cup (0, \infty)$

30.



31.

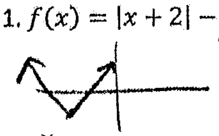


30. a) function b) not bounded
c) neither d) Inc $(-\infty, \infty)$

31. a) function b) bounded below
c) even d) Inc: $(-2, 0) \cup (2, \infty)$
Dec: $(-\infty, -2) \cup (0, 2)$

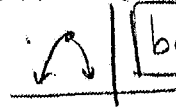
State if the function is bounded above, bounded below, bounded, or not bounded.

1. $f(x) = |x + 2| - 1$



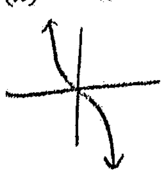
bound below

2. $f(x) = -3(x + 2)^2 + 4$



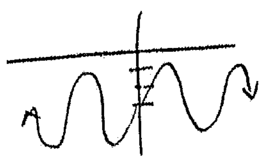
bounded above

3. $f(x) = -x^3$



not bounded

4. $f(x) = 2\sin(\theta) - 3$



bounded

Determine algebraically whether the function is even, odd, or neither.

5. $f(x) = 5x^2$

$f(-x) = 5(-x)^2 = 5x^2$

same, equal

$\boxed{\text{EVEN}}$

6. $f(x) = x^4 + 3x^2 + 8$

$f(-x) = (-x)^4 + 3(-x)^2 + 8$

$= x^4 + 3x^2 + 8$

same, equal

$\boxed{\text{EVEN}}$

7. $f(x) = -5x^3 + 3x^2 - 1$

$f(-x) = -5(-x)^3 + 3(-x)^2 - 1$

$= -5(-x^3) + 3x^2 - 1$

$= 5x^3 + 3x^2 - 1$

not same, not opposite $\boxed{\text{NEITHER}}$

8. $f(x) = \frac{1}{x}$

$f(-x) = \frac{1}{-x} = -\frac{1}{x}$

opposite

$\boxed{\text{ODD}}$

Find all vertical asymptotes, horizontal asymptotes, and/or holes.

9. $f(x) = \frac{x+2}{3-x}$

holes: none

VA: $3 - x = 0$

$3 = x$

HA: deg = deg

$\frac{1}{-1}$

10. $f(x) = \frac{4x-4}{x^2-9} = \frac{4(x-1)}{(x-3)(x+3)}$

holes = none

VA: $x-3=0$ $x+3$

$x = 3$ $x=$

HA: $\deg_N < \deg$

$y = 0$

11. $f(x) = \frac{x^2-2x}{x^3-5x^2+6x} = \frac{\cancel{x}(x-2)}{\cancel{x}(x^2-5x+6)}$

$= \frac{\cancel{x-2}}{(x-3)\cancel{(x-2)}}$

$= \frac{1}{x-3}$

holes: $x=0$ $\frac{1}{0-3} = -\frac{1}{3}$ $(0, -\frac{1}{3})$

$x-2=0$

$x=2$ $\frac{1}{2-3} = \frac{1}{-1}$ $(2, -1)$

VA: $x-3=0$

$x=3$

HA: $\deg_N < \deg_D$ $y=0$

12. $f(x) = \frac{5x^2+2}{3x^2-12} = \frac{5x^2+2}{3(x^2-4)} = \frac{5x^2+2}{3(x-2)(x+2)}$

holes = none

VA: $x-2=0$ $x+2=0$

$x=2$ $x=-2$

HA: deg = deg

$y = \frac{5}{3}$

Key

10.03 Practice

Given the parent function for each, describe the graph of each related function.

1. $f(x) = x^2$

$y = \left[\frac{1}{4}(x-12)\right]^2$ ★ have to factor to see horizontal shift ★

a. $y = -(4x)^2$

- x axis reflection
- horizontal compression by $\frac{1}{4}$

b. $y = 4x^2$

- vertical stretch by 4

c. $y = (.25x - 3)^2$

- horizontal stretch by 4
- right 12

2. $f(x) = |x|$

a. $y = ½\ |x + 2|$

- vertical shrink by ½
- left 2

b. $y = |-x| - 7$

- y axis reflection
- down 7

c. $y = |5/3\ x|$

- horizontal compression by 3/5

Write the function that is obtained from the following transformations.

1. $y = x^3$; shift left 1, down 5, reflected over the x-axis.

$y = -(x+1)^3 - 5$

2. $y = |x|$; vertical stretch of 4, reflected over the y-axis.

$y = 4|-x|$

3. $y = \frac{1}{x}$; shifted right 2, up 9

$y = \frac{1}{x-2} + 9$

4. $y = x^2$; reflected over the x-axis, horizontally stretched 2/3, and down 1

$y = -\left(\frac{3}{2}x\right)^2 - 1$

10.03 Discovery of Transformations Date: Key

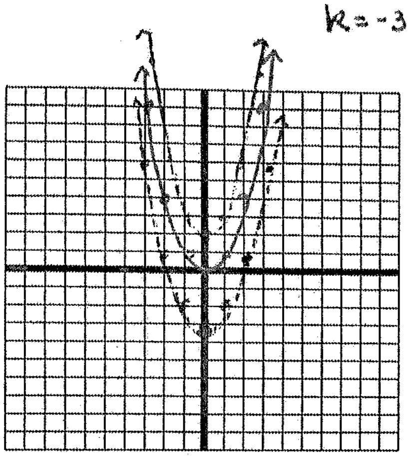
For each function graph the parent and the transformations on the same graph. Use different colors for each. Then compare and describe any changes.

Parent: $y = x^2$

$\boxed{1.\ y = x^2 + 2}$

$\boxed{2.\ y = x^2 - 3}$

How did $y = x^2$ change? k = 2 shift up 2
k = -3 shift down 3

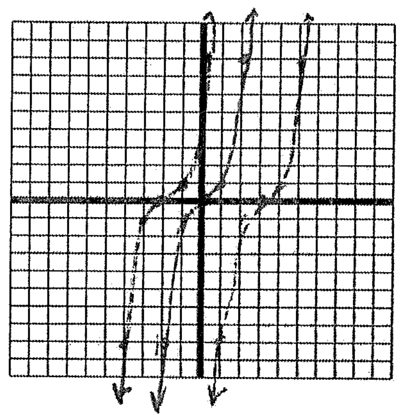


Parent: $y = x^3$

$\boxed{1.\ y = (x-3)^3}$

$\boxed{2.\ y = (x+2)^3}$

How did $y = x^3$ change? h = 3 shift right 3
h = -2 shift left 2

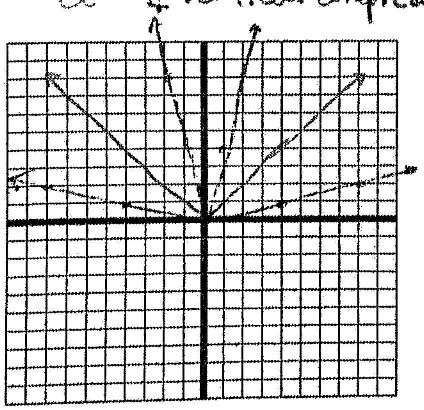


Parent: $y = |x|$

$\boxed{1.\ y = 4|x|}$

$\boxed{2.\ y = 0.25|x|}$

How did $y = |x|$ change?
a = 4 vertical stretch 4 times higher
$a = \frac{1}{4}$ vertical compress to $\frac{1}{4}$ the height

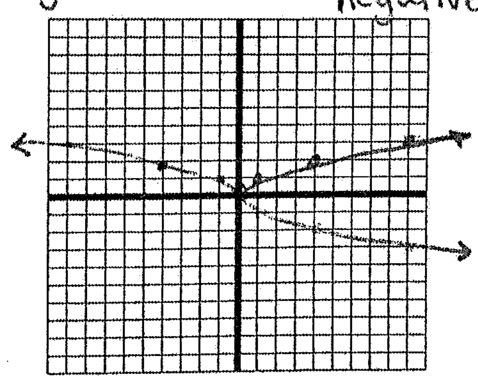


Parent: $y = \sqrt{x}$

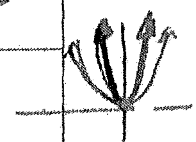
$\boxed{1.\ y = -\sqrt{x}}$

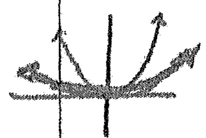
$\boxed{2.\ y = \sqrt{-x}}$

How did $y = \sqrt{x}$ change? negative a reflects over x axis
negative b reflects over y-axis



Type of Transformation	Algebraic Formula	Variable Notes	Example $y = x^2$
Vertical: Translation Shift ↑↓	$f(x)+k$	If $k>0$ shift up If $k<0$ shift down	$y = x^2+3$ $y = x^2-3$
Vertical: Dilation Stretch taller compress shorter	$a\,f(x)$	If $a>1$ stretch If $0<a<1$ compress	$y = 3x^2$ $y = \frac{1}{3}x^2$
Vertical: Reflection flip over x axis	$-f(x)$	Reflect over x-axis	$y = -x^2$
Horizontal: Translation Shift ⇄	$f(x-h)$	If $h>0$ shift right If $h<0$ shift left	$y = (x-3)^2$ $y = (x+3)^2$
Horizontal: Dilation stretch wider compress narrower	$f(bx)$	If $b>1$ compress $(1/b)$ If $0<b<1$ stretch $(1/b)$	$y = (3x)^2$ $y = (\frac{1}{3}x)^2$
Horizontal: Reflection flip over y-axis	$f(-x)$	Reflect over y-axis	$y = (-x)^2$





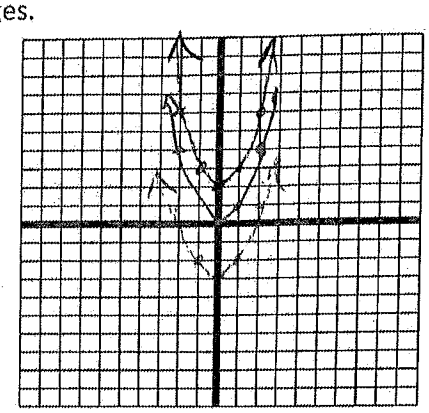
b is tricky

$\frac{1}{3}$ width

3 times wider

Discovery of Transformations

For each section graph the parent function and the other given functions on the same graph. Use different colors for each. Then compare and describe any changes.



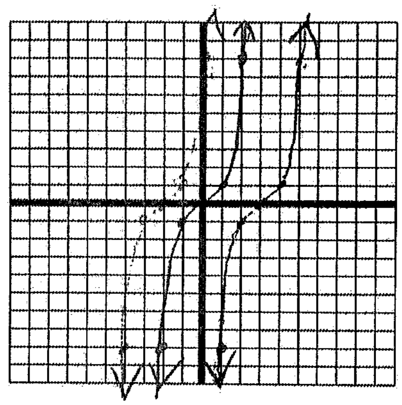
Section A

Parent: $y = x^2$

1. $y = x^2 + 2$ blue
2. $y = x^2 - 3$ red

How did $y = x^2$ change?

