Accelerated Precalculus
Intro to Sine & Cosine Function Graphs

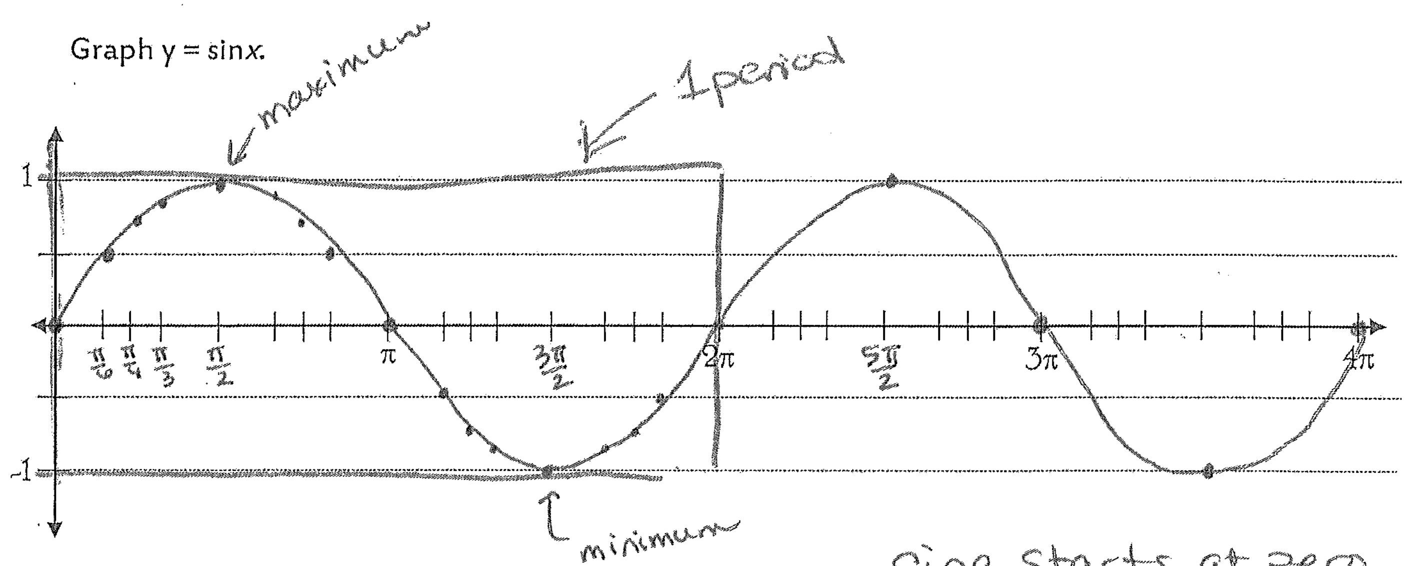
Name Key
Date ________ Period ______

Exploring the Graph of the Sine Function

Using the unit circle, fill in the chart below with the value of sine at each angle.

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Sine (exact)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
Sine (decimal)	0	.5	.707	.866	1	.866	.707	.5	0	-.5	-.707	-.866	-1	-.866	-.707	-.5	0

Graph y = sinx.



Sine starts at zero

1. The period is 2π ← length of the section that repeats
2. The domain is $(-\infty, \infty)$ $\mathbb{R}$ ← x-values
3. The range is $[-1, 1]$ ← y-values
4. The x-intercepts are all multiples of π $\quad \pi k,\ k \in \mathbb{Z}$ ← integers $\quad \pi n,\ n \in \mathbb{Z}$

   epsilon "is an element of"
5. The y-intercept is 0
6. The maximum values are 1 and occur when $\frac{\pi}{2} + 2\pi k,\ k \in \mathbb{Z}$
7. The minimum values are -1 and occur when $\frac{3\pi}{2} + 2\pi k,\ k \in \mathbb{Z}$

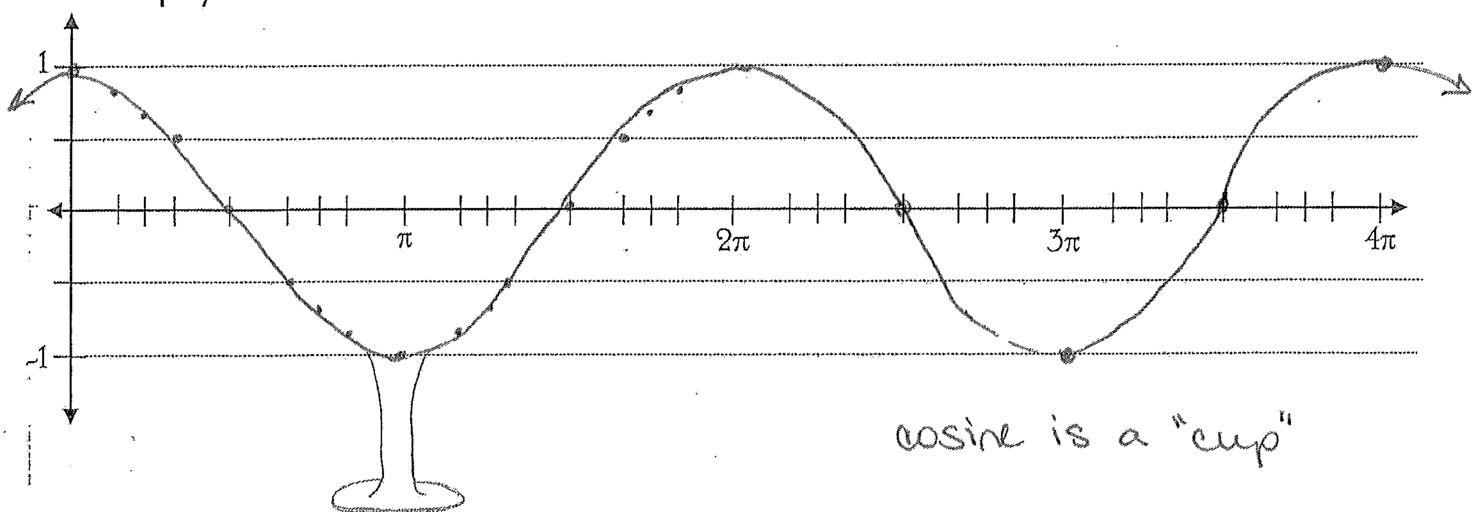
   every 2π later

Exploring the Graph of the Cosine Function

Using the unit circle, fill in the chart below with the value of cosine at each angle.

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Cosine (exact)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosine (decimal)	1	.866	.707	.5	0	-.5	-.707	-.866	-1	-.866	-.707	-.5	0	.5	.707	.866	1

Graph $y = \cos x$.



1. The period is 2π

2. The domain is $(-\infty, \infty)$

3. The range is $[-1, 1]$

4. The x-intercepts are $\frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$

5. The y-intercept is 1

6. The maximum values are 1 and occur when $0 + 2\pi k,\ k \in \mathbb{Z}$

7. The minimum values are -1 and occur when $\pi + 2\pi k,\ k \in \mathbb{Z}$

Accel Pre-Calculus
Graphing Sine and Cosine Functions
2.02 Amplitude and Period Notes

Name: ____________________

Date: ____________

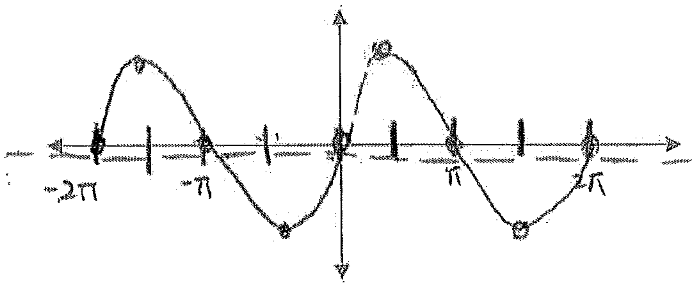
Remember the patterns and the shape of the graphs for the sine and cosine function:

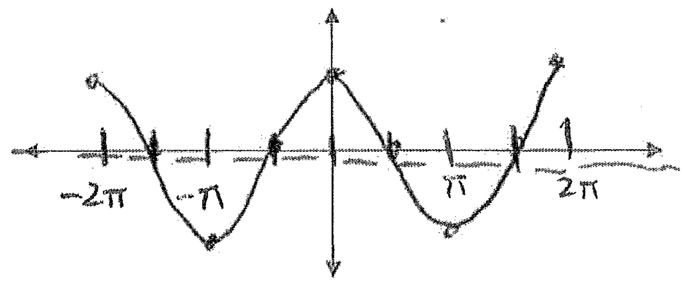
$y = sin\theta$ $y = 1sin\theta$ $a = 1$

θ	0	$\pi/2$	π	$3\pi/2$	2π
$sin\theta$	0	1	0	-1	0

$y = cos\theta$ $y = 1cos\theta$ $a = 1$

θ	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
$cos\theta$	1	0	-1	0	1





Given $y = asinb\theta$ and $y = acosb\theta$ we define the Amplitude as: a-value

* distance from "center" to max height "from rest to crest"

$|a|$

Examples:

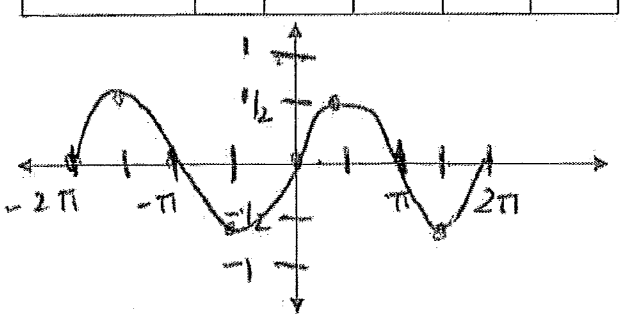
reflection over x-axis

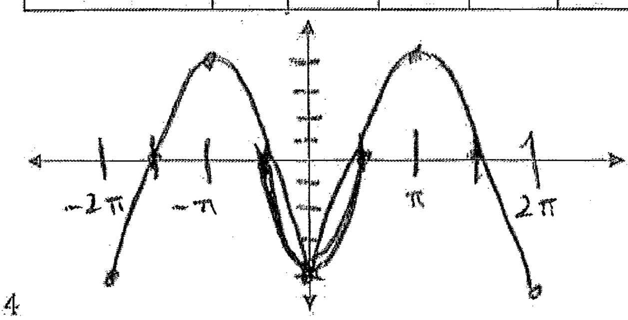
1) $y = \frac{1}{2}sin\theta$ $a = \frac{1}{2}$ Amplitude = $\frac{1}{2}$

