Unit 1 Limits Review

Algebraic Steps (for x approaching Real Number): **1)** Plug in x-value first (IGNORE one-sided limit) **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate the reduced Expression 4) If result is undefined, and it's a one-sided limit, then test using decimals.

Evaluate Limits (for x approaching $\pm\infty$): 1) Compare Degrees: **i)** if Numerator < Denominator , Limit = 0 **ii)** If Numerator = Denominator, Limit = ratio of coefficients **iii)** If Number > Denominator, Limit = DNE $\pm\infty$

L'Hopital's Rule Option: If Evaluating Limits produces $\frac{0}{0}$ then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$

- take derivative of numerator and denominator separately,
- then re-evaluate Limit.

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1. $\lim_{x\to 2}\frac{x^2-4}{x^2+4}$ is

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) ∞

2. $\lim_{x\to\infty}\frac{4-x^2}{x^2-1}$ is

(A) 1 (B) 0 (C) -4 (D) -1 (E) ∞

3. $\lim_{x\to 3^-}\frac{x-3}{x^2-2x-3}$ is

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

4. $\lim_{x\to -3^+}\frac{2x+1}{x+3}$ is

A) 0 B) 2 C) -5 D) ∞ E) $-\infty$

5) $\lim\limits_{x\to\infty} \dfrac{4-x^2}{4x^2-x-2}$ is

(A) -2 (B) $-\dfrac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

6) $\lim\limits_{x\to-\infty} \dfrac{5x^3+27}{20x^2+10x+9}$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 3 (E) ∞

7) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{x}$

(A) $=0$ (B) $=\dfrac{1}{5}$ (C) $=1$ (D) $=5$ (E) does not exist

8) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{3x}$

9) (A) $=0$ (B) $=\dfrac{2}{3}$ (C) $=1$ (D) $=\dfrac{3}{2}$ (E) does not exist

10) The graph of $y=\dfrac{x^2-9}{3x-9}$ has

(A) a vertical asymptote at $x=3$ (B) a horizontal asymptote at $y=\dfrac{1}{3}$
(C) a removable discontinuity at $x=3$ (D) an infinite discontinuity at $x=3$
(E) none of these

11) $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 (B) -2 (C) 1 (D) 2 (E) nonexistent

12) Which statement is true about the curve $y=\dfrac{2x^2+4}{2+7x-4x^2}$?

(A) The line $x=-\frac{1}{4}$ is a vertical asymptote.
(B) The line $x=1$ is a vertical asymptote.
(C) The line $y=-\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptote.
(E) The line $y=2$ is a horizontal asymptote.

Continuity Conditions Review:

1. f(c) is defined (point exists on the graph)
2. The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\right]$
3. f(c) $= \lim_{x \to c} f(x)$

- If function passes all 3 conditions , the function has continuity at x = c
- If condition #2 FAILS, the function has **nonremovable** discontinuity at x = c
- If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at x = c

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13) Let $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1. \end{cases}$

Which of the following statements is (are) true?

I. $\lim_{x \to 1} f(x)$ exists

II. $f(1)$ exists

III. f is continuous at $x = 1$

(A) I only **(B)** II only **(C)** I and II
(D) none of them **(E)** all of them

14) If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, \text{ for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) 1 E) $\frac{7}{5}$

15) Let $f(x) = \begin{cases} \dfrac{x^2+x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists

II. $\lim_{x \to 0} f(x)$ exists

III. f is continuous at $x = 0$

(A) I only **(B)** II only **(C)** I and II only
(D) all of them **(E)** none of them

16)

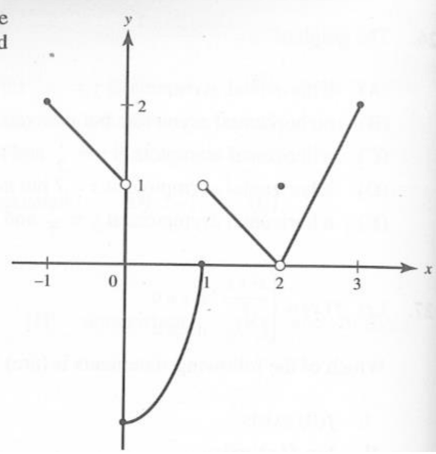
Suppose $\begin{cases} f(x) = \dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ **(B)** except at $x = 2$ **(C)** except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 **(E)** at each real number

Questions 32–36 are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1-x & (-1 \leqslant x < 0) \\ 2x^2-2 & (0 \leqslant x \leqslant 1) \\ -x+2 & (1 < x < 2) \\ 1 & (x = 2) \\ 2x-4 & (2 < x \leqslant 3) \end{cases}$$



32. $\lim_{x \to 2} f(x)$

(A) equals 0 **(B)** equals 1 **(C)** equals 2
(D) does not exist **(E)** none of these

33. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ **(B)** if $x \neq 1$ **(C)** if $x \neq 2$
(D) if $x \neq 3$ **(E)** at each x in $[-1,3]$

34. The function f has a removable discontinuity at

(A) $x = 0$ **(B)** $x = 1$ **(C)** $x = 2$ **(D)** $x = 3$ **(E)** none of these

35. On which of the following intervals is f continuous?

(A) $-1 \leqslant x \leqslant 0$ **(B)** $0 < x < 1$ **(C)** $1 \leqslant x \leqslant 2$
(D) $2 \leqslant x \leqslant 3$ **(E)** none of these

36. The function f has a jump discontinuity at

(A) $x = -1$ **(B)** $x = 1$ **(C)** $x = 2$
(D) $x = 3$ **(E)** none of these