Vertical Shift



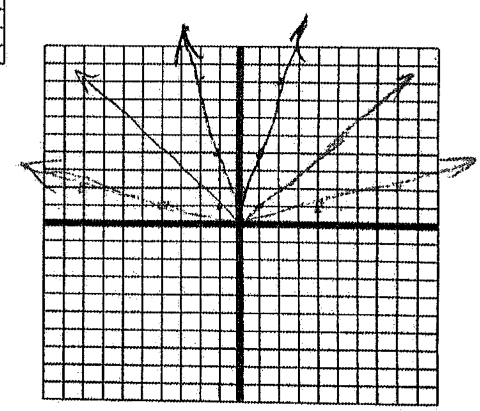
Section B

Parent: $y = x^3$

1. $y = (x - 3)^3$ blue
2. $y = (x + 2)^3$ red

How did $y = x^3$ change?

Horiz shift



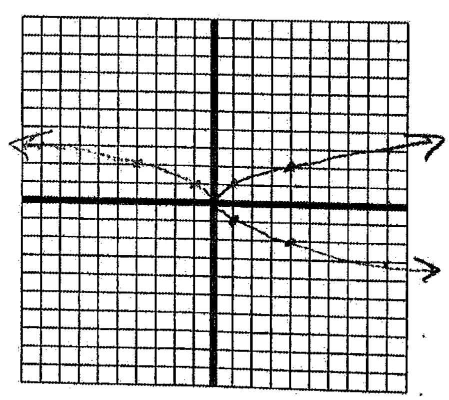
Section C

Parent: $y = |x|$

1. $y = 4|x|$ blue
2. $y = .25|x|$ red

How did $y = |x|$ change?

Vertical Stretch/ Shrink



Section D

Parent: $y = \sqrt{x}$

1. $y = -\sqrt{x}$ blue
2. $y = \sqrt{-x}$ red

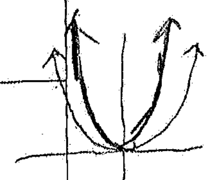
How did $y = \sqrt{x}$ change?

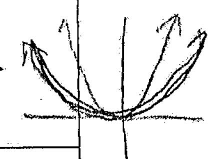
Reflection

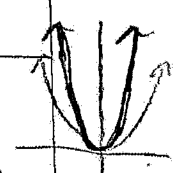
Transformations of Functions

Name ____________________

Type of Transformation	Algebraic Formula	Variable Notes	Example $y=x^2$
Translation Vertical	$f(x)+K$	If $K>0$ up If $K<0$ down	$y=x^2+3$ $y=x^2-3$
Dilation Vertical	$af(x)$	If $a>1$ stretch If $0<a<1$ compression	$y=3x^2$ $y=\frac{1}{3}x^2$
Reflection Vertical	$-f(x)$	Reflects over x-axis	$y=-x^2$
Translation Horizontal	$f(x+h)$	If $h>0$ left If $h<0$ right	$y=(x+3)^2$ $y=(x-3)^2$
Dilation Horizontal	$f(ax)$	If $a>1$ compression If $0<a<1$ stretch	$y=(3x)^2$ $y=(\frac{1}{3}x)^2$
Reflection Horizontal	$f(-x)$	Reflects over y-axis	$y=(-x)^2$





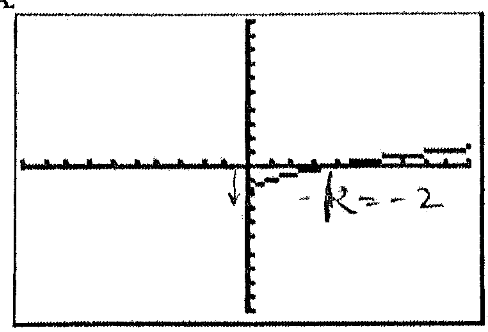


10.04 More Practice with Function Transformations — Date: Key

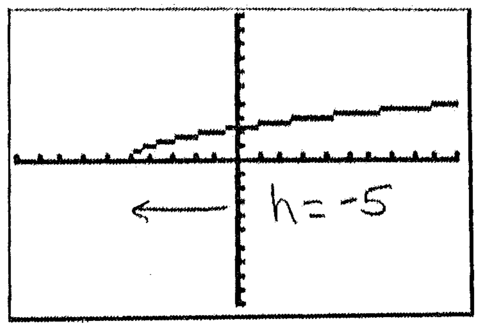
For Questions 1-4: Match each of the following functions to its graph.

1. $y=\sqrt{x+5}$ B 2. $y=\sqrt{x}-2$ A 3. $y=\sqrt{x-3}$ D 4. $y=2\sqrt{-x}$ C

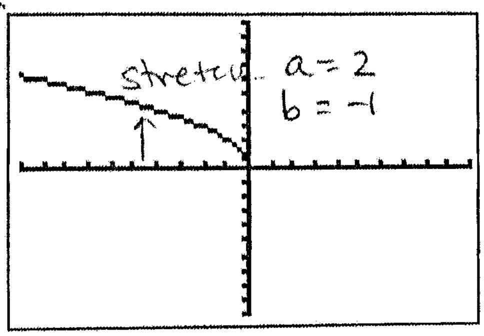
A.



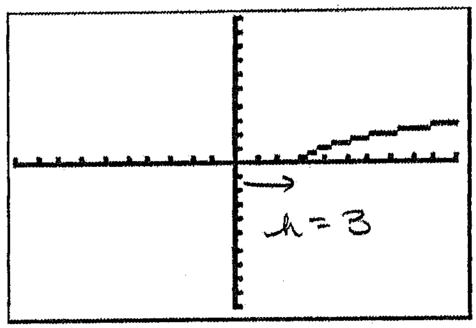
B.



C.



D.



5. Given the function $f(x)=x^2$, write the function whose graph of $f(x)$ is:

a. shifted 6 units to the left $h=-6$

$\boxed{y=(x+6)^2}$

b. reflected about the y-axis $b=-1$

$\boxed{y=(-x)^2}$

c. reflected about the x-axis $a=-1$

$\boxed{y=-(x^2)}$ or $-x^2$

d. shifted 5 units up $k=5$

$\boxed{y=x^2+5}$

e. vertically stretched by a factor of 4 $a=4$

$\boxed{y=4x^2}$

f. vertically stretched (compressed) by a factor of $^1/_3 = a$

$\boxed{y=\frac{1}{3}x^2}$ or $\frac{x^2}{3}$

6. Given the function $f(x)=\frac{1}{x}$; write the function whose graph of $f(x)$ is:

a. shifted 4 units to the right $h=4$

$\boxed{y=\frac{1}{x-4}}$

b. reflected about the y-axis $b=-1$

$\boxed{y=\frac{1}{-x}}$

c. reflected about the x-axis $a=-1$ same

$\boxed{y=-\frac{1}{x}}$

d. shifted 2 units down $k=-2$

$\boxed{y=\frac{1}{x}-2}$

e. vertically stretched by a factor of $3=a$

$\boxed{y=3\left(\frac{1}{x}\right)}$ or $\frac{3}{x}$

f. vertically stretched (compressed) by a factor of $^1/_4 = a$

$\boxed{y=\frac{1}{4}\left(\frac{1}{x}\right)}$ or $\frac{1}{4x}$

7. Use your knowledge of transformations to describe how the parent function, $f(x)$, changes into the related function, $g(x)$.

a. $f(x) = x^2$
$g(x) = x^2 + 5$ Shift up 5 units

b. $f(x) = x^3$
$g(x) = (x-1)^3$ Shift right 1

c. $f(x) = |x|$
$g(x) = -2|x|$ Reflect over x axis Stretch 2 times taller

d. $f(x) = \sqrt{x}$
$g(x) = \frac{1}{3}\sqrt{x}$ Compress to $\frac{1}{3}$ the height

e. $f(x) = \sin x$
$g(x) = \sin(x-2) - 4$ shift right 2 and down 4

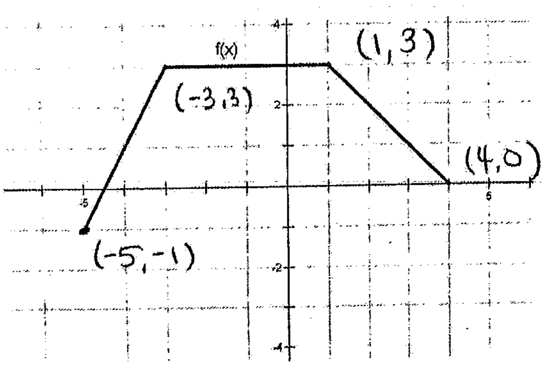
f. $f(x) = \frac{1}{x}$
$g(x) = \frac{1}{x+4} + 2$ Shift left 4, up 2

g. $f(x) = e^x$
$g(x) = -e^{x+2} - 7$ Reflect over X axis Shift left 2, down 7

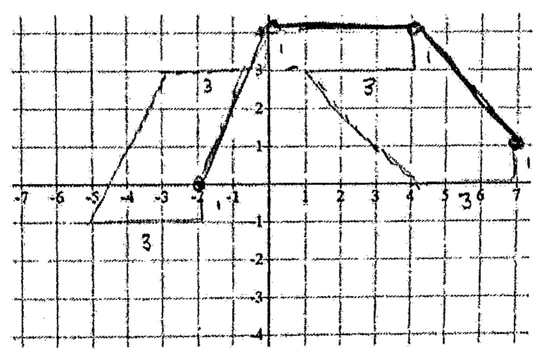
h. $f(x) = \cos x$
$g(x) = 3\cos(2x)$ Stretch 3 times taller, Compress to $\frac{1}{2}$ the width

i. $f(x) = \log x$
$g(x) = -\log(-x)$ Reflect over x axis Reflect over y axis

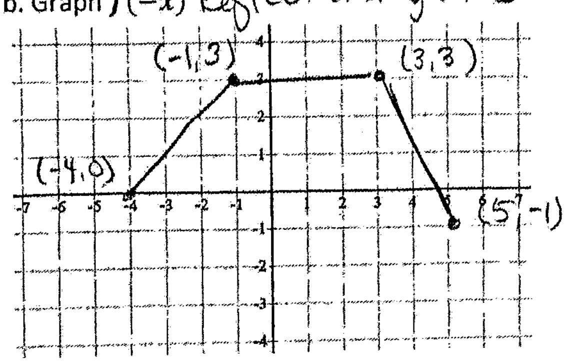
8. Consider the following graph of the function $f(x)$:



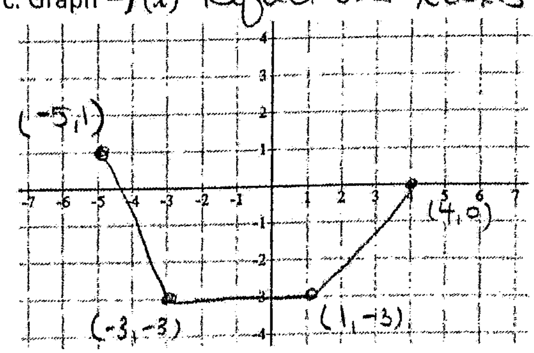
a. Graph $f(x-3)+1$ Shift right 3, u



b. Graph $f(-x)$ Reflect over y axis



c. Graph $-f(x)$ Reflect over x axis



10.05 Function Combination and Composition Date: Key

Notation:		use $f(x) = x+2$ and $g(x) = x-3$	Domain
$(f+g)(x)$	$f(x)+g(x)$ add functions	$(x+2)+(x-3) = 2x-1$	$\mathbb{R}$ $(-\infty,\infty)$
$(f-g)(x)$	$f(x)-g(x)$ subtract functions	$(x+2)-(x-3) = 5$	$\mathbb{R}$ $(-\infty,\infty)$
$(f*g)(x)$	$f(x)\cdot g(x)$ multiply functions	$(x+2)(x-3) = x^2-x-6$	$\mathbb{R}$ $(-\infty,\infty)$
$(f/g)(x)$ or $\left(\frac{f}{g}\right)(x)$	$\frac{f(x)}{g(x)}$ divide functions $g(x)\neq 0$	$\frac{x+2}{x-3}$ ← if you can reduce, do so!	$x-3\neq 0$ $x\neq 3$ $(-\infty,3)\cup(3,\infty)$
$(f\circ g)(x)$ Composition	$f(g(x))$ "f of g of x" put $g(x)$ inside $f(x)$	$f(x-3) = (x-3)+2 = x-1$	$\mathbb{R}$ $(-\infty,\infty)$

Example 1: Find the combinations using $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$. Identify any domain restrictions.