2) $y = -4cos\theta$ $a = -4$ Amplitude = 4

θ	0	$\pi/2$	π	$3\pi/2$	2π
$sin\theta$	0	1	0	-1	0
$y = \frac{1}{2}sin\theta$	0	$\frac{1}{2}$	0	$-1/2$	0

θ	0	$\pi/2$	π	$3\pi/2$	2π
$cos\theta$	1	0	-1	0	1
$y = -4cos\theta$	-4	0	4	0	-4





We defined the period of the graph as: Cycle length of a graph

Given $y = a\sin(b\theta)$ and $y = a\cos(b\theta)$ we use "b" to determine the change to the period of the graph:

$\frac{2\pi}{1/2} \cdot \frac{2}{2} = 4\pi$

Period $(P) = \frac{2\pi}{b}$. Likewise, if you know the Period (P) you can find b: $b = \frac{2\pi}{P}$

Once we determine the period of the graph, we divide the period by 4 to determine the Interval in order to label the x-axis: $\boxed{Interval\ (I) = \frac{P}{4}}$

$\rightarrow y = \cos\left(\frac{1}{2}\theta\right)$

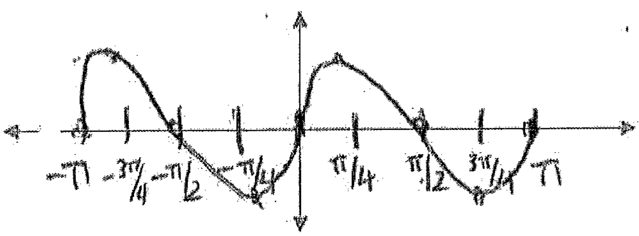
Examples:

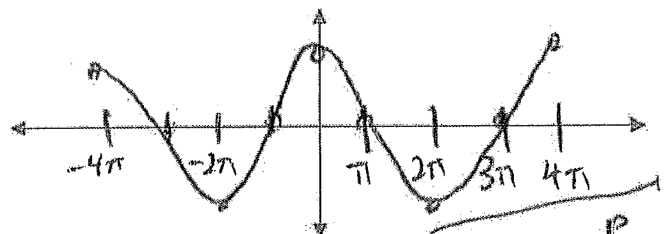
3) $y = \sin(2\theta)$ $b =$ 2
Period = $\frac{2\pi}{2} = \pi$ Interval = $\pi/4$

4) $y = \cos\frac{\theta}{2}$ $b =$ 1/2
Period = 4π Interval = $\frac{4\pi}{4} = \pi$

θ	0	$\pi/4$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = \sin 2\theta$	0	1	0	-1	0

θ	0	π	2π	3π	4π
$y = \cos\frac{\theta}{2}$	1	0	-1	0	1





Let's try to graph sine and cosine with changes to both the amplitude and the period.

$I = \frac{P}{4}$
$I = \frac{1}{4}P$

Examples:

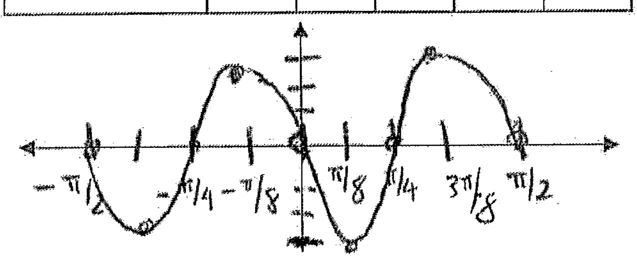
$y = 2\cos\left(\frac{1}{3}\theta\right)$

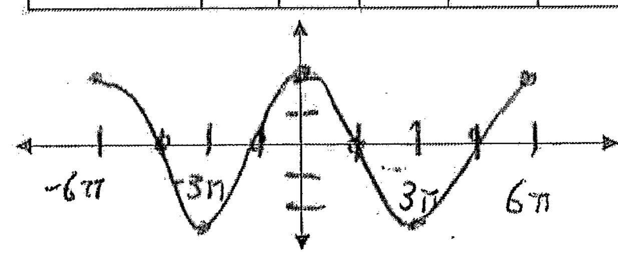
5) $y = -3\sin 4\theta$ $a =$ -3 Amplitude = 3
$b =$ 4 Period = $\frac{2\pi}{4} = \frac{\pi}{2}$ Interval = $\frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

6) $y = 2\cos\frac{\theta}{3}$ $a =$ 2 Amplitude = 2
$b =$ $\frac{1}{3}$ Period = $\frac{2\pi}{1/3} = 6\pi$ Interval = $\frac{6\pi}{4}$ $\frac{3\pi}{2}$

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\pi/2$
$\sin\theta$	0	1	0	-1	0
$y = -3\sin 4\theta$	0	-3	0	3	0

θ	0	$\frac{3}{2}\pi$	3π	$\frac{9}{2}\pi$	6π
$\cos\theta$	1	0	-1	0	1
$y = 2\cos\frac{\theta}{3}$	2	0	-2	0	2





2.02 Practice- Graphing Sin and Cos - Amplitude and Period Date: ______________

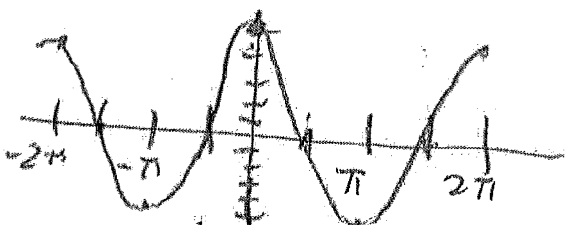
For each function, state the amplitude and period. Then label the axes appropriately and sketch the graph.

1) $y = 5\cos\theta$

Amplitude: 5

Period: 2π

θ	0	$\pi/2$	π	$3\pi/2$	2π
$y=\cos\theta$	1	0	-1	0	1
$y=5\cos\theta$	5	0	-5	0	5

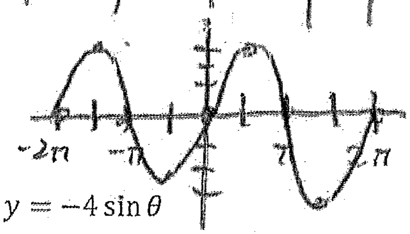


2) $y = 3\sin\theta$

Amplitude: 3

Period: 2π

θ	0	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1	0	-1	0
$3\sin\theta$	0	3	0	-3	0

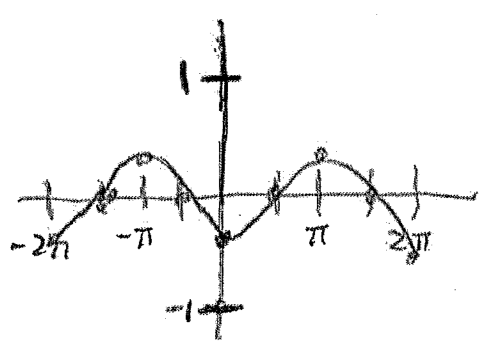


3) $y = -\frac{1}{3}\cos\theta$

Amplitude: $1/3$

Period: 2π

θ	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
$\cos\theta$	1	0	-1	0	1
$-\frac{1}{3}\cos\theta$	$-1/3$	0	$1/3$	0	$-1/3$

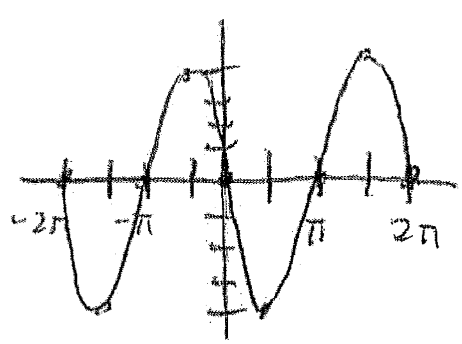


4) $y = -4\sin\theta$

Amplitude: 4

Period: 2π

θ	0	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1	0	-1	0
$-4\sin\theta$	0	-4	0	4	0



$y = a\sin\theta$

$y = a\cos\theta$

Write an equation of the function with the given properties:

5) A sine function with an amplitude of 0.4

$y = 0.4\sin\theta$

6) A cosine function with an amplitude of 7.5

$y = 7.5\cos\theta$

7) A sine function with an amplitude of $\frac{1}{4}$

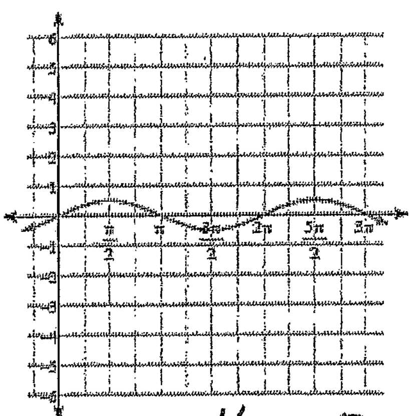
$y = \frac{1}{4}\sin\theta$

8) A cosine function with an amplitude of $\frac{2}{5}$

$y = \frac{2}{5}\cos\theta$

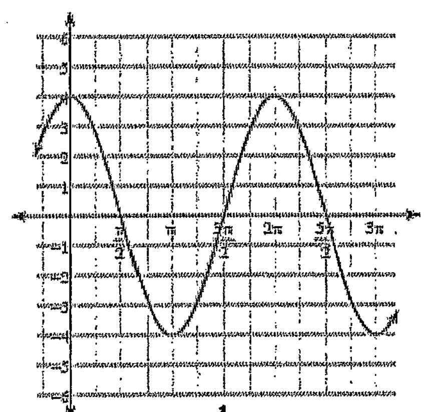
Write an equation for each graph below:

9)



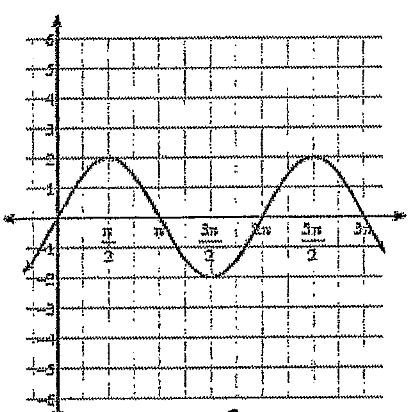
$y = \frac{1}{2}\sin\theta$

10)



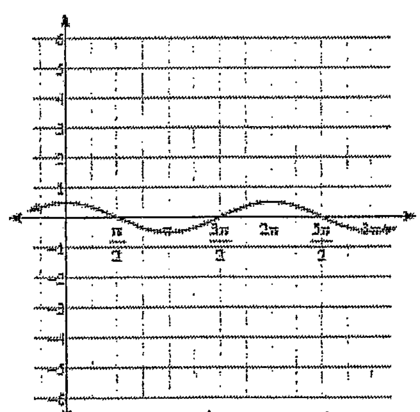
$y = 4\cos\theta$

11)



$y = 2\sin\theta$

12)



$y = \frac{1}{2}\cos\theta$

$y = a\sin(b\theta)$ $y = a\cos(b\theta)$ $\frac{2\pi}{b} = \text{period}$ | $I = \frac{1}{4}P$

2.03 Worksheet: Graphing Sine & Cosine with Amplitude & Period Date: ________

State the amplitude and period for each function.

