Key

Unit 1 Limits Review

Algebraic Steps (for x approaching Real Number): 1) Plug in x-value first (IGNORE one-sided limit) **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate the reduced Expression 4) If result is undefined, and it's a one-sided limit, then test using decimals.

Evaluate Limits (for x approaching ±∞): 1) Compare Degrees: **i)** if Numerator < Denominator , Limit = 0 **ii)** If Numerator = Denominator, Limit = ratio of coefficients **iii)** If Number > Denominator, Limit = DNE ±∞

L'Hopital's Rule Option: If Evaluating Limits produces $\frac{0}{0}$ then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$

- take derivative of numerator and denominator separately,
- then re-evaluate Limit.

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1. $\lim_{x\to 2}\frac{x^2-4}{x^2+4}$ is $\frac{2^2-4}{2^2+4} = \frac{0}{8} = \boxed{0}$

(A) 1 $\quad$ **(B)** 0 $\quad$ (C) $-\frac{1}{2}$ $\quad$ (D) −1 $\quad$ (E) ∞

2. $\lim_{x\to\infty}\frac{4-x^2}{x^2-1}$ is *compare degrees* $\frac{-1x^2}{1x^2} = \frac{-1}{1} = \boxed{-1}$

(A) 1 $\quad$ (B) 0 $\quad$ (C) −4 $\quad$ **(D)** −1 $\quad$ (E) ∞

3. $\lim_{x\to 3^-}\frac{x-3}{x^2-2x-3}$ is $\frac{3-3}{3^2-2(3)-3} = \frac{0}{0} \to \lim_{x\to 3^-}\frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+1)} \to \frac{1}{3+1} = \boxed{\frac{1}{4}}$

* Note we don't test using decimals unless limit is undefined (ex: $\frac{4}{0}$)

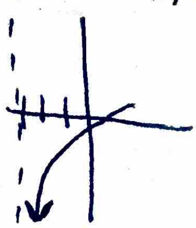
(A) 0 $\quad$ (B) 1 $\quad$ **(C)** $\frac{1}{4}$ $\quad$ (D) ∞ $\quad$ (E) none of these

4. $\lim_{x\to -3^+}\frac{2x+1}{x+3}$ is $\frac{2(-3)+1}{-3+3} = \frac{-5}{0}$ limit d.n.e. (but since one-sided limit, we test decimals) $\lim_{x\to -3^+}\frac{2x+1}{x+3}$

test $x = -2.9$

$\frac{2(-2.9)+1}{-2.9+3} = \frac{-}{+} = \boxed{-\infty}$

A) 0 $\quad$ B) 2 $\quad$ C) -5 $\quad$ D) ∞ $\quad$ **E) −∞**



5) $\lim\limits_{x\to\infty} \frac{4-x^2}{4x^2-x-2}$ is

*compare degrees

$$\frac{-1x^2}{4x^2} = \boxed{-\frac{1}{4}}$$

(A) -2 $\boxed{\text{(B)}}$ $-\frac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

6) $\lim\limits_{x\to-\infty} \frac{5x^3+27}{20x^2+10x+9}$ is

*compare degrees
* Numerator > Denominator
limit d.n.e., test ±100

$$\lim_{x\to-\infty} \frac{5(-100)^3+27}{20(-100)^2+10(-100)+9} = \frac{-}{+} = \boxed{-\infty}$$

test $x=-100$

$\boxed{\text{(A)} \ -\infty}$ (B) -1 (C) 0 (D) 3 (E) ∞

7) $\lim\limits_{x\to0} \frac{\sin 5x}{x} \to \frac{0}{0}$ L'Hopital's Rule (take derivative)

$$\lim_{x\to0} \frac{\cos(5x)\cdot5}{1} = \frac{\cos(0)\cdot5}{1} = \boxed{5}$$

(A) $=0$ (B) $=\frac{1}{5}$ (C) $=1$ $\boxed{\text{(D)} \ =5}$ (E) does not exist