$(f+g)(x) = f(x)+g(x) = \boxed{\sqrt{x}+x^2-4 \text{ where } x\neq 0}$

$(f-g)(x) = f(x)-g(x) = \sqrt{x}-(x^2-4)$
$= \boxed{\sqrt{x}-x^2+4 \text{ where } x\neq 0}$

$(f*g)(x) = (\sqrt{x})(x^2-4) = \boxed{x^2\sqrt{x}-4\sqrt{x} \text{ where } x\neq 0}$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x^2-4}$ → $(x-2)(x+2)$
$x-2\neq 0$ $x+2\neq 0$
where $x\neq 2,-2$

Example 2: Evaluate the combinations using $f(x) = 4x^2 + 9$ and $g(x) = x^2 - 3x + 1$.

$(f-g)(1) = f(1)-g(1)$
$= 4(1)^2+9-[1^2-3(1)+1]$
$= 4+9-[1-3+1]$
$= 13-(-1) = \boxed{14}$

either evaluate + then subtract OR subtract + then evaluate!

$(f*g)(4) = f(4)\cdot g(4) = (4(4)^2+9)(4^2-3(4)+1)$
$= (4\cdot 16+9)(16-12+1)$
$= (73)(5)$
$= \boxed{365}$

Example 3: Find the function compositions using $f(x) = 2x^2 - x$ and $g(x) = -12x + 7$.

$(f\circ g)(x) = f(g(x)) = f(-12x+7)$
$= 2(-12x+7)^2-(-12x+7)$
$= 2(144x^2-168x+49)+12x-7$
$= \boxed{288x^2-324x+91}$ ($\mathbb{R}$)

336 98
-12 -7

$(g\circ f)(x) = g(f(x)) = g(2x^2-x)$
$= -12(2x^2-x)+7$
$= \boxed{-24x^2+12x+7}$ ($\mathbb{R}$)

Example 4: Find the function compositions using $f(x) = \log x$ and $g(x) = 3 - x$.

$(f\circ g)(x) = f(g(x)) = f(3-x)$
$= \boxed{\log(3-x)}$
where $3-x>0$
($x<3$)

* positive arguments *

$(g\circ f)(x) = g(f(x)) = g(\log x)$
$= \boxed{3-\log x \text{ where } x>0}$

Example 5: Evaluate the function compositions using $f(x) = 5x + 3$ and $g(x) = 3x^2$.

$(f\circ g)(4) = f(g(4))$
$= f(3\cdot 4^2)$
$= f(48)$
$= 5(48)+3$
$= 240+3 = \boxed{243}$

$g(f(2)) = g(5\cdot 2+3)$
$= g(13)$
$= 3(13)^2$
$= 3\cdot 169$
$= \boxed{507}$

$(f\circ f)(-3) = f(f(-3))$
$= f(5\cdot -3+3)$
$= f(-15+3)$
$= f(-12)$
$= 5\cdot -12+3 = \boxed{-57}$

10.05 Practice Date: Key

State the domain of each given function. Then, evaluate each combination at the given value.

1. $f(x) = x^3 - 3$ D: $(-\infty, \infty)$ $\mathbb{R}$

   $g(x) = 2x + 1$ D: $(-\infty, \infty)$ $\mathbb{R}$

   Find $(f \cdot g)(-1)$

   $= f(-1) \cdot g(-1)$

   $= ((-1)^3 - 3) \cdot (2 \cdot -1 + 1)$

   $= -4 \cdot -1$

   $= \boxed{4}$

2. $w(x) = \frac{9}{x+10}$ D: $(-\infty, -10) \cup (-10, \infty)$ $x \neq -10$

   $z(x) = \log_4 x$ D: $(0, \infty)$ $x > 0$

   Find $(w + z)(2)$

   $= w(2) + z(2)$

   $= \frac{9}{2+10} + \log_4(2)$ $4^? = 2$

   $= \frac{9}{12} + \frac{1}{2}$

   $= \frac{3}{4} + \frac{1}{2} = \boxed{\frac{5}{4} \text{ or } 2.25}$

3. $h(x) = 5x - 2$ D: $(-\infty, \infty)$ $\mathbb{R}$

   $j(x) = -2x^2 + 7$ D: $(-\infty, \infty)$ $\mathbb{R}$

   Find $(h - j)(0)$

   $h(0) - j(0)$

   $(5 \cdot 0 - 2) - (-2 \cdot 0^2 + 7)$

   $(0 - 2) - (0 + 7)$

   $-2 - 7$

   $\boxed{-9}$

4. $m(x) = x^2$ D: $(-\infty, \infty)$

   $t(x) = \sqrt{x - 4}$ D: $[4, \infty)$ $x \geq 4$ $x - 4 \geq 0$

   Find $(t/m)(5)$

   $\frac{t(5)}{m(5)} = \frac{\sqrt{5-4}}{5^2}$

   $= \frac{\sqrt{1}}{25} = \boxed{\frac{1}{25}}$

5. $j(x) = \frac{x}{x+1}$ D: $(-\infty, -1) \cup (-1, \infty)$ $x + 1 \neq 0$ $x \neq -1$

   $k(x) = 9 - x^2$ D: $(-\infty, \infty)$ $\mathbb{R}$

   Find $(j \circ k)(4)$

   $= j(k(4))$

   $= j(9 - 4^2)$

   $= j(-7)$

   $= \frac{-7}{-7+1}$

   $= \frac{-7}{-6} = \boxed{\frac{7}{6}}$

6. $a(x) = x^2 + 7$ D: $(-\infty, \infty)$ $\mathbb{R}$

   $b(x) = \sqrt{x + 13}$ D: $[-13, \infty)$ $x \geq -13$ $x + 13 \geq 0$

   Find $(b \circ a)(-4)$

   $= b(a(-4))$

   $= b((-4)^2 + 7)$

   $= b(16 + 7)$

   $= b(23)$

   $= \sqrt{23 + 13}$

   $= \sqrt{36}$

   $= \boxed{6}$

State the domain of each given function. Then, perform the indicated operation and determine the domain of the new function.

7. $f(x) = x - 2$ D: $(-\infty, \infty)$ $\mathbb{R}$

$g(x) = x^2 + x$ D: $(-\infty, \infty)$ $\mathbb{R}$

Find $(f+g)(x)$

$= f(x) + g(x)$

$= (x-2) + (x^2+x)$

$= \boxed{x^2+2x-2}$ D: $(-\infty, \infty)$

8. $m(x) = (x-1)^2$ D: $(-\infty, \infty)$

$p(x) = 3 - x$ D: $(-\infty, \infty)$

Find $(m-p)(x)$

$= m(x) - p(x)$

$= (x-1)^2 - (3-x)$

$= x^2 - 2x + 1 - 3 + x$

$= \boxed{x^2 - x - 2}$ D: $(-\infty, \infty)$

9. $u(x) = \frac{1}{x-2}$ D: $x \neq 2$ $(-\infty, 2) \cup (2, \infty)$

$v(x) = x^2 - 4$ D: $(-\infty, \infty)$

Find $(uv)(x)$

$= u(x) \cdot v(x)$

$= \left(\frac{1}{x-2}\right) \cdot (x^2-4)$

$= \frac{1}{\cancel{x-2}} \cancel{(x-2)}(x+2)$

$= \boxed{x+2}$ D: $(-\infty, 2) \cup (2, \infty)$

made from $\frac{1}{x-2}$

10. $c(x) = \sqrt{x+3}$ D: $x \geq -3$ $[-3, \infty)$

$d(x) = 4x^2 + 1$ D: $(-\infty, \infty)$

Find $\left(\frac{d}{c}\right)(x)$

$= \frac{d(x)}{c(x)} = \boxed{\frac{4x^2+1}{\sqrt{x+3}}}$

where $\sqrt{x+3} \neq 0$ $x+3 \geq 0$

$x \neq -3$ $x \geq -3$

D: $(-3, \infty)$

Combine restrictions!

11. $n(x) = x^2 + 4x + 3$ D: $(-\infty, \infty)$ $\mathbb{R}$

$z(x) = \log(x+1)$ D: $x+1 > 0$ $(-1, \infty)$

$x > -1$

Find $(z \circ n)(x)$

$= z(n(x))$

$= z(x^2+4x+3)$

$= \log((x^2+4x+3)+1)$

$= \boxed{\log(x^2+4x+4)}$ argument must be positive

where $x^2+4x+4 > 0$

$(x+2)(x+2) > 0$

pos $\emptyset$ pos

-2

$x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

12. $r(x) = \sqrt{x}$ D: $x \geq 0$ $[0, \infty)$

$t(x) = \frac{1}{x-1}$ D: $x - 1 \neq 0$ $(-\infty, 1) \cup (1, \infty)$

$x \neq 1$

Find $t(r(x))$

$t(\sqrt{x}) = \frac{1}{\sqrt{x}-1} \cdot \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$= \boxed{\frac{\sqrt{x}+1}{x-1}}$

where $\sqrt{x} - 1 \neq 0$ $x \geq 0$

$\sqrt{x} \neq 1$

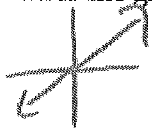
$x \neq 1$

$[0, 1) \cup (1, \infty)$

10.06 Piecewise Function Notes

Date: Key

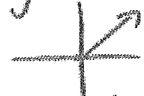
What is the graph of $y = x$?



linear

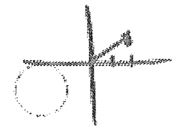
What if you only want a ray?

$y = x \quad x \geq 0$



Or only want a segment?