1. $y = -\frac{2}{5}\sin 9\theta$

$\boxed{a = \frac{2}{5} \quad \text{period}: \frac{2\pi}{9}}$

2. $y = \frac{2}{3}\cos\frac{3}{7}\theta$

$\frac{2\pi}{3/7}$ $2\pi \cdot \frac{7}{3} = \frac{14\pi}{3}$

$\boxed{a = 2/3 \quad p = \frac{14\pi}{3}}$

3. $y = -2.5\cos\frac{\theta}{5}$ $y = -2.5\cos\frac{1}{5}\theta$

$\boxed{a = 2.5 \quad p = 10\pi}$

4. $y = \frac{1}{3}\sin\frac{\theta}{3}$

$\boxed{a = \frac{1}{3} \quad p = 6\pi}$

Write an equation of the sine function with each amplitude and period.

5. amplitude = 3, period = $\frac{\pi}{6}$

$a = 3$

$\frac{2\pi}{b} = \frac{\pi}{6}$ | $b\pi = 12\pi$ | $b = 12$

$y = 3\sin 12\theta$

$\boxed{y = \pm 3\sin(12\theta)}$

6. amplitude = 6, period = 3π

$y = 6\sin(\frac{2}{3}\theta)$ $\boxed{y = \pm 6\sin(\frac{2}{3}\theta)}$

7. amplitude = 2, period = 10π

$p = \frac{2\pi}{b}$ $10\pi = \frac{2\pi}{b} \rightarrow 10\pi b = 2\pi \rightarrow b = \frac{2\pi}{10\pi} = \frac{1}{5}$

$\boxed{y = \pm 2\sin(\frac{1}{5}\theta)}$

8. amplitude = 5, period = 7π

$7\pi = \frac{2\pi}{b}$

$b = 2/7$

$\boxed{y = \pm 5\sin\frac{2}{7}\theta}$

9. amplitude = 4, period = 8π

$p = \frac{2\pi}{b} \rightarrow 8\pi = \frac{2\pi}{b} \rightarrow b = \frac{1}{4}$

$\boxed{y = \pm 4\sin(\frac{1}{4}\theta)}$

10. amplitude = $\frac{3}{5}$, period = $\frac{\pi}{3}$

$p = \frac{2\pi}{b}$

$\pi/3 = \frac{2\pi}{b}$

$b = 6$

$\boxed{y = \pm\frac{3}{5}\sin 6\theta}$

Write an equation of the cosine function with each amplitude and period.

11. amplitude = $\frac{1}{2}$, period = 2π $b = 1$

$\boxed{y = \pm\frac{1}{2}\cos\theta}$

12. amplitude = 7, period = 4π $b = 1/2$

$\boxed{y = \pm 7\cos(\frac{1}{2}\theta)}$

13. amplitude = $\frac{2}{3}$, period = 6π $b = 1/3$

$\boxed{y = \pm\frac{2}{3}\cos(\frac{1}{3}\theta)}$

14. amplitude = $\frac{1}{4}$, period = $\frac{8\pi}{3}$ $b = 3/4$

$\boxed{y = \pm\frac{1}{4}\cos(\frac{3}{4}\theta)}$

15. amplitude = 6, period = $\frac{\pi}{4}$ $b = 8$

$\boxed{y = \pm 6\cos(8\theta)}$

16. amplitude = 11, period = $\frac{\pi}{2}$ $b = 4$

$\boxed{y = \pm 11\cos(4\theta)}$

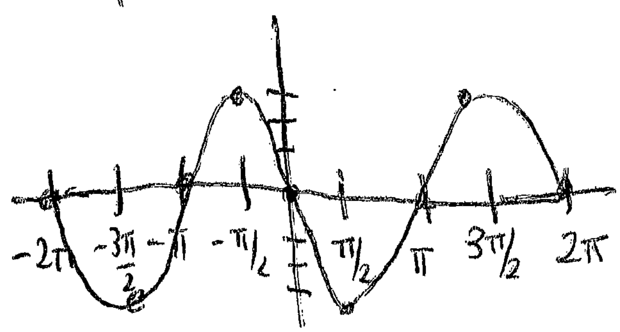
State the amplitude and period of each function. Then graph at least one period of the function.

17. $y = -3\sin\theta$

Amplitude: 3

Period: 2π

	0	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1	0	-1	0
$-3\sin\theta$	0	-3	0	3	0



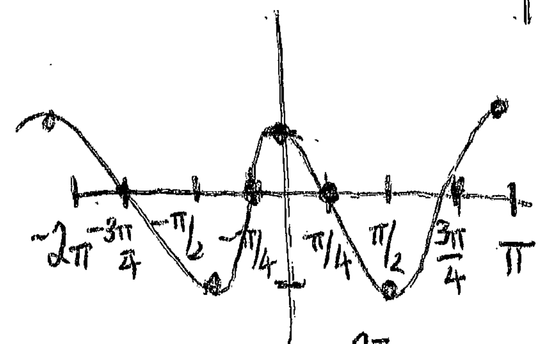
$p = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

18. $y = \cos 2\theta$

Amplitude: 1

Period: π Interval $= \frac{\pi}{4}$

	0	$\pi/4$	$\pi/2$	$\frac{3\pi}{4}$	π
$\cos\theta$					
$\cos 2\theta$	1	0	-1	0	1



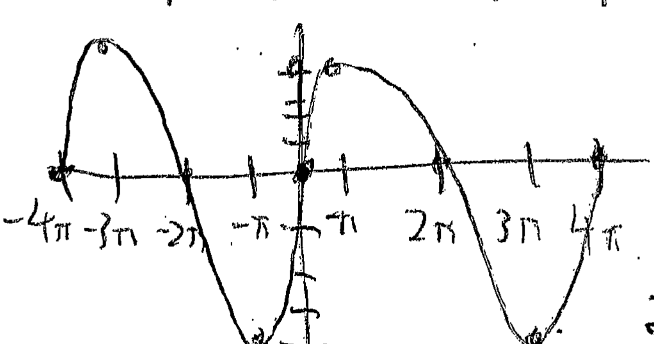
19. $y = 4\sin\frac{\theta}{2}$

$I = P \cdot \frac{1}{4}$

Amplitude: 4

Period: $\frac{2\pi}{1/2} = 4\pi$ $I = \frac{P}{4} = \pi$

	0	π	2π	3π	4π
$y = \sin\frac{\theta}{2}$	0	1	0	-1	0
$y = 4\sin\frac{\theta}{2}$	0	4	0	-4	0



20. $y = -\frac{1}{2}\cos 3\theta$

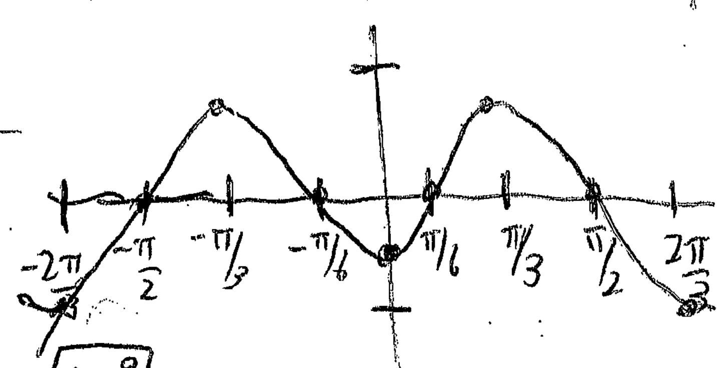
$p = \frac{2\pi}{3}$ $I = \frac{1}{4}P$

$I = \frac{1}{4} \cdot \frac{2}{3}\pi = \frac{\pi}{6}$

Amplitude: $1/2$

Period: $2\pi/3$

	0	$\pi/6$	$\pi/3$	$\pi/2$	$\frac{2\pi}{3}$
$y = \cos 3\theta$	1	0	-1	0	1
$y = -\frac{1}{2}\cos 3\theta$	$\frac{-1}{2}$	0	$1/2$	0	$-1/2$



2.04 Practice Problems: Graphing Sine & Cosine Functions with Amplitude & Period

For #1 & 2, state the amplitude and period of each function. Then graph at least two periods of the function.

1. $y = -3\cos 6\theta$

Table/Chart:

Amplitude: ______________________

Period: ______________________

Graph:

2. $y = 5\sin\frac{2\theta}{3}$

Table/Chart:

Amplitude: ______________________

Period: ______________________

Graph:

3. Write the equation of a cosine function with amplitude of $\frac{6}{11}$ and period of $\frac{7\pi}{4}$.

____

* $y = a\sin(b\theta)$, $y = a\cos(b\theta)$ | $\text{period} = \frac{2\pi}{b}$ | $\text{Interval} = \frac{p}{4}$ or $I = \frac{1}{4} \cdot p$

Key

2.04 Practice Problems: Graphing Sine & Cosine Functions with Amplitude & Period

For #1 & 2, state the amplitude and period of each function. Then graph at least two periods of the function.

