Unit 2 AP Practice pg. 220-221

AP® Review Problems: Chapter 2

1. $f'(x) = \frac{d}{dx}(\sec x) = \sec x \tan x$

   $f'\left(\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right) = \sqrt{2}\cdot 1 = \boxed{\sqrt{2}}$

   CHOICE D

2. Three forms of the derivative are

   $f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$, $f(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$, and $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

   Only answers I and III match any of these.

   CHOICE D

3. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{x^2-5}\right) = \frac{\left[\frac{d}{dx}(3)\right](x^2-5)-\left[\frac{d}{dx}(x^2-5)\right]\cdot 3}{(x^2-5)^2} = \frac{0\cdot(x^2-5)-2x\cdot 3}{(x^2-5)^2} = \boxed{-\frac{6x}{(x^2-5)^2}}$

   CHOICE B

4. (A) FALSE. f is not differentiable where there are vertical tangents, which occur at $x = -3, -1, 1$, and 3; or where there are corners, which occur at those same x-values and also at $x = -2$, 0, and 2.

   (B) FALSE. It appears that $f'(x) < 0$ on the intervals: $(-2,\ -1)$, $(0,\ 1)$, and $(2, 3)$.

   (C) TRUE. It appears that for all $c \in \mathbb{R}$, $\lim_{x\to c} f(x)$exists, $f(c)$ is defined, and $\lim_{x\to c} f(x) = f(c)$.

   (D) FALSE. For $f(x)$ to be an even function, it must be true that$f(-x) = f(x)$. However, here it appears that, for example,$f\left(-\frac{1}{2}\right) \neq f\left(\frac{1}{2}\right)$.

   CHOICE C

5. $m_{\sec} = \frac{f(2.002)-f(2)}{2.002-2} = \frac{3.207-3.194}{0.002} = \frac{0.013}{0.002} = \boxed{6.5}$

   CHOICE A

6. $y' = \frac{d}{dx}(\sin x + xe^x + 6) = \frac{d}{dx}(\sin x) + \frac{d}{dx}(xe^x) + \frac{d}{dx}(6) = \cos x + \left[\frac{d}{dx}(x)\right]e^x + x\left[\frac{d}{dx}e^x\right] + 0 = \cos x + e^x + xe^x$

   When $x = 5$, $y' = \cos(5) + e^{(5)} + (5)e^{(5)} = \boxed{\cos 5 + 6e^5}$

   CHOICE A

7. $f'(x) = \frac{d}{dx}(3xe^x + 5) = \left[\frac{d}{dx}(3x)\right]e^x + 3x\left[\frac{d}{dx}(e^x)\right] + \frac{d}{dx}(5) = 3e^x + 3xe^x$

   The slope of the tangent line to $f(x)$ at $x = 0$ is $f'(0) = 3e^{(0)} + 3\cdot(0)\cdot e^{(0)} = 3$.

   The slope of the normal line at $x = 0$ is $\frac{-1}{f'(0)} = -\frac{1}{3}$

   $f(0) = 3\cdot(0)\cdot e^{(0)} + 5 = 5$

   $y - 5 = -\frac{1}{3}(x - 0)$

   $\boxed{y = -\frac{1}{3}x + 5}$

   CHOICE B

8. $v(t) = s'(t) = \frac{d}{dt}\left(t^4 - 6t^3 - 2t - 1\right) = \frac{d}{dt}\left(t^4\right) - 6\frac{d}{dt}\left(t^3\right) - 2\frac{d}{dt}(t) - \frac{d}{dt}(1) =$
$4t^3 - 18t^2 - 2$

$a(t) = v'(t) = s''(t) = \frac{d}{dt}\left(4t^3 - 18t^2 - 2\right) = 4\frac{d}{dt}\left(t^3\right) - 18\frac{d}{dt}\left(t^2\right) - \frac{d}{dt}(2) = 12t^2 - 36t$

$$a(t) = 0$$
$$12t^2 - 36t = 0$$
$$12t(t-3) = 0$$

$\boxed{t = 0 \text{ or } t = 3}$

CHOICE D

9. $f'(x) = \frac{d}{dx}[e^x(\sin x + \cos x)] = \left[\frac{d}{dx}(e^x)\right](\sin x + \cos x) + e^x\left[\frac{d}{dx}(\sin x + \cos x)\right] =$
$e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x = \boxed{2e^x \cos x}$

CHOICE C

10. $f'(x) = \frac{d}{dx}\left(\frac{x+3}{x^2+2}\right) = \frac{\left[\frac{d}{dx}(x+3)\right](x^2+2) - \left[\frac{d}{dx}(x^2+2)\right](x+3)}{(x^2+2)^2} = \frac{1\cdot(x^2+2) - 2x(x+3)}{(x^2+2)^2} = \frac{x^2+2-2x^2-6x}{(x^2+2)^2} =$
$\frac{-x^2-6x+2}{(x^2+2)^2}$

$f'(1) = \frac{-(1)^2 - 6(1) + 2}{\left[(1)^2+2\right]^2} = -\frac{5}{9}$

$f(1) = \frac{(1)+3}{(1)^2+2} = \frac{4}{3}$

$$y - \frac{4}{3} = -\frac{5}{9}(x-1)$$
$$9y - 12 = -5x + 5$$

$\boxed{5x + 9y = 17}$

CHOICE A

11. CHOICE C

AP® Cumulative Review Problems: Chapters 1–2

1. $\lim_{x\to 4} \frac{x-4}{4-x} = \lim_{x\to 4} \frac{x-4}{-1(x-4)} = -1$

   $\boxed{\text{CHOICE B}}$.