$y = x \quad 0 \leq x \leq 2$



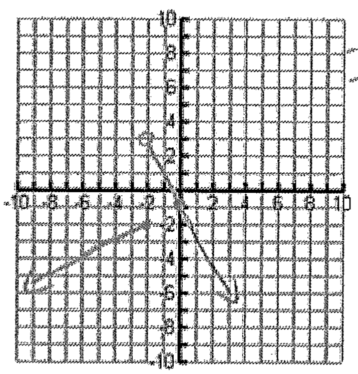
Evaluate the function at the given values. Then, graph the piecewise function.

1. $f(x) = \begin{cases} \frac{1}{2}x - 1 & if \ \underline{x \leq -2} \\ -2x - 1 & if \ \underline{x > -2} \end{cases}$

Evaluate: a) $f(0)$ b) $f(4)$

$f(0) = -2(0) - 1 = -1$

$f(4) = -2(4) - 1 = -9$



x	$\frac{1}{2}x - 1$
-4	-3
-2	-2

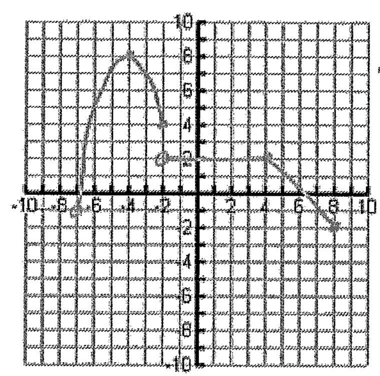
x	$-2x - 1$
-2	3
0	-1

2. $g(x) = \begin{cases} -(x+4)^2 + 8 & if \quad -7 < x \leq -2 \\ 2 & if \quad -2 < x \leq 4 \\ 6 - x & if \quad 4 < x \leq 8 \end{cases}$

Evaluate: a) $g(-2)$ b) $g(5)$

$g(-2) = -(-2+4)^2 + 8 = -4 + 8 = 4$

$g(5) = 6 - 5 = 1$



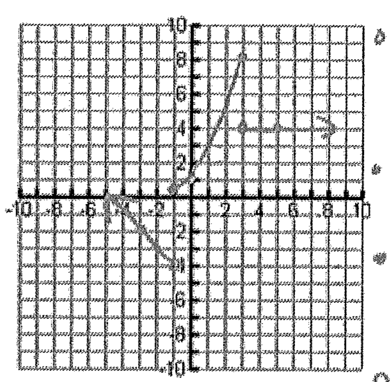
x	$-(x+4)^2 + 8$
-7	-1
-4	8
-2	4

x	2
-2	2
4	2

x	$6 - x$
4	2
8	-2

3. $q(x) = \begin{cases} -x - 5 & if \quad x < -1 \\ 2^x & if \quad -1 \leq x \leq 3 \\ 4 & if \quad x > 3 \end{cases}$

x	$-x - 5$
-3	-2
-2	-3
-1	-4



x	2^x
-1	0.5
0	1
3	8

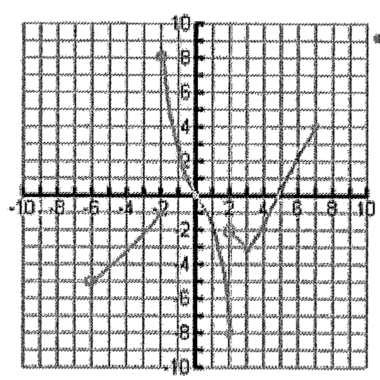
x	4
3	4
5	4

Domain: $(-\infty, \infty)$

Range: $(-4, \infty)$

4. $r(x) = \begin{cases} \frac{3}{2}x + 4 & if \quad -6 < x \leq -2 \\ -x^3 & if \quad -2 < x \leq 2 \\ 2|x - 3| - 4 & if \quad 2 < x \leq 7 \end{cases}$

x	$\frac{3}{2}x + 4$
-6	-5
-2	1



x	$-x^3$
-2	8
0	0
2	-8

x	$2\|x-3\|-4$
2	-2
3	-4
4	-2
7	4

Domain: $(-6, 7]$

Range: $[-8, 8)$

Key

10.06 Practice

Date: ______________

Evaluate the piecewise function when (a) $x = -1$, (b) $x = 0$, and (c) $x = 2$.

1. $f(x) = \begin{cases} x^2 - 3 & \text{if} \quad x < 0 \\ 8 & \text{if} \quad x \geq 0 \end{cases}$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = 8$

$f(2) = 8$

2. $g(x) = \begin{cases} \frac{3x+1}{x-5} & \text{if} \quad x < -3 \\ \log(-x) & \text{if} \quad -3 \leq x < 0 \\ 3^x & \text{if} \quad x \geq 0 \end{cases}$

$f(-1) = \log(1) = 0$

$f(0) = 3^0 = 1$

$f(2) = 3^2 = 9$

Graph the piecewise function. State the domain and range of the function.

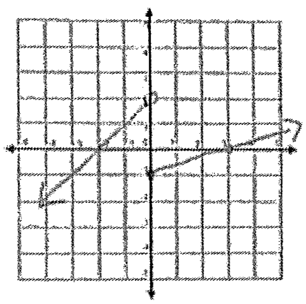
3. $t(x) = \begin{cases} x + 2 & \text{if} \quad x < 0 \\ \frac{1}{3}x - 1 & \text{if} \quad x \geq 0 \end{cases}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x	$x+2$
-2	0
0	2

x	$\frac{1}{3}x - 1$
0	-1
3	0



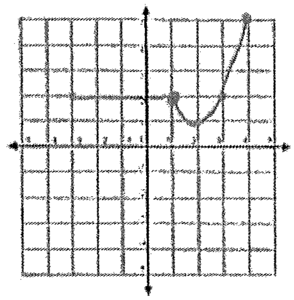
4. $v(x) = \begin{cases} 2 & \text{if} \quad -3 \leq x < 1 \\ (x-2)^2 + 1 & \text{if} \quad 1 \leq x \leq 4 \end{cases}$

Domain: $[-3, 4]$

Range: $[1, 5]$

x	2
-3	2
1	2

x	$(x-2)^2+1$
1	2
2	1
3	2
4	5



5. $a(x) = \begin{cases} -x - 4 & \text{if} \quad x \leq -2 \\ 3^x & \text{if} \quad -2 < x < 1 \\ \sqrt{x - 1} & \text{if} \quad x \geq 1 \end{cases}$

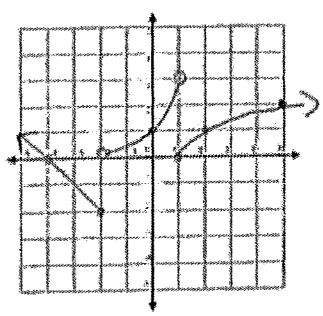
Domain: $(-\infty, \infty)$

Range: $[-2, \infty)$

x	$-x-4$
-4	0
-2	-2

x	3^x
-2	$\frac{1}{9} \approx .11$
0	1
1	3

x	$\sqrt{x-1}$
1	0
2	1
5	2



6. $r(x) = \begin{cases} \log_3(x + 4) & \text{if} \quad -4 < x \leq -1 \\ \lvert x - 1 \rvert - 1 & \text{if} \quad -1 < x \leq 2 \\ -\frac{3}{2}x + 3 & \text{if} \quad x > 2 \end{cases}$

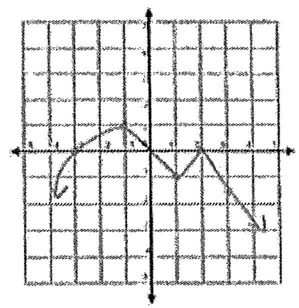
Domain: $(-4, \infty)$

Range: $(-\infty, 1]$

x	$\log_3(x+4)$
-4	und
-3	0
-1	1

x	$\lvert x-1 \rvert - 1$
-1	1
1	-1
2	0

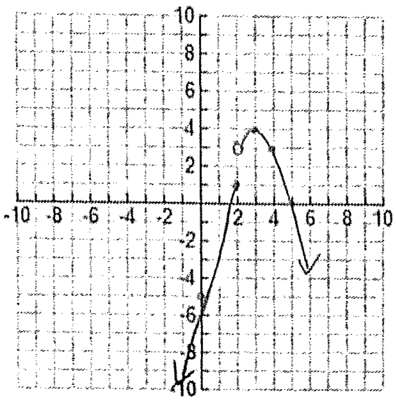
x	$-\frac{3}{2}x + 3$
2	0
3	-1.5



10.07 Practice Date: Key

Graph the following piecewise functions.

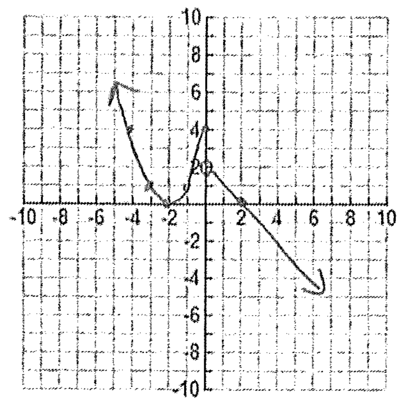
1. $f(x) = \begin{cases} 3x-5, & if\ x \le 2 \\ -(x-3)^2+4 & if\ x > 2 \end{cases}$



x	3x-5
0	-5
2	1

x	$-(x-3)^2+4$
2	3
3	4
4	3

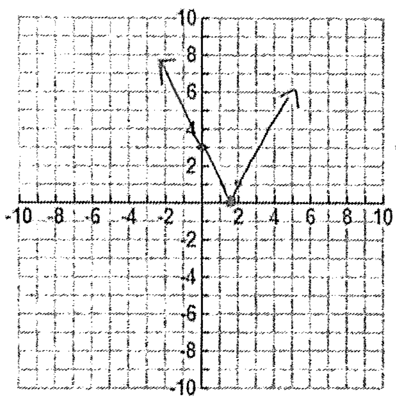
2. $g(x) = \begin{cases} (x+2)^2 & if\ x \le 0 \\ 2-x & if\ x > 0 \end{cases}$



X	$(x+2)^2$
-2	0
-1	1
0	4

x	2-x
0	2
2	0

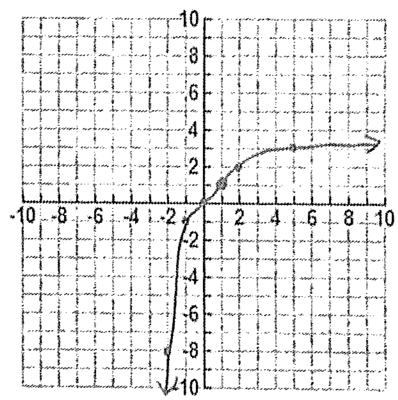
3. $f(x) = \begin{cases} 3-2x & if\ x < \frac{3}{2} \\ 2x-3 & if\ x \ge \frac{3}{2} \end{cases}$



x	3-2x
0	3
$\frac{3}{2}$	0

X	2x-3
$\frac{3}{2}$	0
2	1

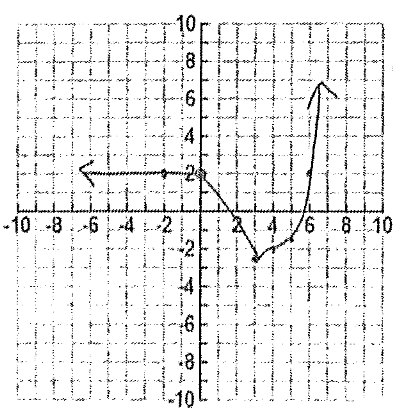
4. $g(x) = \begin{cases} x^3 & if\ x < 1 \\ \sqrt{(x-1)}+1 & if\ x \ge 1 \end{cases}$