1. $y = -3\cos 6\theta$ $a = -3$, $b = 6$

Amplitude: 3

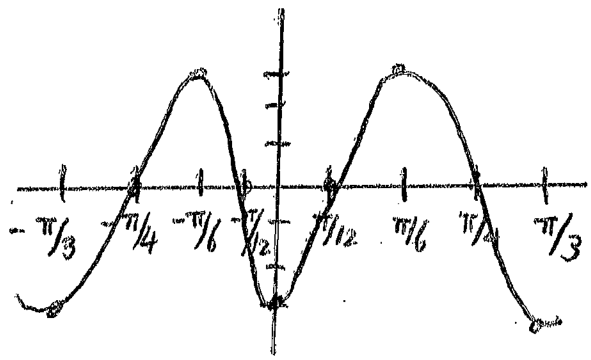
Period: $\frac{\pi}{3}$

$\text{period} = \frac{2\pi}{b} \rightarrow \frac{2\pi}{6} = \frac{\pi}{3}$

Graph: $I = \frac{1}{4}p \rightarrow I = \frac{1}{4} \cdot \frac{\pi}{3} = \frac{\pi}{12}$

Table/Chart:

θ	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
$\cos 6\theta$	1	0	-1	0	1
$-3\cos 6\theta$	-3	0	3	0	-3



2. $y = 5\sin\frac{2\theta}{3}$ $a = 5$, $b = 2/3$

Amplitude: 5

Period: 3π

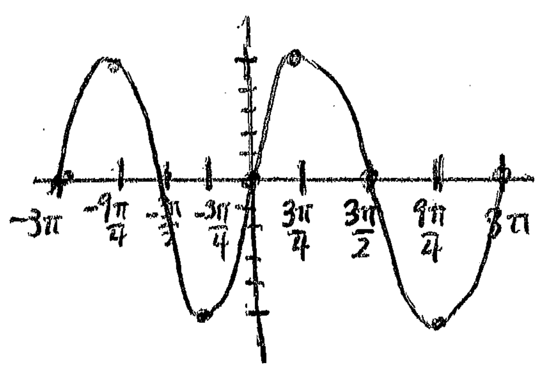
$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{2/3} \rightarrow 2\pi \cdot \frac{3}{2} = 3\pi$

$I = \frac{p}{4} = \frac{3\pi}{4}$

Table/Chart:

θ	0	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	$\frac{9\pi}{4}$	3π
$\sin\left(\frac{2\theta}{3}\right)$	0	1	0	-1	0
$5\sin\left(\frac{2\theta}{3}\right)$	0	5	0	-5	0

Graph:



3. Write the equation of a cosine function with amplitude of $\frac{6}{11}$ and period of $\frac{7\pi}{4}$.

$a = \pm\frac{6}{11}$ $\text{period} = \frac{2\pi}{b}$

$\frac{7\pi}{4} = \frac{2\pi}{b}$

$7\pi b = 8\pi$

$b = \frac{8\pi}{7\pi}$

$b = 8/7$

$y = a\cos(b\theta)$

$\boxed{y = \pm\frac{6}{11}\cos\left(\frac{8}{7}\theta\right)}$

Accel Pre-Calculus Name: ________________

2.05 Graphing Sine and Cosine- Phase Shift Notes

For $y = a\sin[b(\theta - c)] + d$ or $y = a\cos[b(\theta - c)] + d$

Phase Shift- horizontal shift; 'c' tells you the phase shift; direction is opposite of what you think; <u>you must factor out 'b' to see the phase shift!</u>

Examples: Identify the transformations that occur on the parent function and graph the function.

1. $y = \sin(\theta - \frac{3\pi}{4})$ shift right by $\frac{3\pi}{4}$ units

2. $y = \cos(2\theta + \pi)$ * shift left by $\frac{\pi}{2}$ units

$y = \cos[2(\theta + \frac{\pi}{2})]$ * shrink (period) by factor of 2

2.05 Practice: Graphing Sine & Cosine with Amplitude, Period, and Phase Shift

Write an equation of the sine function given the following information.

period $= \frac{2\pi}{b}$

$\frac{14\pi}{1} = \frac{2\pi}{b}$

$14\pi b = 2\pi$

1) amp. = 3.5 and a period of 14π

$b = \frac{2}{14} = \frac{1}{7}$

$\boxed{y = 3.5\sin(\frac{1}{7}\theta)}$

2) amp. = $\frac{1}{2}$, period = $\frac{1}{3}$, and phase shift right $\frac{1}{2}$

$\frac{1}{3} = \frac{2\pi}{b}$

$b = 6\pi$

$y = \frac{1}{2}\sin[6\pi(\theta - \frac{1}{2})]$

Write an equation of the cosine function given the following information.

3) amp = 1.25, period=6π, & phase shift right 3π down 3

$\frac{6\pi}{1} = \frac{2\pi}{b}$

$6\pi b = 2\pi$

$b = \frac{1}{3}$

$y = 1.25\cos[\frac{1}{3}(\theta - 3\pi)] - 3$

4) amp. = $\frac{2}{3}$, period = $\frac{3}{4}$, and phase shift left 2

$\frac{3}{4} = \frac{2\pi}{b}$

$3b = 8\pi$

$b = \frac{8\pi}{3}$

$y = \frac{2}{3}\cos[\frac{8\pi}{3}(\theta + 2)]$

For each function, state the amplitude and period. Then label the axes appropriately and sketch the graph. $\frac{2\pi}{\pi/2}$

5) $y = 3\cos(4\theta)$ $a = 3$ $b = 4$

Amp: 3 Per: ______ PS: —

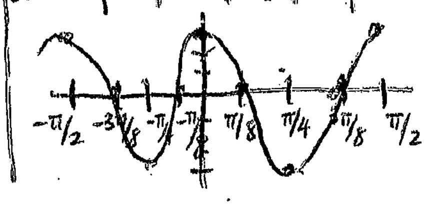
period $= \frac{2\pi}{b}$

period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

θ	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
$\cos 4\theta$	1	0	-1	0	1
$3\cos 4\theta$	3	0	-3	0	3



6) $y = -4\sin\frac{\pi\theta}{2}$ $a = -4$ $b = \frac{\pi}{2}$

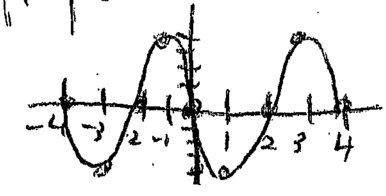
period $= \pi/2$

$p = 2\pi \cdot \frac{2}{\pi} = 4$

Amp: 4 Per: 4 PS: —

$I = \frac{4}{4} = 1$

θ	0	1	2	3	4
$\sin(\frac{\pi\theta}{2})$	0	1	0	-1	0
$-4\sin(\frac{\pi\theta}{2})$	0	-4	0	4	0



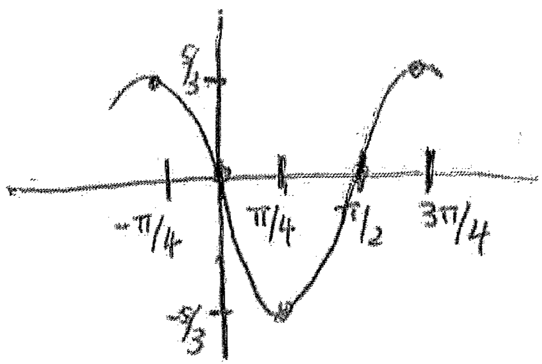
7) $y = \frac{5}{3}\cos\left(2\theta + \frac{\pi}{2}\right)$

$y = \frac{5}{3}\cos\left[2\left(\theta + \frac{\pi}{4}\right)\right]$

$a = 5/3 \quad b = 2 \quad$ PS: left $\pi/4$

period $= \frac{2\pi}{b} \rightarrow \frac{2\pi}{2} = \pi \qquad I = \frac{\pi}{4}$

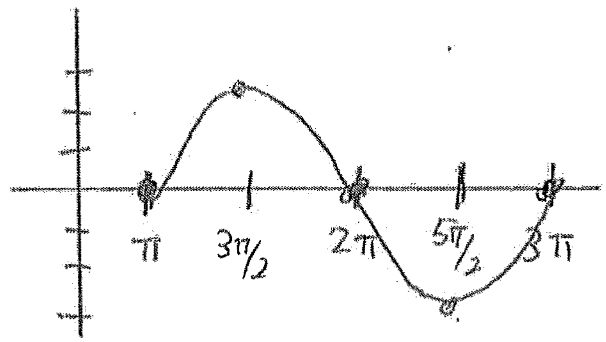
	$\boxed{-\pi/4}$	$\boxed{0}$	$\boxed{\pi/4}$	$\boxed{\pi/2}$	$\boxed{3\pi/4}$
θ	$0-\frac{\pi}{4}$	$\pi/4-\frac{\pi}{4}$	$\frac{\pi}{2}-\pi/4$	$\frac{3\pi}{4}-\pi/4$	$\pi-\pi/4$
$\cos(2\theta)$	1	0	−1	0	1
$\frac{5}{3}\cos(2\theta)$	$\frac{5}{3}$	0	$-\frac{5}{3}$	0	$5/3$



8) $y = 2.5\sin(\theta - \pi)$

$a = 2.5 \quad b = 1 \quad c = \pi \quad$ P.S. right π units

	$\boxed{\pi}$	$\boxed{3\pi/2}$	$\boxed{2\pi}$	$\boxed{5\pi/2}$	$\boxed{3\pi}$
θ	$0+\pi$	$\pi/2+\pi$	$\pi+\pi$	$3\pi/2+\pi$	$2\pi+\pi$
$\sin(\theta-\pi)$	0	1	0	−1	0
$2.5\sin(\theta-\pi)$	0	2.5	0	−2.5	0



9) $y = 6\cos\left(\frac{\theta}{4} - \frac{\pi}{8}\right)$

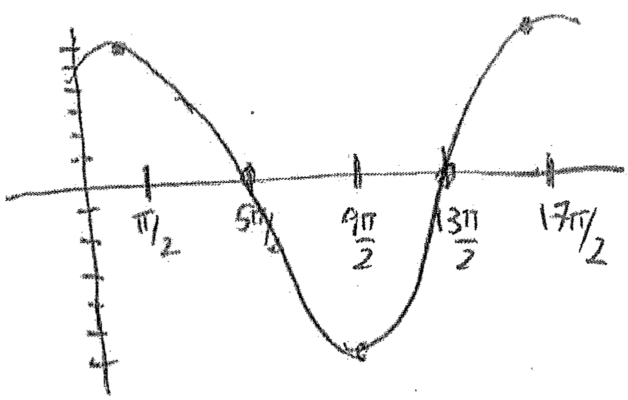
$y = 6\cos\left[\frac{1}{4}\left(\theta - \frac{\pi}{2}\right)\right]$

Amp: $\underline{6}$ Per: $\underline{8\pi}$ P.S: $\underline{\text{right } \pi/2}$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/4} = 2\pi \cdot 4 = 8\pi$

$I = \frac{8\pi}{4} = 2\pi$

θ	$\boxed{\pi/2}$ $0 + \frac{\pi}{2}$	$\boxed{\frac{5\pi}{2}}$ $2\pi + \frac{\pi}{2}$	$\boxed{\frac{9\pi}{2}}$ $4\pi + \frac{\pi}{2}$	$\boxed{\frac{13\pi}{2}}$ $6\pi + \frac{\pi}{2}$	$\boxed{\frac{17\pi}{2}}$ $8\pi + \frac{\pi}{2}$
$\cos\left(\frac{\theta}{4}\right)$	1	0	-1	0	1
$6\cos\left(\frac{\theta}{4}\right)$	6	0	-6	0	6