2. $\lim_{x\to 0} \frac{3x+\sin x}{2x}$

   $= \lim_{x\to 0} \frac{3x}{2x} + \lim_{x\to} \frac{\sin x}{2x}$

   $= \frac{3}{2} \lim_{x\to 0} (1) + \frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x}$

   $= \frac{3}{2}(1) + \frac{1}{2}(1)$

   $= 2$

   $\boxed{\text{CHOICE C}}$.

3. $\lim_{x\to 1} f(x)\, g(x) = k + x$

   $\lim_{x\to 1} f(x) \lim_{x\to 1} g(x) = k + 1$

   $(2)(-2) = k + 1$

   $k = -5$

   $\boxed{\text{CHOICE A}}$.

4. Examine each choice in turn.

   Examine Choice A: $f(x) = \frac{2}{\pi}\tan^{-1} x$ has horizontal asymptotes at $y = \frac{2}{\pi}\left(\frac{\pi}{2}\right) = 1$ and at $y = \frac{2}{\pi}\left(\frac{-\pi}{2}\right) = -1$.

   Examine Choice B: $f(x) = e^{-x} + 1$ is a decreasing exponential function with a horizontal asymptote at $y = 1$.

   Examine Choice C: $f(x) = \frac{1-x^2}{1+x^2}$ has a horizontal asymptote at $y = -1$ determined by taking the ratio of the coefficient of the highest power in the numerator over the coefficient of the highest power in the denominator since the powers of the numerator and denominator are equal.

   Examine Choice D: $f(x) = \frac{2x^2-1}{2x^2+x}$ has a horizontal asymptote at $y = 1$ determined by taking the ratio of the coefficient of the highest power in the numerator over the coefficient of the highest power in the denominator since the powers of the numerator and denominator are equal.

   $\boxed{\text{CHOICE A}}$.

5. The Intermediate Value Theorem provides that for a function f that is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$ that for N, any number between $f(a)$ and $f(b)$, there is at least one number c in the open interval (a, b) for which $f(c) = N$. Here, since $g(x) = f^{-1}(x)$, in addition to the defined functions $f(-2) = 1$ and $f(5) = -3$ we know that $g(1) = -2$ and $g(-3) = 5$. So on the graph of $f(x)$ we have a continuous graph extending from $(-2, 1)$ to $(5, -3)$ inclusive. On the graph of $g(x)$ we have a continuous graph extending from $(1, -2)$ to $(5, -3)$ inclusive. Accordingly, the only correct option is $\boxed{\text{CHOICE A}}$.

6. Let us consider each choice in turn. Choice A can be true as x gets closer to c from the negative side the y coordinates could increase to very large numbers, to ∞, because $x = c$ is a vertical asymptote. Choice B can also be true if $y = c$ is a horizontal asymptote which could occur if with the equation $f(x) = \dfrac{c}{x - c}$ where $x = c$ is the vertical asymptote. Choice C could be true if $f(c)$ is defined which would be the case in an example such as $f(x) = \begin{cases} \dfrac{1}{x - c} & \text{if } x \neq c \\ 5 & \text{if } x = c \end{cases}$. Choice D cannot be true since the statement $\lim\limits_{x \to c} f(x)$ is false. $\boxed{\text{CHOICE D}}$.

7. $s(t) = \dfrac{t^3}{15} - \dfrac{t^2}{2} + \dfrac{5}{t}$

   $v(t) = s'(t) = \dfrac{t^2}{5} - t - \dfrac{5}{t^2}$

   $a(t) = v'(t) = s''(t) = \dfrac{2t}{5} - 1 + \dfrac{10}{t^3}$

   $a(5) = \dfrac{2(5)}{5} - 1 + \dfrac{10}{5^3} = \dfrac{27}{25}$

   $\boxed{\text{CHOICE D}}$.

8. $2ax^2 + bx - 1 = bx^2 + bx - a$

   $2a = b \; a = 1$

   $18a + 3b - 1 = 9b + 3b - a$

   $19a - 9b = 1$

   $\boxed{\text{CHOICE C}}$.

9. $f(x) = xe^x$

   $f'(x) = e^x + xe^x$

   $f'(1) = e + e = 2e$

   $\boxed{\text{CHOICE C}}$.

10. $s(t) = 3t^4 - 8t^3 - 6t^2 + 24t$

$v(t) = s'(t) = 12t^3 - 24t^2 - 12t + 24$
$2t^3 - 24t^2 - 12t + 24 = 0$
$t^3 - 12t^2 - 6t + 12 = 0$
$\left(t^2 - 1\right)(t - 2) = 0$
$(t - 1)(t + 1)(t - 2) = 0$
$v'(t) = a(t) = 36t^2 - 48t - 12$
$a(1) = 36 - 48 - 12 = -24$
$a(2) = 36(2^2) - 48(2) - 12 = 36$
$t = -1$ is not in the domain.

$\boxed{\text{CHOICE D}}$.