X	x^3
-2	-8
-1	-1
0	0
1	1

x	$\sqrt{x-1}+1$
1	1
2	2
5	3

5. $f(x) = \begin{cases} 2 & x < 0 \\ -\frac{3}{2}x+2 & 0 \le x \le 3 \\ \frac{1}{2}(x-4)^3-2 & x > 3 \end{cases}$

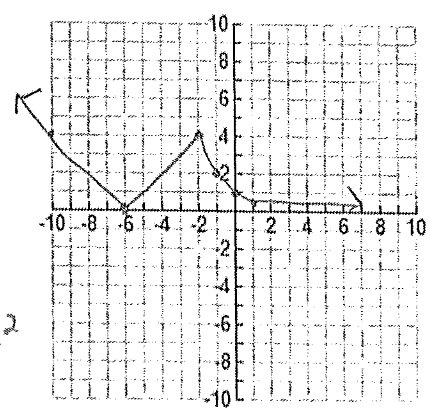


X	2
-2	2
0	2

X	$-\frac{3}{2}x+2$
0	2
3	-2.5

X	$\frac{1}{2}(x-4)^3-2$
3	-2.5
4	-2
5	-1.5

6. $g(x) = \begin{cases} \lvert x+6 \rvert & x \le -2 \\ \left(\frac{1}{2}\right)^x & x > -2 \end{cases}$

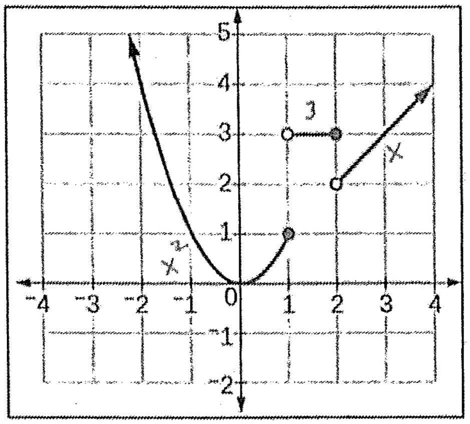


X	$\lvert x+6 \rvert$
-10	4
-6	0
-2	4

X	$\left(\frac{1}{2}\right)^x$
-2	4
-1	2
0	1
1	$\frac{1}{2}$

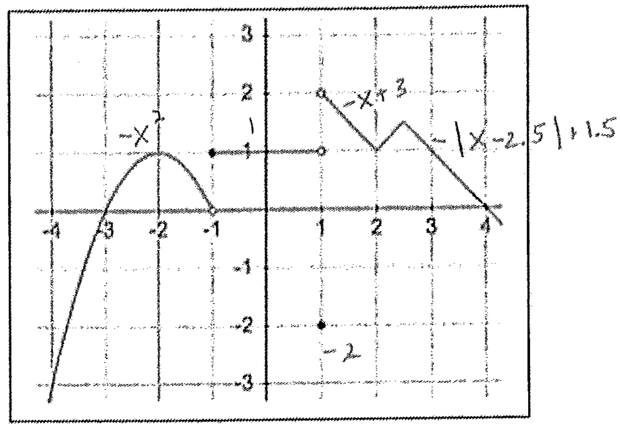
Write the equation for the piecewise function that is graphed. Assume that the domain for each is $(-\infty, \infty)$.

7. There are 3 pieces!



$$f(x) = \begin{cases} x^2 & x \leq 1 \\ 3 & 1 < x \leq 2 \\ x & x > 2 \end{cases}$$

8. There are at least 5 pieces!

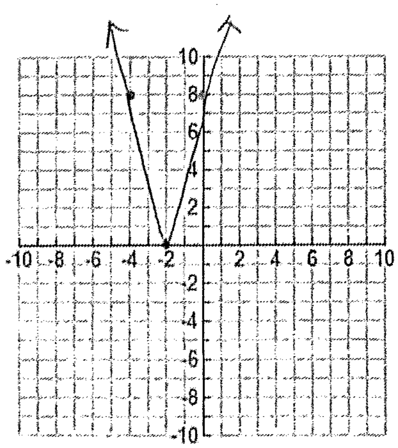


$$f(x) = \begin{cases} -x^2 & x < -1 \\ 1 & -1 \leq x < 1 \\ -2 & x = 1 \\ -x+3 & 1 < x \leq 2 \\ -|x-2.5|+1.5 & x > 2 \end{cases}$$

Absolute value functions can be expressed as a piecewise function with two linear pieces, one for each ray that meet at the vertex.

11. Express $f(x) = 4|x+2|$ as a piecewise function and graph $f(x)$.

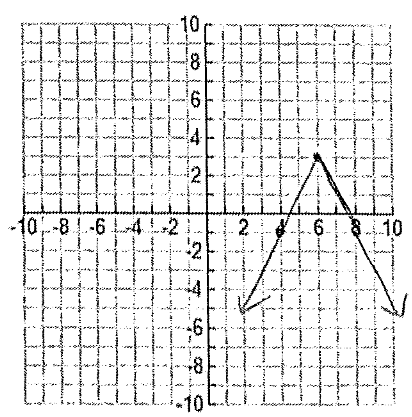
$$f(x) = \begin{cases} -4x-8 & x < -2 \\ 4x+8 & x \geq -2 \end{cases}$$



x	y
-4	8
-2	0
0	8

12. Express $f(x) = -2|x-6|+3$ as a piecewise function and graph $f(x)$.

$$f(x) = \begin{cases} 2x-9 & x < 6 \\ -2x+15 & x \geq 6 \end{cases}$$



x	y
4	-1
6	3
8	-1

10.07 Piecewise Functions

Graph the following on GRAPH PAPER!!!

1. $f(x) = \begin{cases} 3x-5, & if\ x \le 2 \\ (x-3)^2+4 & if\ x > 2 \end{cases}$

$3x-5$

x	y
2	1
0	-5

$(x-3)^2+4$

x	y
2	5
3	4
4	5

2. $f(x) = \begin{cases} (x-2)^2 & x \le 2 \\ 2-x & x > 2 \end{cases}$

$(x-2)^2$

x	y
2	0
0	4

$2-x$

x	y
2	0
4	-2

3. $f(x) = \begin{cases} 3-2x & if\ x < \frac{3}{2} \\ 2x-3 & if\ x \ge \frac{3}{2} \end{cases}$

$3-2x$

x	y
3/2	0
0	3

$2x-3$

x	y
3/2	0
3	3

4. $g(x) = \begin{cases} x^3 & x < 1 \\ \sqrt{(x-1)} & x \ge 1 \end{cases}$

x^3

x	y
1	1
0	0
-1	-1

$\sqrt{x-1}$

x	y
1	0
2	1

5. $f(x) = \begin{cases} -x-3 & if\ x \le -5 \\ -5 & if\ -1 \le x \le 3 \\ 3x-14 & if\ x > 3 \end{cases}$

$-x-3$

x	y
-5	2
-7	4

-5

x	y
-1	-5
3	-5

$3x-14$

x	y
3	-5
5	1

6. $f(x) = \begin{cases} -x-3 & if\ x < -2 \\ x & if\ -1 \le x \le 2 \\ 1 & if\ x > 3 \end{cases}$

$-x-3$

x	y
-2	-1
-4	1

x

x	y
-1	-1
2	2

1

x	y
3	1
5	1

7. $h(x) = \begin{cases} 2 & x < 0 \\ 4x-1 & 0 \le x \le 3 \\ (x-3)^3 & x > 3 \end{cases}$

2

x	y
0	2
-2	2

$4x-1$

x	y
0	-1
3	11

$(x-3)^3$

x	y
3	0
4	1
5	8

8. $g(x) = \begin{cases} |x+5| & x \le 1 \\ -x^2+4x+3 & x > 1 \end{cases}$

$|x+5|$

x	y
1	6
-1	4

$-x^2+4x+3$

x	y
1	6
2	7

9. $q(x)=\begin{cases}2x+1 & \text{if } x\le 0\\ x^2-5x-2 & \text{if } x>0\end{cases}$

$2x+1$

x	y
0	1 •
-2	-3 •

x^2-5x-2

x	y
0	-2 ∘
2.5	-8.25 •

10. $f(x)=\begin{cases}x^2-1 & \text{if } x\le -1\\ 2 & \text{if } -1<x<3\\ \sqrt{x+1} & \text{if } x\ge 3\end{cases}$

x^2-1

x	y
-1	0 •
-2	3 •

2

x	y
-1	2 ∘
3	2 ∘

$\sqrt{x+1}$

x	y
3	2 •
8	3 •

11. Express $f(x) = |4x+7|$ as a piecewise function and hence graph $f(x)$.

$f(x)=\begin{cases}4x+7 & x\ge -7/4\\ -4x-7 & x< -7/4\end{cases}$

12. Express $f(x) = |4x+3|-4$ as a piecewise function and hence graph $f(x)$.

$f(x)=\begin{cases}4x-1 & x\ge -\frac{3}{4}\\ -4x-7 & x<-\frac{3}{4}\end{cases}$

9. $q(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ x^2 - 5x - 2 & \text{if } x > 0 \end{cases}$

$2x+1$

x	y
0	1 •
-2	-3 •

x^2-5x-2

x	y
0	-2 o
2.5	-8.25 •

10. $f(x) = \begin{cases} x^2 - 1 & \text{if } x \le -1 \\ 2 & \text{if } -1 < x < 3 \\ \sqrt{x+1} & \text{if } x \ge 3 \end{cases}$

x^2-1

x	y
-1	0 •
-2	3 •

2

x	y
-1	2 o
3	2 o

$\sqrt{x+1}$

x	y
3	2 •
8	3 •

11. Express $f(x) = |4x + 7|$ as a piecewise function and hence graph $f(x)$.

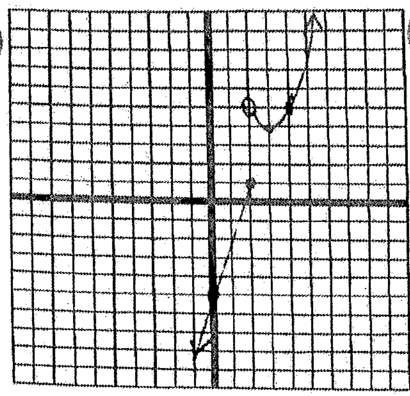
$$f(x) = \begin{cases} 4x+7 & x \ge -7/4 \\ -4x-7 & x < -7/4 \end{cases}$$

12. Express $f(x) = |4x + 3| - 4$ as a piecewise function and hence graph $f(x)$.

$$f(x) = \begin{cases} 4x-1 & x \ge \frac{-3}{4} \\ -4x-7 & x < -\frac{3}{4} \end{cases}$$