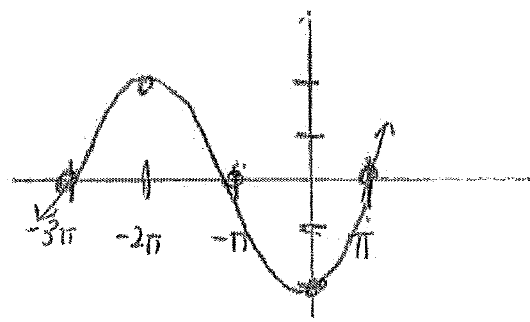
10) $y = 2\sin\left(\frac{\theta}{2} + \frac{3\pi}{2}\right)$

$y = 2\sin\left[\frac{1}{2}\left(\theta + 3\pi\right)\right]$

$a = 2$ $b = \frac{1}{2}$ P.S. $\underline{\text{shift left } 3\pi}$

$\text{period} = \frac{2\pi}{1/2} = 2\pi \cdot \frac{2}{1} = 4\pi$ $I = \frac{4\pi}{4} = \pi$

θ	$\boxed{-3\pi}$ $0 - 3\pi$	$\boxed{-2\pi}$ $\pi - 3\pi$	$\boxed{-\pi}$ $2\pi - 3\pi$	$\boxed{0}$ $3\pi - 3\pi$	$\boxed{\pi}$ $4\pi - 3\pi$
$\sin(\theta)$	0	1	0	-1	0
$2\sin(\theta)$	0	2	0	-2	0



11) $y = -3\cos(3\pi\theta + \pi)$

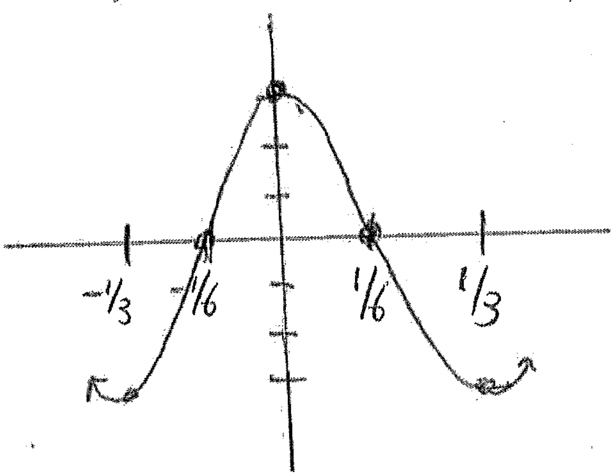
$y = -3\cos\left[3\pi\left(\theta + \frac{1}{3}\right)\right]$

$a = -3 \quad b = 3\pi \quad$ P.S. left $\frac{1}{3}$

period $= \frac{2\pi}{b} \rightarrow \frac{2\pi}{3\pi} = \frac{2}{3} \qquad I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$

	$\boxed{-1/3}$	$\boxed{-1/6}$	$\boxed{0}$	$\boxed{+1/6}$	$\boxed{1/3}$
θ	$0 - \frac{1}{3}$	$\frac{1}{6} - \frac{2}{6}$	$\frac{2}{6} - \frac{2}{6}$	$\frac{3}{6} - \frac{2}{6}$	$\frac{4}{6} - \frac{2}{6}$
$\cos\theta$	1	0	-1	0	1
$-3\cos\theta$	-3	0	3	0	-3



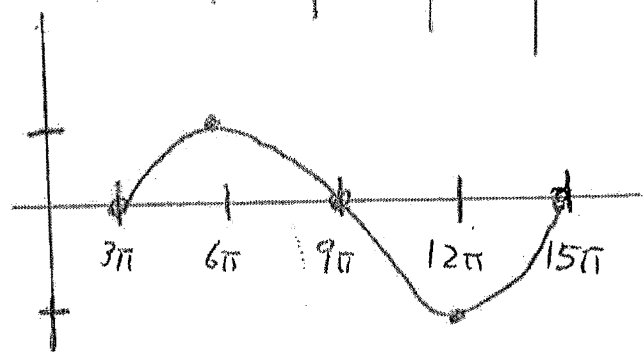
12) $y = \sin\left(\frac{\theta}{6} - \frac{\pi}{2}\right)$

$y = \sin\left[\frac{1}{6}(\theta - 3\pi)\right]$

$a = 1 \quad b = \frac{1}{6} \quad$ P.S. right 3π units

period $= \frac{2\pi}{1/6} \rightarrow 2\pi \cdot \frac{6}{1} = 12\pi \qquad I = \frac{12\pi}{4} = 3\pi$

	$\boxed{3\pi}$	$\boxed{6\pi}$	$\boxed{9\pi}$	$\boxed{12\pi}$	$\boxed{15\pi}$
θ	$0 + 3\pi$	$3\pi + 3\pi$	$6\pi + 3\pi$	$9\pi + 3\pi$	$12\pi + 3\pi$
$\sin\theta$	0	1	0	-1	0
$1\sin\theta$	0	1	0	-1	0



7) $y = \frac{5}{3}\cos\left(2\theta + \frac{\pi}{2}\right)$

Amp: ______ Per: ______ PS: ______

8) $y = 2.5\sin(\theta - \pi)$

Amp: ______ Per: ______ PS: ______

9) $y = 6\cos\left(\frac{\theta}{4} - \frac{\pi}{8}\right)$

Amp: ______ Per: ______ PS: ______

10) $y = 2\sin\left(\frac{\theta}{2} + \frac{3\pi}{2}\right)$

Amp: ______ Per: ______ PS: ______

11) $y = -3\cos(3\pi\theta + \pi)$

Amp: ______ Per: ______ PS: ______

12) $y = \sin\left(\frac{\theta}{6} - \frac{\pi}{2}\right)$

Amp: ______ Per: ______ PS: ______

$a=-4$
period $=\pi$

Write a sine equation <u>and</u> a cosine equation for each graph below:

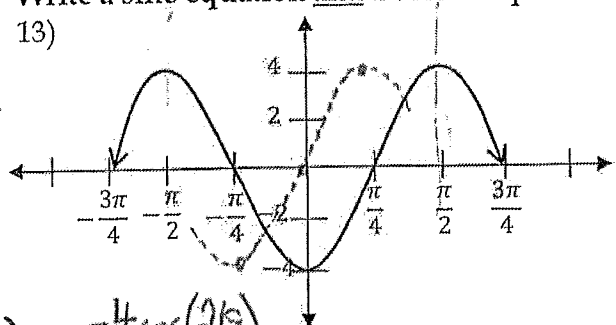
$a = 0.5$ | period $= 6\pi$ | $\frac{6\pi}{1} = \frac{2\pi}{b}$ | $6\pi b = 2\pi$ | $b = \frac{2\pi}{6\pi} = \frac{1}{3}$

13)

$\frac{\pi}{1} = \frac{2\pi}{b}$

$b\pi = 2\pi$

$b = 2$



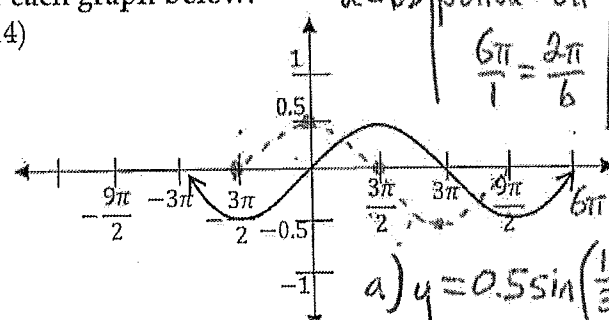
a) $y = -4\cos(2\theta)$

b) phase shift right $\pi/4$

$a = 4 \quad b = 2$

$y = 4\sin\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

14)



a) $y = 0.5\sin\left(\frac{1}{3}\theta\right)$

b) shift cosine graph $\frac{3\pi}{2}$ to the right

$y = 0.5\cos\left[\frac{1}{3}\left(\theta - \frac{3\pi}{2}\right)\right]$

Accel Pre-Calculus Name: ______________

2.06 Graphing Sine and Cosine- Vertical Shift Notes

For $y = a\sin[b(\theta - c)] + d$ or $y = a\cos[b(\theta - c)] + d$

Vertical Shift- amount graph moves up or down; 'd' tells you the vertical shift; direction is what you assume

Examples: Identify the transformations that occur on the parent function and graph the function.

1. $y = \sin\left(\theta - \frac{\pi}{4}\right) + 3$

$a = 1$ P.S. right $\pi/4$ units

$b = 1$ $d = 3 \rightarrow$ vertical shift up 3 units.

2. $y = \cos\left(3\theta + \frac{\pi}{2}\right) - 1$

$y = \cos\left[3\left(\theta + \frac{\pi}{6}\right)\right] - 1$ | $a = 3$, $b = 3 \rightarrow$ period $= \frac{2\pi}{3}$ | shift left $\pi/6$ units, $d = -1 \rightarrow$ vertical shift down 1 unit.

2.06 Worksheet: Graphing Sine and Cosine with Amplitude, Period, Phase Shift and Vertical Shift Date: ________

Write the function described:

1. A sine function with amplitude = 15, with a reflection, period = 4 π, phase shift right $\frac{\pi}{2}$ and vertical shift down 10.

$a = 15$, $4\pi = \frac{2\pi}{b}$ | $4\pi b = 2\pi$, $b = \frac{2\pi}{4\pi} = \frac{1}{2}$ | $c = \pi/2$, $d = -10$ | $y = -15\sin\left[\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)\right] - 10$

2. A cosine function having an amplitude = $\frac{1}{2}$, period = $\frac{\pi}{3}$, phase shift left $\frac{\pi}{3}$ and vertical shift up 5.

$a = 1/2$, $\frac{\pi}{3} = \frac{2\pi}{b}$ | $b\pi = 6\pi$, $b = 6$ | $c = +\pi/3$, $d = 5$ | $y = \frac{1}{2}\cos\left[6\left(\theta + \frac{\pi}{3}\right)\right] + 5$

3. A cosine function that has been vertically stretched by a factor of 4, has been reflected over the x-axis, has been horizontally compressed to a period of $\frac{2\pi}{3}$, and has been shifted right π units and down 3 units.

$a = 4$, $\frac{2\pi}{3} = \frac{2\pi}{b}$ | $2\pi b = 6\pi$, $b = \frac{6\pi}{2\pi} = 3$ | $c = -\pi$, $d = -3$ | $y = -4\cos[3(\theta - \pi)] - 3$

4. A sine function that has been horizontally stretched to a period of 10, vertically compressed by a factor of $\frac{2}{3}$, shifted left $\frac{5}{2}$, and up 1 unit.