8) $\lim\limits_{x\to0} \frac{\sin 2x}{3x}$ $\frac{0}{0} \to$ L'Hopital's

$$\lim_{x\to0} \frac{\cos(2x)\cdot2}{3} = \frac{\cos(0)\cdot2}{3} = \boxed{\frac{2}{3}}$$

(A) $=0$ $\boxed{\text{(B)} \ =\frac{2}{3}}$ (C) $=1$ (D) $=\frac{3}{2}$ (E) does not exist

10) The graph of $y=\frac{x^2-9}{3x-9}$ has

$$y=\frac{\cancel{(x-3)}(x+3)}{3\cancel{(x-3)}}$$

hole at $x=3$ (removable discontinuity)

(A) a vertical asymptote at $x=3$
(B) a horizontal asymptote at $y=\frac{1}{3}$
$\boxed{\text{(C)}}$ a removable discontinuity at $x=3$
(D) an infinite discontinuity at $x=3$
(E) none of these

11) $\lim\limits_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ is

expand, then compare degrees

$$\lim_{x\to\infty} \frac{2x^2+1}{4-x^2} = \frac{2}{-1} = \boxed{-2}$$

(A) -4 $\boxed{\text{(B)}}$ -2 (C) 1 (D) 2 (E) nonexistent

12) Which statement is true about the curve $y=\frac{2x^2+4}{2+7x-4x^2}$?

$$y=\frac{2(x^2+2)}{-(4x^2-7x-2)}$$

$$y=\frac{2(x^2+2)}{-(4x+1)(x-2)}$$

V.A.

$4x+1=0$ $x-2=0$

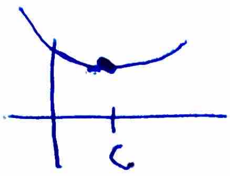
$\boxed{\text{V.A. } x=-1/4 \quad x=2}$

$\boxed{\checkmark\text{(A)}}$ The line $x=-\frac{1}{4}$ is a vertical asymptote.
~~(B)~~ The line $x=1$ is a vertical asymptote.
~~(C)~~ The line $y=-\frac{1}{4}$ is a horizontal asymptote.
~~(D)~~ The graph has no vertical or horizontal asymptote.
~~(E)~~ The line $y=2$ is a horizontal asymptote.

horizontal asymptote $\frac{2x^2}{-4x^2}$

H.A. $y=-1/2$

Continuity Conditions Review:



1. f(c) is defined (point exists on the graph)
2. The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\right]$
3. f(c) $= \lim_{x \to c} f(x)$

- If function passes all 3 conditions, the function has continuity at x = c
- If condition #2 FAILS, the function has **nonremovable** discontinuity at x = c
- If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at x = c

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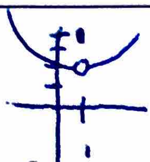
13) Let $f(x)=\begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$



Which of the following statements is (are) true?

I. $\lim_{x \to 1} f(x)$ exists ✓

II. $f(1)$ exists ✓

III. f is continuous at $x = 1$ ✗

(A) I only (B) II only **$\boxed{\text{(C) I and II}}$**

(D) none of them (E) all of them

*Step through continuity conditions:

i) $f(1) = 4$

ii) $\lim_{x \to 1} \frac{x^2-1}{x-1} = \frac{0}{0} \to \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 1+1 = 2$

iii) $f(1) \neq \lim_{x \to 1} f(x)$, $2 \neq 4$

*limit exists, but not continuous at x=1 because removable discontinuity

14) If $\begin{cases} f(x) = \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, \text{ for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

A) 0 $\boxed{\text{B) } \frac{1}{6}}$ C) $\frac{1}{3}$ D) 1 E) $\frac{7}{5}$

*step through continuity conditions:

i) $f(2) = K$

ii) $\lim_{x \to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = \frac{0}{0}$

conjugate method

$\lim_{x \to 2} \frac{(\sqrt{2x+5}-\sqrt{x+7})}{(x-2)} \cdot \frac{(\sqrt{2x+5}+\sqrt{x+7})}{(\sqrt{2x+5}+\sqrt{x+7})}$

$2x+5-x-7$

$\lim_{x \to 2} \frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$

$\lim_{x \to 2} \frac{\cancel{x-2}\ 1}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})} = \frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{6}$

iii) $f(2) = k = \boxed{\frac{1}{6}}$

15) Let $f(x)=\begin{cases} \frac{x^2+x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

Which of the following statements is (are) true?