10.07 Piecewise

1



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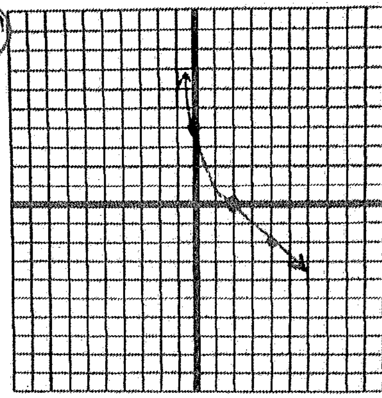

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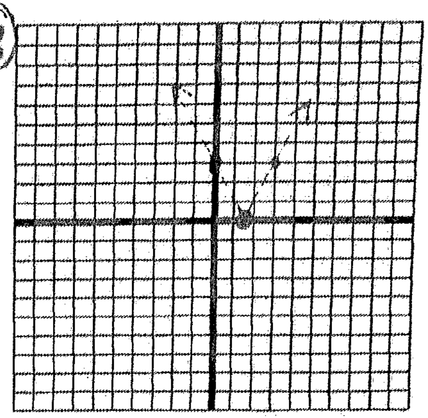

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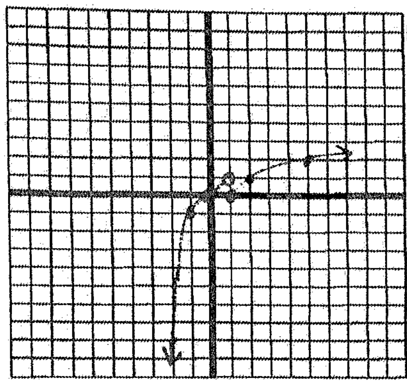

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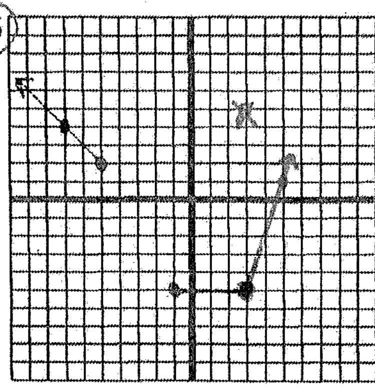

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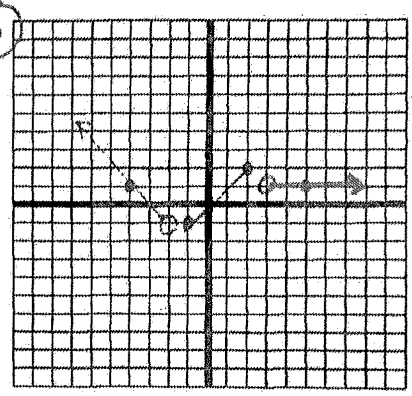

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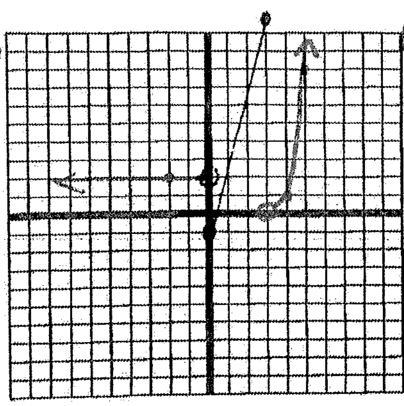

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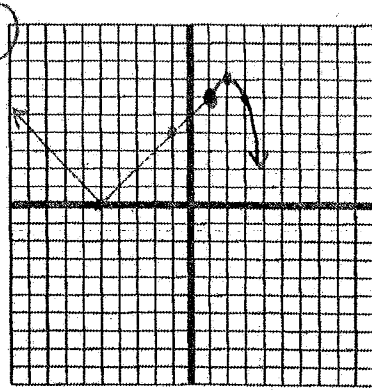

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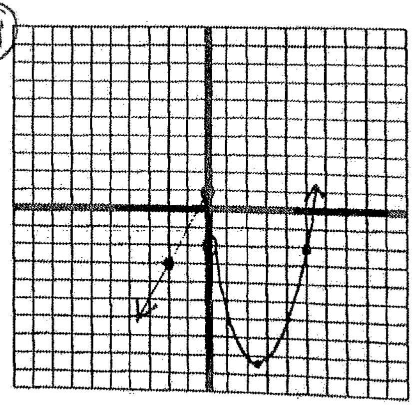

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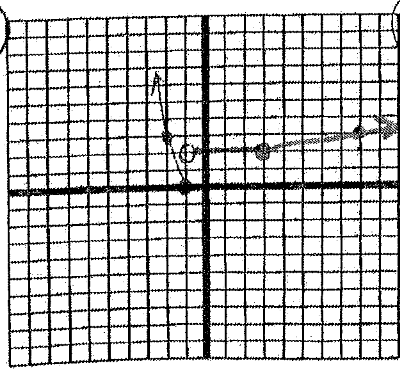

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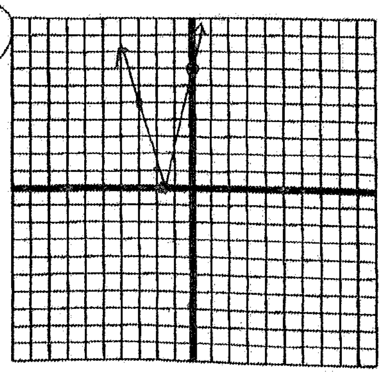
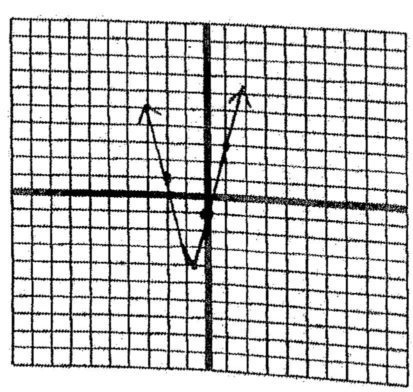




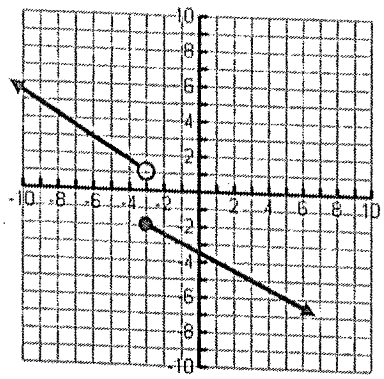
10.07 Writing the Piecewise Notes Date: Key

Write the equation for the piecewise function that is shown. Remember to include the domain restrictions for each piece. Then answer the questions about the characteristics of the function.

1.

① $m = -\frac{2}{3}$, $b = -1$

② $m = -\frac{1}{2}$, $b = -3.5$



$$f(x) = \begin{cases} -\frac{2}{3}x - 1 & x < -3 \\ -\frac{1}{2}x - \frac{7}{2} & x \geq -3 \end{cases}$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -2] \cup (1, \infty)$

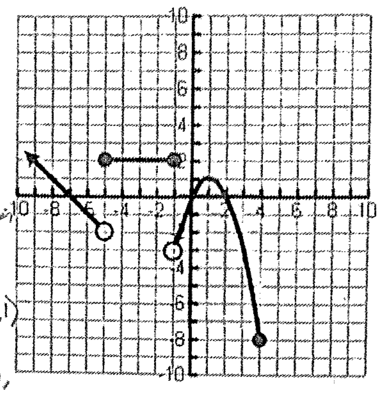
Is the graph continuous? no; discontinuous @ $x = -3$

2.

① $m = -1$, $b = -7$

② horizontal line

③ vertex: (1,1), $a = -1$, reflection, no stretch



$$f(x) = \begin{cases} -x - 7 & x < -5 \\ 2 & -5 \leq x \leq -1 \\ -(x-1)^2 + 1 & -1 < x \leq 4 \end{cases}$$

Intervals of Increase: $(-1, 1)$

Intervals of Decrease: $(-\infty, -5) \cup (1, 4]$

Intervals of Constant: $[-5, -1]$

3.

① $m = 2$, $b = 8$

② $m = -2$, $b = -12$

③ x^3

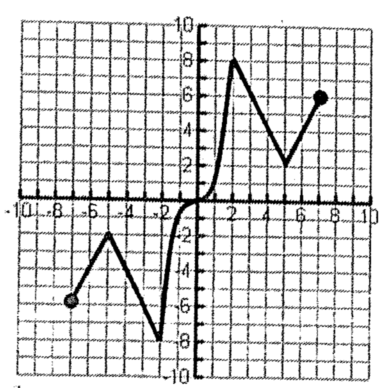
④ $m = -2$, $b = 12$

⑤ $m = 2$, $b = -8$

OR ① abs value, x axis reflection, vert. stretch by 2, vertex (-5, -2)

② x^3

③ abs value, vertical stretch by 2, vertex (5, 2)



$$f(x) = \begin{cases} 2x + 8 & -7 \leq x \leq -5 \\ -2x - 12 & -5 < x \leq -2 \\ x^3 & -2 < x < 2 \\ -2x + 12 & 2 \leq x < 5 \\ 2x - 8 & 5 \leq x \leq 7 \end{cases}$$

$-2|x+5| + 2 \quad -7 \leq x < -2$

$x^3 \quad -2 < x < 2$

$2|x-5| + 2 \quad 2 \leq x \leq 7$

Symmetry: Odd

Boundedness: Bounded

Extrema: Abs min: (-2, -8) Abs max: (2, 8)

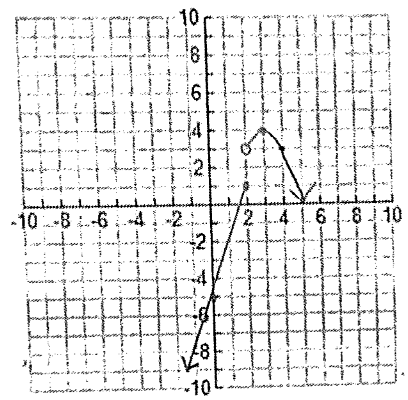
local min: (5, 2) (-7, -6) local max: (-5, -2) (7, 6)

10.07 Practice

Date: Key

Graph the following piecewise functions.