$10 = \frac{2\pi}{b}$, $10b = 2\pi$, $b = \frac{2\pi}{10} = \frac{\pi}{5}$ | $a = 2/3$, $c = +5/2$, $d = 1$ | $y = \frac{2}{3}\sin\left[\frac{\pi}{5}\left(\theta + 5/2\right)\right] + 1$

5) $y = -2\cos[8(\theta)] - 3$

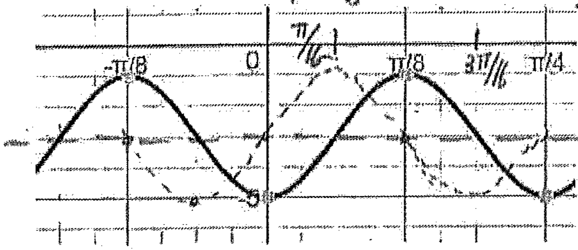
$y = 2\sin[8(\theta - \pi/16)] - 3$

6) a) $y = \sin\left[2\left(\theta - \frac{\pi}{2}\right)\right] + 1$

b) $y = \cos\left[2\left(\theta - \frac{3\pi}{4}\right)\right] + 1$

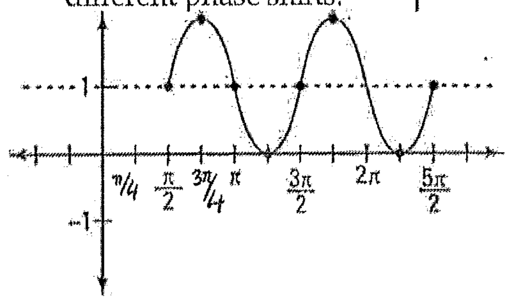
5. The wave graphed below. It is **both:** a sine function and a cosine function, with different phase shifts.

period $= \pi/4$ | $\frac{\pi}{4} = \frac{2\pi}{b}$ | $b\pi = 8\pi$ | $b = 8$ | $I = \frac{1}{4} \cdot \frac{\pi}{4}$ | $I = \pi/16$



6. The wave graphed below. It is **both:** a sine function and a cosine function, with different phase shifts.

period $= \pi$ | $\pi = \frac{2\pi}{b}$ | $b\pi = 2\pi$ | $b = 2$



Given $y = a\sin[b(\theta - c)] + d$ and $y = a\cos[b(\theta - c)] + d$. For each function, state the amplitude, period, interval, phase shift (PS), and vertical shift (VS). Sketch the graph and label the axes.

7) $y = -2\sin\frac{\theta}{2} + 1$

Amp: 2 Per: ______ Int: ______

PS: none VS: up 1

8) $y = 2\cos\left(\theta + \frac{\pi}{2}\right) - 3$

Amp: ______ Per: ______ Int: ______

PS: ______ VS: ______

9) $y = \frac{1}{2}\sin\left(\frac{\theta}{6} + \frac{\pi}{3}\right) + 4$

Amp: ______ Per: ______ Int: ______

PS: ______ VS: ______

10) $y = -\cos(6\pi\theta + \pi) - 1$

Amp: ______ Per: ______ Int: ______

PS: ______ VS: ______

11) $y = 3\sin\left(2\theta + \frac{3\pi}{2}\right) + 0.5$

Amp: ______ Per: ______ Int: ______

PS: ______ VS: ______

12) $y = -\cos\left(\frac{\pi}{4}\theta + \frac{\pi}{2}\right) - 1.5$

Amp: ______ Per: ______ Int: ______

PS: ______ VS: ______

7) $y = -2\sin\left(\frac{\theta}{2}\right) + 1$

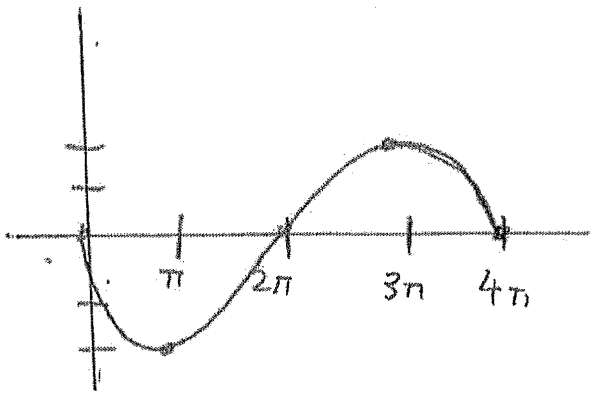
$a = -2 \quad b = 1/2 \quad d = 1$

period $= \frac{2\pi}{1/2} \rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$

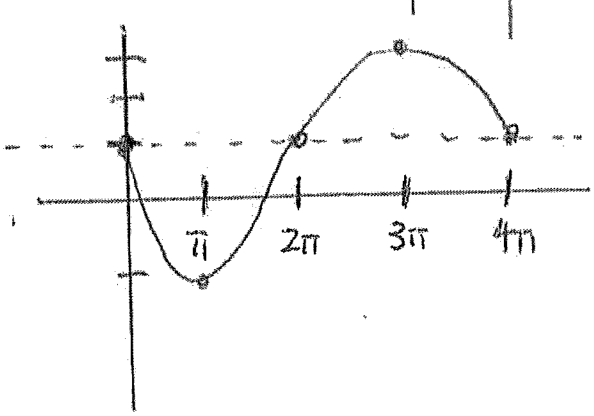
$I = \frac{4\pi}{4} = \pi$

Amp: 2 Per: 4π Int: π

PS none VS up 1 unit



θ	0	π	2π	3π	4π
$\sin\left(\frac{\theta}{2}\right)$	0	1	0	-1	0
$-2\sin\left(\frac{\theta}{2}\right)$	0	-2	0	2	0

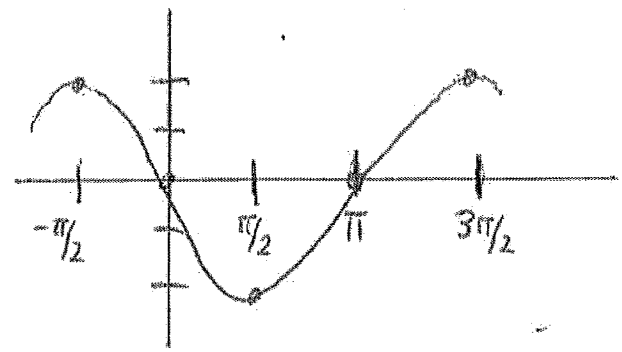


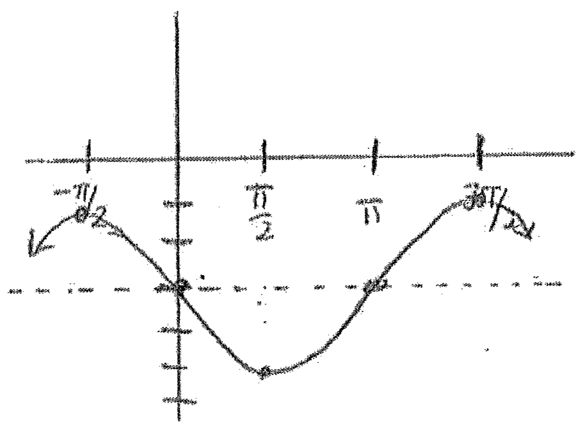
8) $y = 2\cos\left(\theta + \frac{\pi}{2}\right) - 3$

$a = 2 \quad b = 1$ P.S. left $\pi/2$ units

$d = -3$ VS: down 3 units period: 2π

	$\boxed{-\pi/2}$	$\boxed{0}$	$\boxed{\pi/2}$	$\boxed{\pi}$	$\boxed{3\pi/2}$
θ	$0 - \pi/2$	$\pi/2 - \pi/2$	$\pi - \pi/2$	$3\pi/2 - \pi/2$	$2\pi - \pi/2$
$\cos(\theta)$	1	0	-1	0	1
$2\cos(\theta)$	2	0	-2	0	2





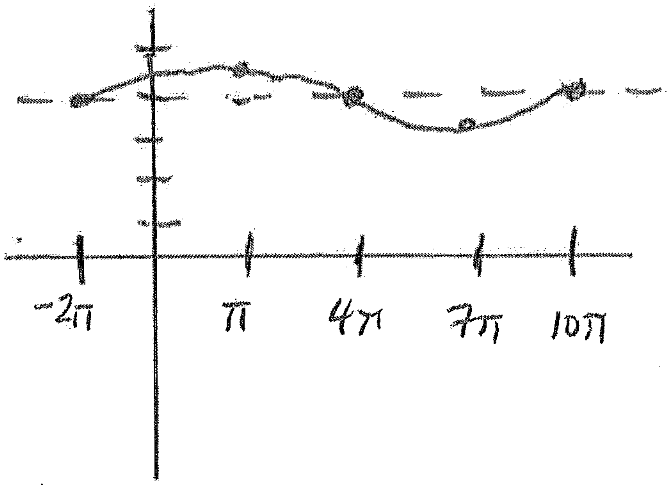
9) $y=\frac{1}{2}\sin\left(\frac{\theta}{6}+\frac{\pi}{3}\right)+4$

$y=\frac{1}{2}\sin\left[\frac{1}{6}(\theta+2\pi)\right]+4$

period $=\frac{2\pi}{b}\rightarrow\frac{2\pi}{1/6}=2\pi\cdot 6=12\pi$

$I=\frac{12\pi}{4}=3\pi$

θ	$\boxed{-2\pi}$ $0-2\pi$	$\boxed{\pi}$ $3\pi-2\pi$	$\boxed{4\pi}$ $6\pi-2\pi$	$\boxed{7\pi}$ $9\pi-2\pi$	$\boxed{10\pi}$ $12\pi-2\pi$
$\sin\left(\frac{\theta}{6}\right)$	0	1	0	-1	0
$\frac{1}{2}\sin\left(\frac{\theta}{6}\right)$	0	$1/2$	0	$-1/2$	0



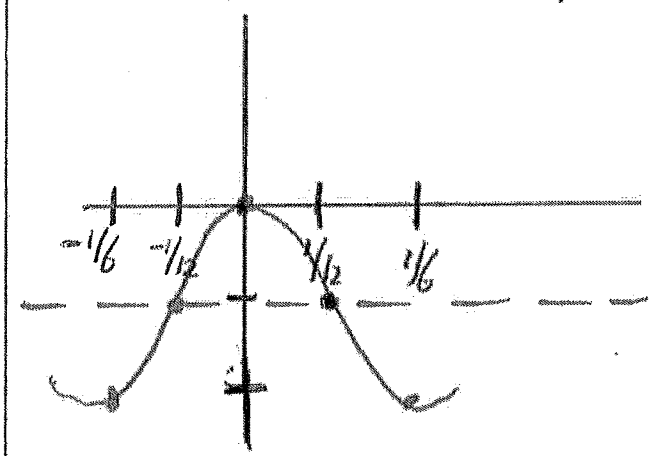
10) $y=-\cos(6\pi\theta+\pi)-1$

$y=-\cos\left[6\pi\left(\theta+\frac{1}{6}\right)\right]-1$

period $=\frac{2\pi}{6\pi}=\frac{1}{3}$ $\quad I=\frac{1}{4}\cdot P\rightarrow\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$

x shift left $1/6$ or $\frac{2}{12}$

θ	$\boxed{-1/6}$ $0-\frac{1}{6}$	$\boxed{-1/12}$ $\frac{1}{12}-\frac{2}{12}$	$\boxed{0}$ $\frac{2}{12}-\frac{2}{12}$	$\boxed{1/12}$ $\frac{3}{12}-\frac{2}{12}$	$\boxed{1/6}$ $\frac{4}{12}-\frac{2}{12}$
$\cos\theta$	1	0	-1	0	1
$-\cos\theta$	-1	0	1	0	-1