I. $f(0)$ exists ✓

II. $\lim_{x \to 0} f(x)$ exists ✓

III. f is continuous at $x = 0$ ✓

(A) I only (B) II only (C) I and II only

$\boxed{\text{(D) all of them}}$ (E) none of them

Continuity conditions:

✓ i) $f(0) = 1$

✓ ii) $\lim_{x \to 0} \frac{x^2+x}{x} = \frac{0}{0} \to \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} = 1$

✓ iii) $f(0) = \lim_{x \to 0} f(x) = 1$

16) Suppose $\begin{cases} f(x) = \dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

continuity condition at $x=1$

i) $f(1)=-3$

ii) $\lim_{x\to1} \frac{3x\cancel{(x-1)}}{(x-2)\cancel{(x-1)}} = \frac{3}{-1} = -3$

iii) $f(1) = \lim_{x\to1} f(x) = -3$

continuous at $x=1$

continuity condition at $x=2$

i) $f(2)=4$

ii) $\lim_{x\to2} \frac{3x\cancel{(x-1)}}{(x-2)\cancel{(x-1)}} = \frac{6}{0}$

limit does not exist at $x=2$

VA. at $x=2$

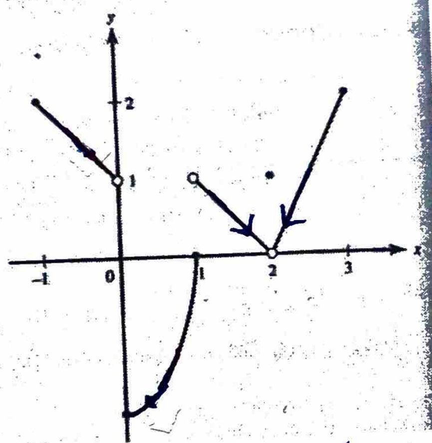
Not continuous at $x=2$

Then $f(x)$ is continuous

(A) except at $x=1$ $\boxed{\text{(B) except at } x=2}$ (C) except at $x=1$ or 2
(D) except at $x=0, 1,$ or 2 (E) at each real number

Questions 32–36 are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1-x & (-1 \le x < 0) \\ 2x^2-2 & (0 \le x \le 1) \\ -x+2 & (1 < x < 2) \\ 1 & (x=2) \\ 2x-4 & (2 < x \le 3) \end{cases}$$



32. $\lim_{x\to2} f(x)$ the graph approaches 0 from both sides of $x=2$

$\boxed{\text{(A) equals 0}}$ (B) equals 1 (C) equals 2
(D) does not exist (E) none of these

33. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ (B) if $x \neq 1$ (C) if $x \neq 2$
(D) if $x \neq 3$ $\boxed{\text{(E) at each } x \text{ in } [-1,3]}$

34. The function f has a removable discontinuity at (means limit exists but not continuous)

(A) $x=0$ (B) $x=1$ $\boxed{\text{(C) } x=2}$ (D) $x=3$ (E) none of these

35. On which of the following intervals is f continuous?

(A) $-1 \le x \le 0$ $\boxed{\text{(B) } 0 < x < 1}$ (C) $1 \le x \le 2$
(D) $2 \le x \le 3$ (E) none of these

36. The function f has a jump discontinuity at (nonremovable discontinuity)

(A) $x=-1$ $\boxed{\text{(B) } x=1}$ (C) $x=2$
(D) $x=3$ (E) none of these

$\lim_{x\to1^-} f(x) = 0$

$\lim_{x\to1^+} f(x) = 1$