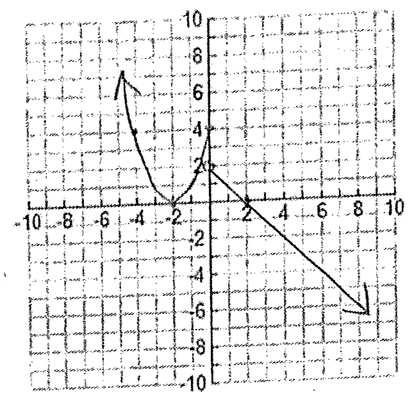
1. $f(x) = \begin{cases} 3x-5, & if\ x \le 2 \\ -(x-3)^2+4 & if\ x > 2 \end{cases}$



x	$3x-5$
0	-5
2	1

x	$-(x-3)^2+4$
2	3
3	4

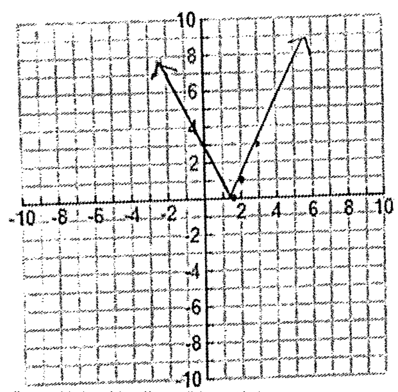
2. $g(x) = \begin{cases} (x+2)^2 & if\ x \le 0 \\ 2-x & if\ x > 0 \end{cases}$



x	$(x+2)^2$
-2	0
0	4

x	$2-x$
0	2
2	0

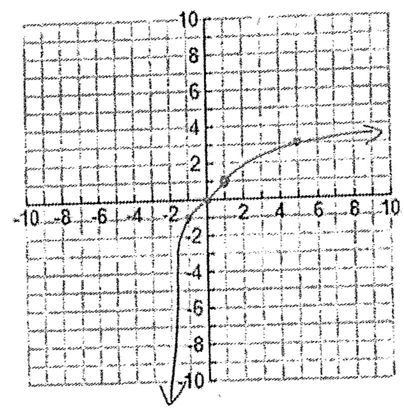
3. $f(x) = \begin{cases} 3-2x & if\ x < \frac{3}{2} \\ 2x-3 & if\ x \ge \frac{3}{2} \end{cases}$



x	$3-2x$
0	3
$\frac{3}{2}$	0

x	$2x-3$
$\frac{3}{2}$	0
2	1

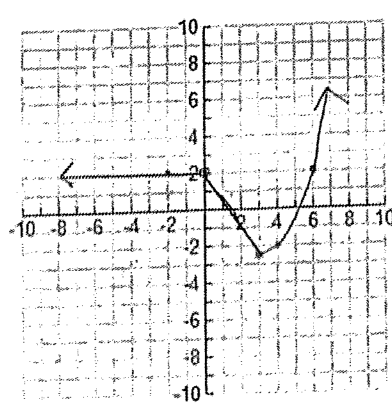
4. $g(x) = \begin{cases} x^3 & if\ x < 1 \\ \sqrt{(x-1)}+1 & if\ x \ge 1 \end{cases}$



x	x^3
-1	-1
0	0
1	1

x	$\sqrt{(x-1)}+1$
1	1
5	3

5. $f(x) = \begin{cases} 2 & x < 0 \\ -\frac{3}{2}x+2 & 0 \le x \le 3 \\ \frac{1}{2}(x-4)^3-2 & x > 3 \end{cases}$

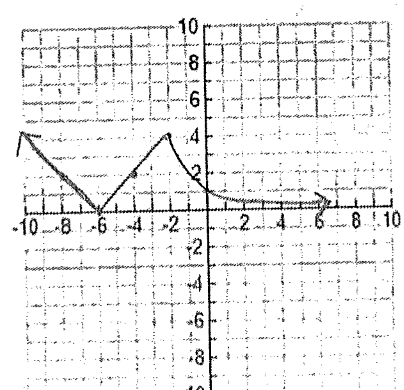


x	2
-2	2
0	2

x	$-\frac{3}{2}x+2$
0	2
3	-2.5

x	$\frac{1}{2}(x-4)^3-2$
3	-2.5
4	-2
6	2

6. $g(x) = \begin{cases} \lvert x+6 \rvert & x \le -2 \\ \left(\frac{1}{2}\right)^x & x > -2 \end{cases}$



x	$\lvert x+6 \rvert$
-4	2
-2	4

x	$\left(\frac{1}{2}\right)^x$
-2	4
0	1

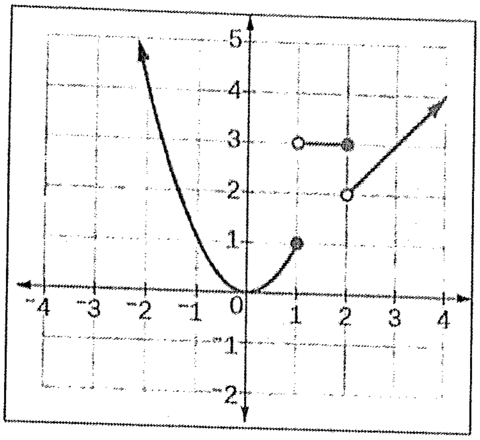
Write the equation for the piecewise function that is graphed. Assume that the domain for each is $(-\infty, \infty)$.

7. There are 3 pieces!

① x^2

② $y=3$

③ $m=1$
$b=0$



$$f(x) = \begin{cases} x^2 & x \le 1 \\ 3 & 1 < x \le 2 \\ x & x > 2 \end{cases}$$

8. There are at least 5 pieces!

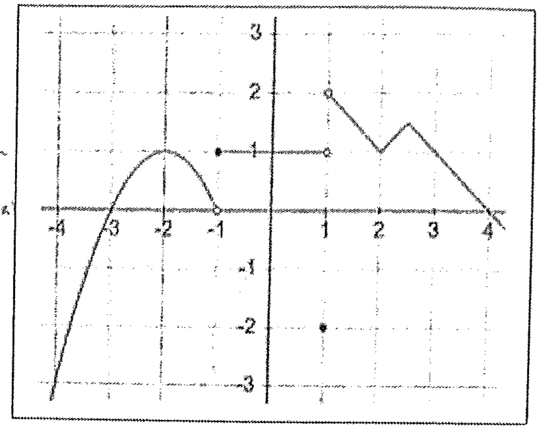
① vertex: $(-2,1)$
x-axis reflection

② horizontal line

③ point @ $(1,-2)$

④ $m=-1$
$b=3$

⑤ abs value
vertex: $(2.5, 1.5)$
x axis reflection



answers may vary

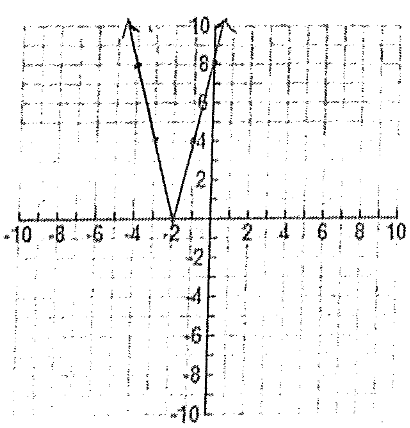
$$f(x) = \begin{cases} -(x+2)^2+1 & x < -1 \\ 1 & -1 \le x < 1 \\ -2 & x = 1 \\ -x+3 & 1 < x < 2 \\ -|x-2.5|+1.5 & x \ge 2 \end{cases}$$

Absolute value functions can be expressed as a piecewise function with two linear pieces, one for each ray that meet at the vertex.

11. Express $f(x) = 4|x+2|$ as a piecewise function and graph $f(x)$.

vertex: $(-2,0)$
vertical stretch by 4

$$f(x) = \begin{cases} -4x-8 & x \le -2 \\ 4x+8 & x > -2 \end{cases}$$



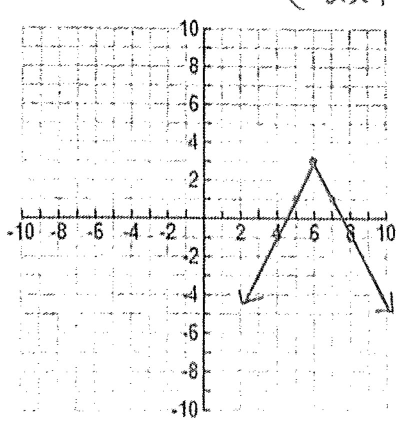
① $m=-4$
$b=-8$

② $m=4$
$b=8$

12. Express $f(x) = -2|x-6|+3$ as a piecewise function and graph $f(x)$.

vertex: $(6,3)$
x axis reflection
vertical stretch by 2

$$f(x) = \begin{cases} 2x-9 & x \le 6 \\ -2x+15 & x > 6 \end{cases}$$



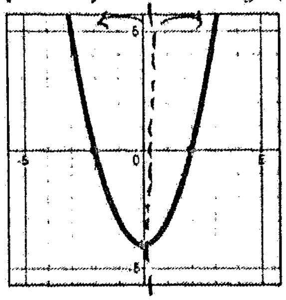
① $m=2$
$b=-9$

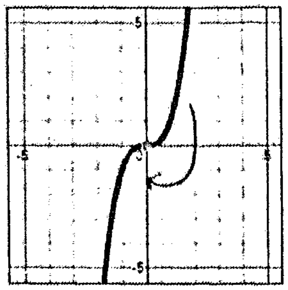
② $m=-2$
$b=15$

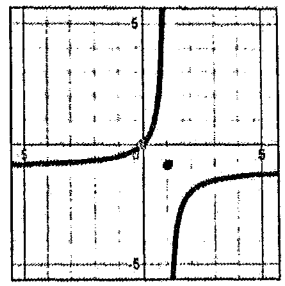
10.08 Analysis of Functions Review Date: Key

Functions: be able to name and sketch each of the 10 basic (parent) functions.

Symmetry: label each graph as even, odd, or neither.







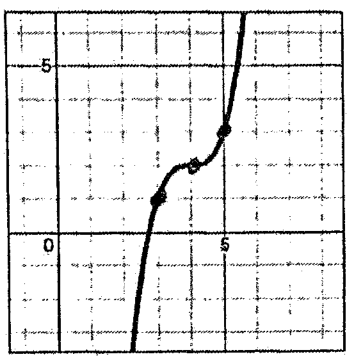
1. Even
2. ODD
3. neither symmetric about (1,-1)

4. How can you tell from the graph if it is even or odd? How can you determine this algebraically?

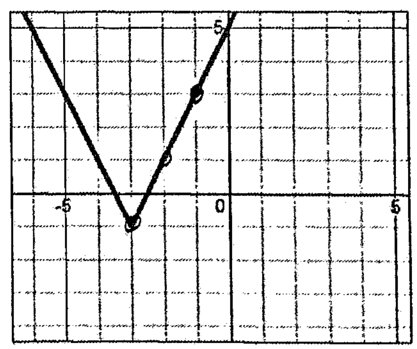
An even function - Reflection over y-axis. Plug in $-x$, is $f(-x)=f(x)$

An odd function - Rotate 180° around origin. Plug in $-x$, is $f(-x)=-f(x)$

Transformations: write the function of each graph by identifying the parent and the transformations.



x^3
shift right 4
shift up 2

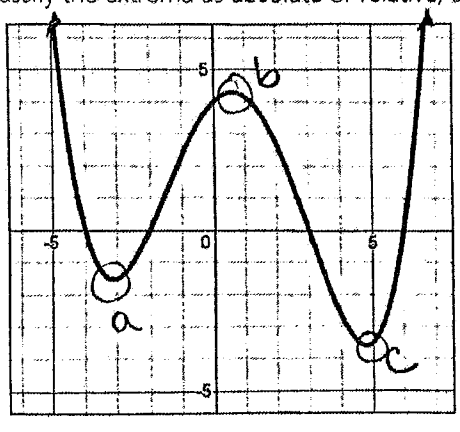


$|x|$
$a=2$ stretched taller x2
shift left 3
down 1

5. $y=(x-4)^3+2$

6. $y=2|x+3|-1$

Extrema: Circle the extrema of on the graph of the function, $f(x)$. Then, label each extrema as a, b, or c. Finally, classify the extrema as absolute or relative, and maximum or minimum for the function..