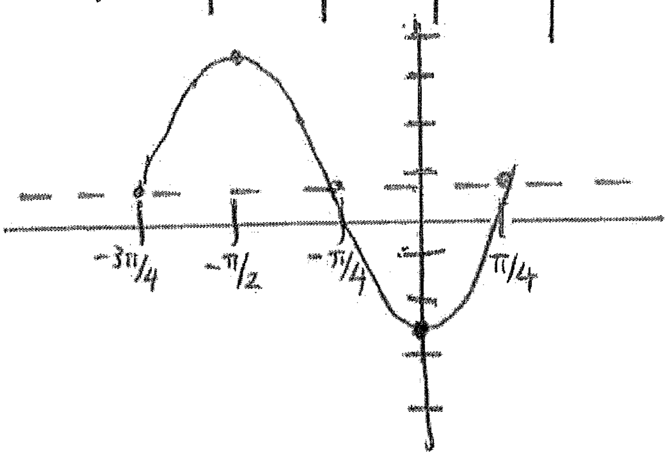


11) $y=3\sin\left(2\theta+\frac{3\pi}{2}\right)+0.5$

$y=3\sin\left[2\left(\theta+\frac{3\pi}{4}\right)\right]+0.5$

period $=\frac{2\pi}{2}\to\pi$ $\quad I=\frac{\pi}{4}$

θ	$\boxed{-3\pi/4}$ $0-\frac{3\pi}{4}$	$\boxed{-\pi/2}$ $\pi/4-\frac{3\pi}{4}$	$\boxed{-\pi/4}$ $\pi/2-3\pi/4$	$\boxed{0}$ $3\pi/4-3\pi/4$	$\boxed{\pi/4}$ $\pi-3\pi/4$
$\sin(2\theta)$	0	1	0	-1	0
$3\sin(2\theta)$	0	3	0	-3	0

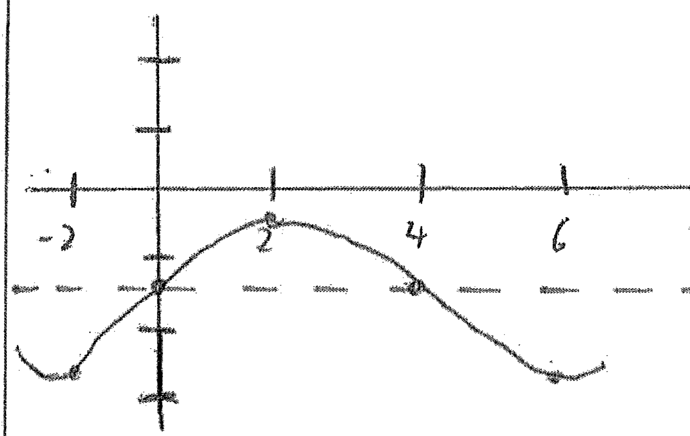


12) $y=-\cos\left(\frac{\pi}{4}\theta+\frac{\pi}{2}\right)-1.5$

$y=-\cos\left[\frac{\pi}{4}(\theta+2)\right]-1.5$

period $=\frac{2\pi}{\pi/4}\to 2\pi\cdot\frac{4}{\pi}=8$ $\quad I=\frac{8}{4}=2$

θ	$\boxed{-2}$ $0-2$	$\boxed{0}$ $2-2$	$\boxed{2}$ $4-2$	$\boxed{4}$ $6-2$	$\boxed{6}$ $8-2$
$\cos(\frac{\pi}{4}\theta)$	1	0	-1	0	1
$-\cos(\frac{\pi}{4}\theta)$	-1	0	1	0	-1



Key

2.07 Additional Review WS #2 (Graphing Sine and Cosine Functions)

State the equations for the following graphs.

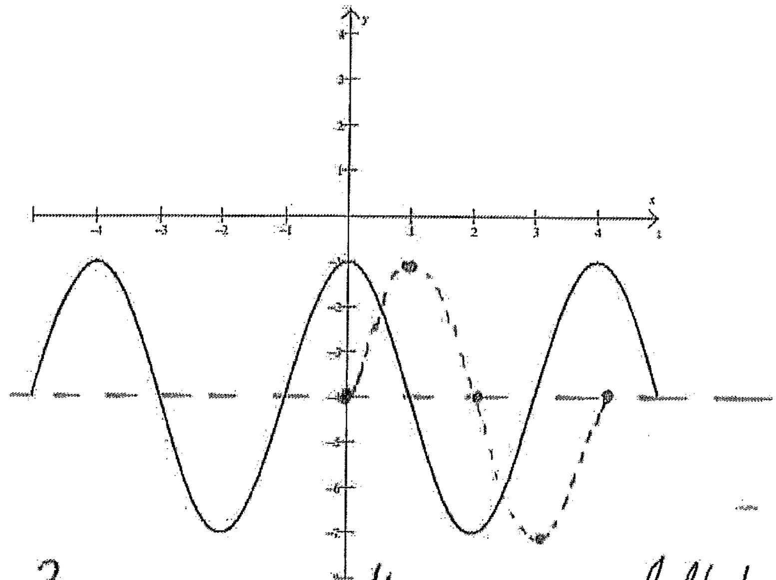
1)

period $= \frac{2\pi}{b}$

$\frac{4}{1} = \frac{2\pi}{b}$

$4b = 2\pi$

$b = \frac{\pi}{2}$



$a = 3$
$b = \pi/2$
$c = -1$
$d = -4$

Amplitude = 3 Period = 4 Phase Shift = left 1 unit

Using Sine function: $y = 4\sin\left[\frac{\pi}{2}(\theta + 1)\right] - 4$

2)

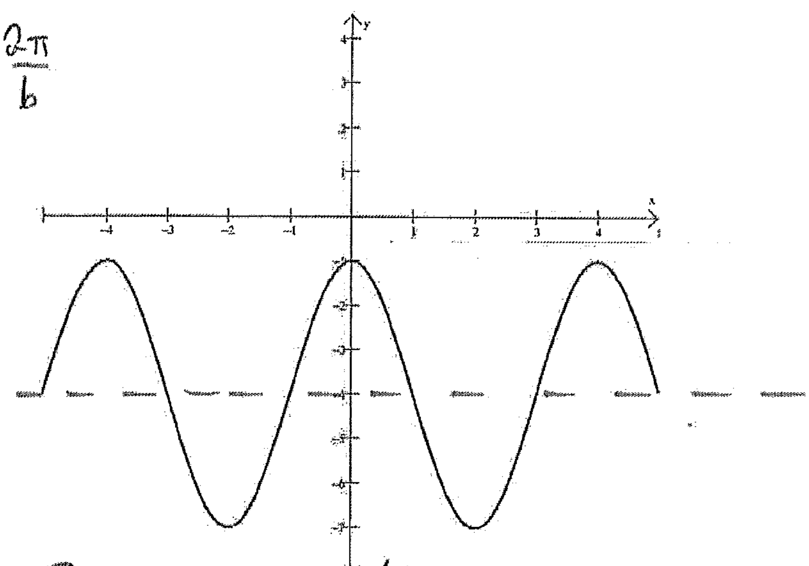
period $= \frac{2\pi}{b}$

$\frac{4}{1} = \frac{2\pi}{b}$

$4b = 2\pi$

$b = \frac{2\pi}{4}$

$b = \frac{\pi}{2}$



Amplitude = 3 Period = 4 Phase Shift = none

Using Cosine function: $y = 4\cos\left[\frac{\pi}{2}(\theta)\right] - 4$

$a = 3$ $d = -4$
$b = \pi/2$
$c = 0$

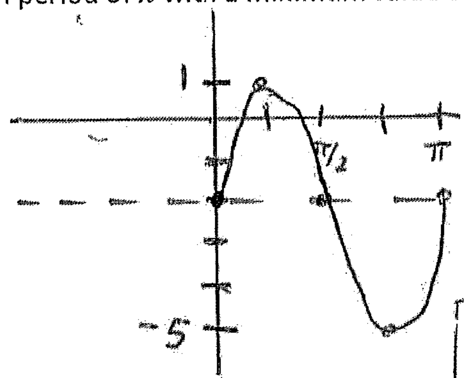
3) Write the sine function with period of π with a minimum value of -5 and a maximum value of 1

period $= \pi$

$\frac{\pi}{1} = \frac{2\pi}{b}$

$b\pi = 2\pi$

$b = 2$



$a = 3$

$b = 2$

$c = 0$

$d = -2$

$\boxed{y = 3\sin[2(\theta)] - 2}$

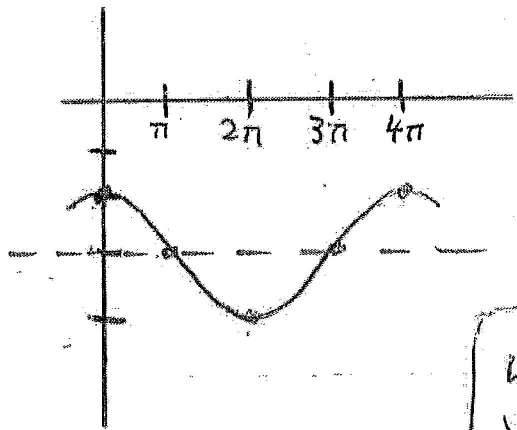
4) Write a cosine function with a period of 4π whose maximum value is -2

period $= 4\pi$

$\frac{4\pi}{1} = \frac{2\pi}{b}$

$4\pi b = 2\pi$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$



$a = 1$

$b = \frac{1}{2}$

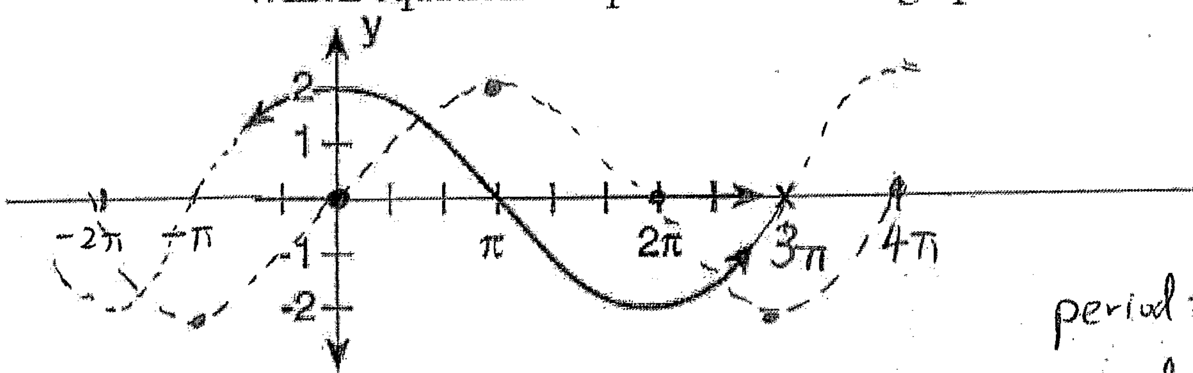
$c = 0$

$d = -3$

$\boxed{y = 1\cos[\tfrac{1}{2}(\theta)] - 3}$

5)