7.

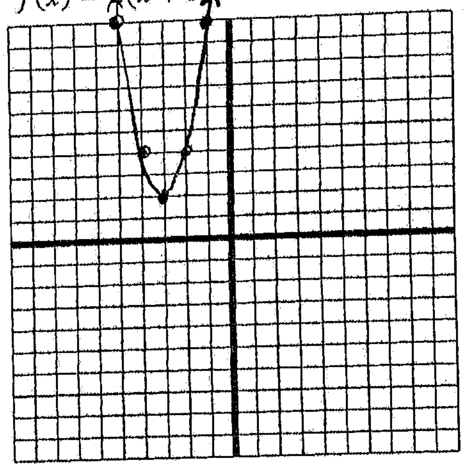
a) Relative min

b) Relative max

c) Absolute min

Graph characteristics: for the following functions, graph it, describe the transformations that occur from the parent graph, and idenfity the characteristics:

8. $f(x) = 2(x+3)^2 + 2$



$a=2$
$h=-3$
$k=2$

over 1
up 2

Transformations: Shift left 3, up 2, Vert stretch by 2

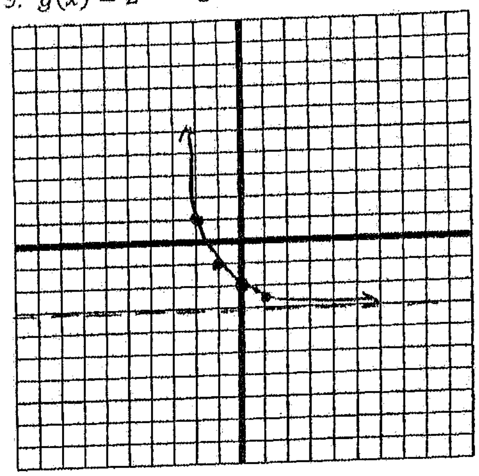
Domain $(-\infty,\infty)$ Range $[2,\infty)$

Extrema Abs min $(-3,2)$ Bounded below

Increase $(-3,\infty)$ Decrease $(-\infty,-3)$

End Behavior: $\lim_{x\to-\infty} f(x)=\infty$ $\lim_{x\to\infty} f(x)=\infty$

9. $g(x) = 2^{-x} - 3$



$b=-1$
$k=-3$ rA

x	2^{-x} -3
-2	$2^2-3=1$
-1	$2^1-3=-1$
0	$2^0-3=-2$
1	$2^{-1}-3=-2.5$

Transformations: Reflect over y axis

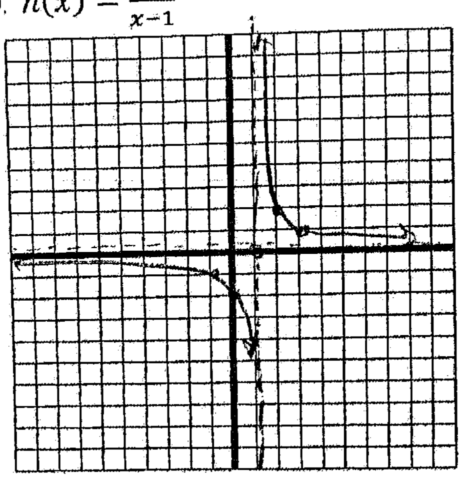
Domain $(-\infty,\infty)$ Range $(-3,\infty)$

Extrema none Bounded below

Increase none Decrease $(-\infty,\infty)$

End Behavior: $\lim_{x\to-\infty} f(x)=\infty$ $\lim_{x\to\infty} f(x)=-3$

10. $h(x) = \frac{2}{x-1}$



$h=1$
shift right 1

$a=2$
vert stretch
over 1
up 2

$x-1\neq 0$
$x\neq 1$

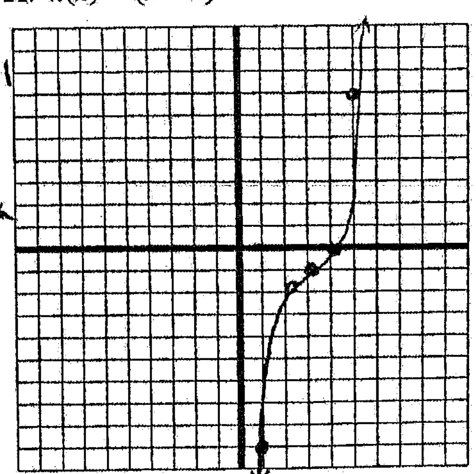
Domain $(-\infty,1)\cup(1,\infty)$ Range $(-\infty,0)\cup(0,\infty)$

Continuous? no. infinite disc @ $x=1$ non removable

Asymptotes: VA $x=1$ HA $y=0$

End Behavior: $\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to\infty} f(x)=0$

11. $k(x) = (x-3)^3 - 1$



$h=3$
right 3

$k=-1$
down 1

$a=1$
over 1
up 1

Domain $(-\infty,\infty)$ Range $(-\infty,\infty)$

Continuous? yes

Bounded? no unbounded

End Behavior: $\lim_{x\to-\infty} f(x)=-\infty$ $\lim_{x\to\infty} f(x)=\infty$

left right

Asymptotes: state the vertical and/or horizontal asymptotes for the graphs of each function.

12. $f(x) = \frac{5x}{3x-6}$ $3x-6 \neq 0$ $3x \neq 6$ $x \neq 2$

VA: $x = 2$

HA: $y = \frac{5}{3}$

$deg_N = deg_D$

$1 = 1$

13. $g(x) = \log_2(x+1)$

VA: $x = -1$ $x+1 > 0$ $x > -1$

HA: none

14. $h(x) = e^x + 3$

VA: none

HA: $y = 3$ $e^x > 0$ $e^x + 3 > 3$

Function Combination & Composition: given f(x) = $x^2 - 1$ and g(x) = $3 - x$, find each. Then state the domain of the resulting function.

15. f − g

$= (x^2-1) - (3-x)$

$= x^2 - 1 - 3 + x$

$= x^2 + x - 4$

Domain: $(-\infty, \infty)$

16. f/g $= \frac{x^2-1}{3-x}$

$= \frac{x^2-1}{3-x}$

Domain: $(-\infty, 3) \cup (3, \infty)$

$3 - x \neq 0$

$3 \neq x$

17. f * g

$= (x^2-1)(3-x)$

$= 3x^2 - x^3 - 3 + x$

$= -x^3 + 3x^2 + x - 3$

Domain: $(-\infty, \infty)$

18. f (g(x)) $= f(3-x)$

$= (3-x)^2 - 1$

$= 9 - 6x + x^2 - 1$

$= x^2 - 6x + 8$

Domain: $(-\infty, \infty)$

Given f(x) = $\sqrt{x-4}$ and g(x) = $x^2 - 5$, find each.

19. f (g(x)) $= f(x^2-5)$

$= \sqrt{(x^2-5)-4}$

$= \sqrt{x^2-9}$

$= \sqrt{x^2-9}$

Domain $x^2 - 9 \geq 0$

$x \geq 3$ or $x \leq -3$

20. f (g(5)) use #19 or

$f(5^2-5) = f(20)$

$= \sqrt{20-4} = \sqrt{16}$

$\boxed{4}$

21. g (f(x)) $= g(\sqrt{x-4})$

$= (\sqrt{x-4})^2 - 5$

$= x - 4 - 5$

$= x - 9$

Domain $x - 4 \geq 0$

$x \geq 4$

22. g (f(1)) use #21 or

$g(\sqrt{1-4}) = g(\sqrt{-3})$

not possible

x=1 is not in the domain of g(f(x)) b/c 1 is not in domain of f(x)

Given h(x), find f(x) and g(x) such that h(x) = f (g(x)). Hint: Look for the function on the "inside", that's g(x)!

23. h(x) = $(x+1)^3 - 4$

f(x) = $x^3 - 4$

g(x) = $x + 1$

24. h(x) = 5^{3x}

f(x) = 5^x

g(x) = $3x$

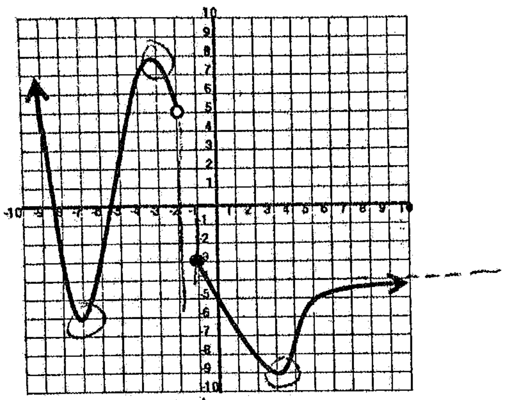
25. h(x) = $|x-3| + 1$

f(x) = $|x| + 1$

g(x) = $x - 3$

Piecewise functions: be able to graph, write the function from the graph, and identify the characteristics.

26. Analyze the following graph of a piecewise function.



gap!

Domain: $(-\infty,-2)\cup(-1,\infty)$ Range: $(-9,\infty)$

Increase ______ Decrease: ______

Extrema: Rel min $(-7,-6)$, Rel max $(-3,8)$, [1st] Abs Max @ $(3,-9)$

Bounded? below Continuous? no

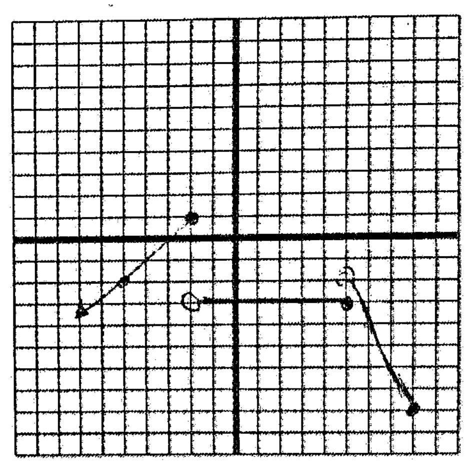
End Behavior: $\lim_{x\to-\infty} f(x)=\infty$ ← left $\lim_{x\to\infty} f(x)= -4$ ← right

27. Graph: $g(x)=\begin{cases} x+3 & if \quad x\le -2 \\ -3 & if \quad -2<x\le 5 \\ 8-2x & if \quad 5<x\le 8 \end{cases}$

x	x+3	
-2	-2+3=1	•
-5	-5+3=-2	←

x	-3	
-2	-3	o
5	-3	•

x	8-2x	
5	8-2(5)=-2	o
8	8-2(8)=-8	•



28. Write the function graphed in 3 pieces:

$h(x)=\begin{cases} -1 & -5\le x<-2 \\ -x^2+3 & -2\le x\le 1 \\ -x+3 & x>2 \end{cases}$