Which equation is represented in the graph below?



cosine function: $y = 2\cos(\frac{1}{2}\theta)$

sine function: $y = 2\sin[\frac{1}{2}(\theta + \pi)]$

$a = 2$ $\quad$ $c = +\pi$ left

$b = \frac{1}{2}$ $\quad$ $d = 0$

period $= 4\pi$

period $= \frac{2\pi}{b}$

$\frac{4\pi}{1} = \frac{2\pi}{b}$

$4\pi b = 2\pi$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$

$a = 2$

$b = \frac{1}{2}$

$c = 0$

$d = 0$

Key

2.07c Review WS #3 Graphing Sine and Cosine $y=-2\cos\left[\frac{1}{4}(\theta+12\pi)\right]-5$

1. Describe the transformations that change the graph of the first function into the graph of the second. $y = \cos\theta$ and $y = -2\cos\left(\frac{\theta}{4}+3\pi\right)-5$

$a=-2$ $\quad c=3\pi$
$b=1/4$ $\quad d=-5$
$\text{period}=\frac{2\pi}{1/4}=2\pi\cdot\frac{4}{1}=8\pi$

* reflection over x-axis
* vertical stretch factor of 2
* horizontal stretch factor of 4
* shift left 12π units
* shift down 5 units

2. Write a cosine function with amplitude = 2, reflection over x-axis, period = $\frac{2\pi}{3}$, phase shift left $\frac{\pi}{2}$, and vertical shift up 12 units

$\frac{2\pi}{3}=\frac{2\pi}{b}$
$2\pi b=6\pi$

$b=\frac{6\pi}{2\pi}=3$

$a=-2$
$b=3$
$c=\pi/2$
$d=12$

$\boxed{y=-2\cos\left[3\left(\theta+\pi/2\right)\right]+12}$

3. **Write** a sine equation that completes one quarter of its period in 3π, has been shifted right $\frac{3\pi}{4}$ units, has a minimum value of -7, and a maximum value of 3.

$3\pi \quad 3\pi \quad 3\pi \quad 3\pi$
period $=12\pi$

$\text{period}=\frac{2\pi}{b}$
$12\pi=\frac{2\pi}{b}$

$12\pi b=2\pi$
$b=\frac{2\pi}{12\pi}$
$b=\frac{1}{6}$

$c=\frac{3\pi}{4}$

3
-2
-7

$a=5$
$d=-2$

$y=a\sin[b(\theta+c)]+d$

$\boxed{y=5\sin\left[\frac{1}{6}\left(\theta-\frac{3\pi}{4}\right)\right]-2}$

4. Write the amplitude, period, phase shift, and vertical shift of each function. Then graph at least one period of the function.

a. $y=-3\cos\left(3\theta+\frac{3\pi}{4}\right)-1$

$y=-3\cos\left[3\left(\theta+\frac{\pi}{4}\right)\right]-1$

$b=3$
$\text{period}=\frac{2\pi}{3}$
$I=\frac{1}{4}\cdot\frac{2\pi}{3}=\frac{2\pi}{12}=\frac{\pi}{6}$

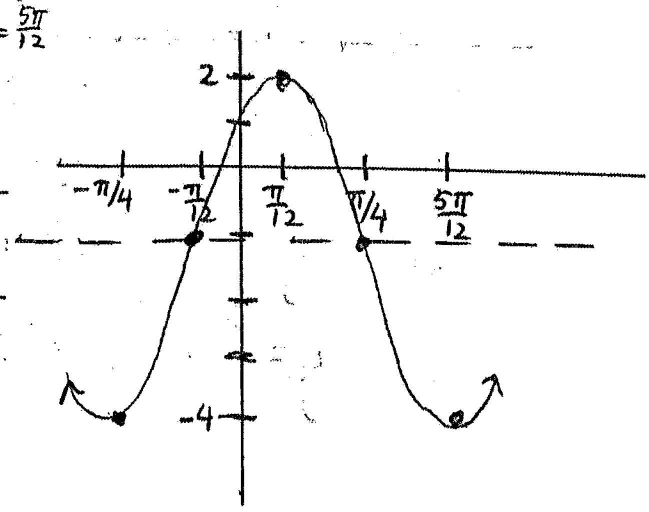
amplitude: 3 period: $2\pi/3$ phase shift: left $\pi/4$ vertical shift: down 1

Domain: $(-\infty,\infty)$ Range: $[-4,2]$

Graph:

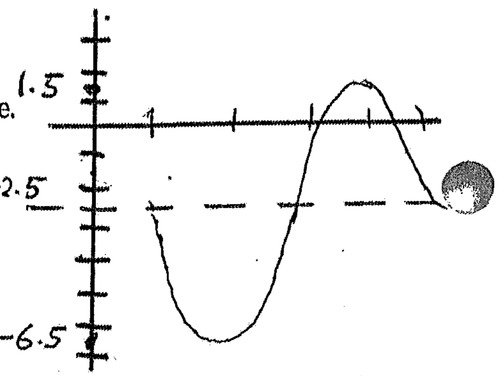
$\frac{8\pi}{12}-\frac{3\pi}{12}=\frac{5\pi}{12}$

	$\boxed{-\pi/4}$	$\frac{2\pi}{12}-\frac{3\pi}{12}=\boxed{\frac{-\pi}{12}}$	$\frac{4\pi}{12}-\frac{3\pi}{12}=\boxed{\frac{\pi}{12}}$	$\frac{6\pi}{12}-\frac{3\pi}{12}=\boxed{\frac{3\pi}{12}}$	
	$0-\frac{\pi}{4}$	$\pi/6-\pi/4$	$\frac{2\pi}{6}-\pi/4$	$\frac{3\pi}{6}-\pi/4$	$\frac{4\pi}{6}-\pi/4$
$\cos(3\theta)$	1	0	-1	0	1
$-3\cos(3\theta)$	-3	0	3	0	-3



5. For $y = -4\sin(5\theta - 2\pi) - 2.5$, determine the domain & range.

Domain: $(-\infty, \infty)$ Range: $[-6.5, 1.5]$

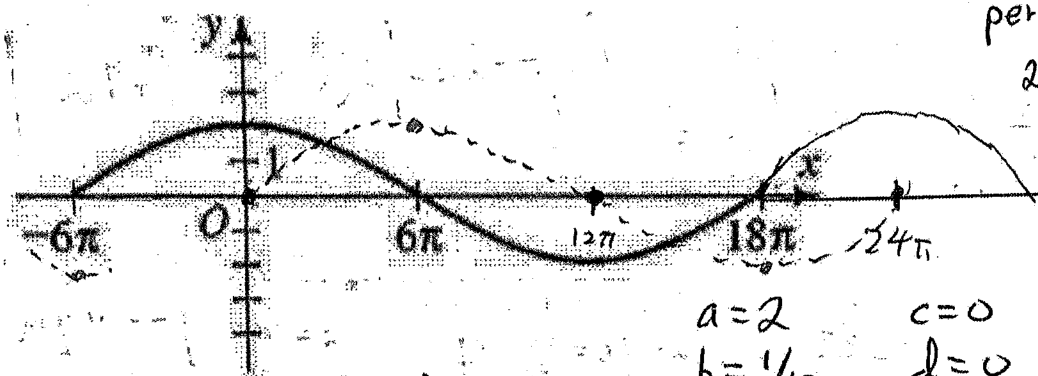


$a = -4$

$d = -2.5$

6. State both a cosine function and sine function that would represent the graph below.

a)



period $= 24\pi$

$24\pi = \frac{2\pi}{b}$

$24\pi b = 2\pi$

$b = \frac{2\pi}{24\pi} = \frac{1}{12}$

$a = 2$ $c = 0$

$b = 1/12$ $d = 0$

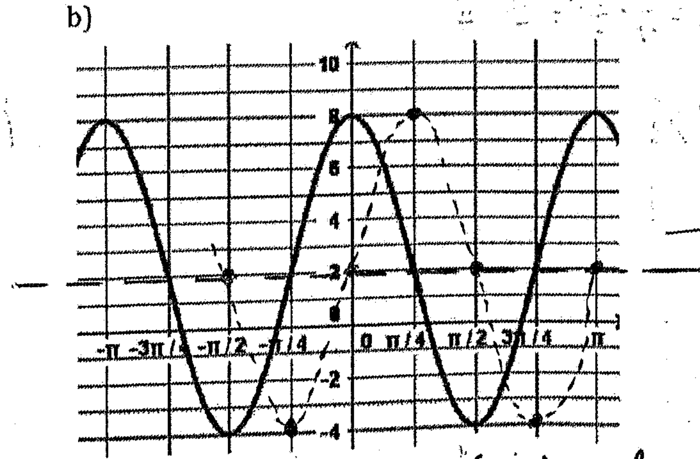
Using cosine: $y = 2\cos\left(\frac{1}{12}\theta\right)$

$a = 2$ $c = +6\pi$

$b = \frac{1}{12}$ $d = 0$

Using sine: $y = 2\sin\left[\frac{1}{12}(\theta + 6\pi)\right]$

b)



period $= \pi$

$\pi = \frac{2\pi}{b} \rightarrow b\pi = 2\pi \rightarrow b = 2$

$a = 6$ $c = 0$

$b = 2$ $d = 2$

Using cosine: $y = 6\cos(2\theta) + d$

$a = 6$ $c = +\pi/4$

$b = 2$ $d = 2$

Using sine: $y = 6\sin\left[2\left(\theta + \frac{\pi}{4}\right)\right] + 2$

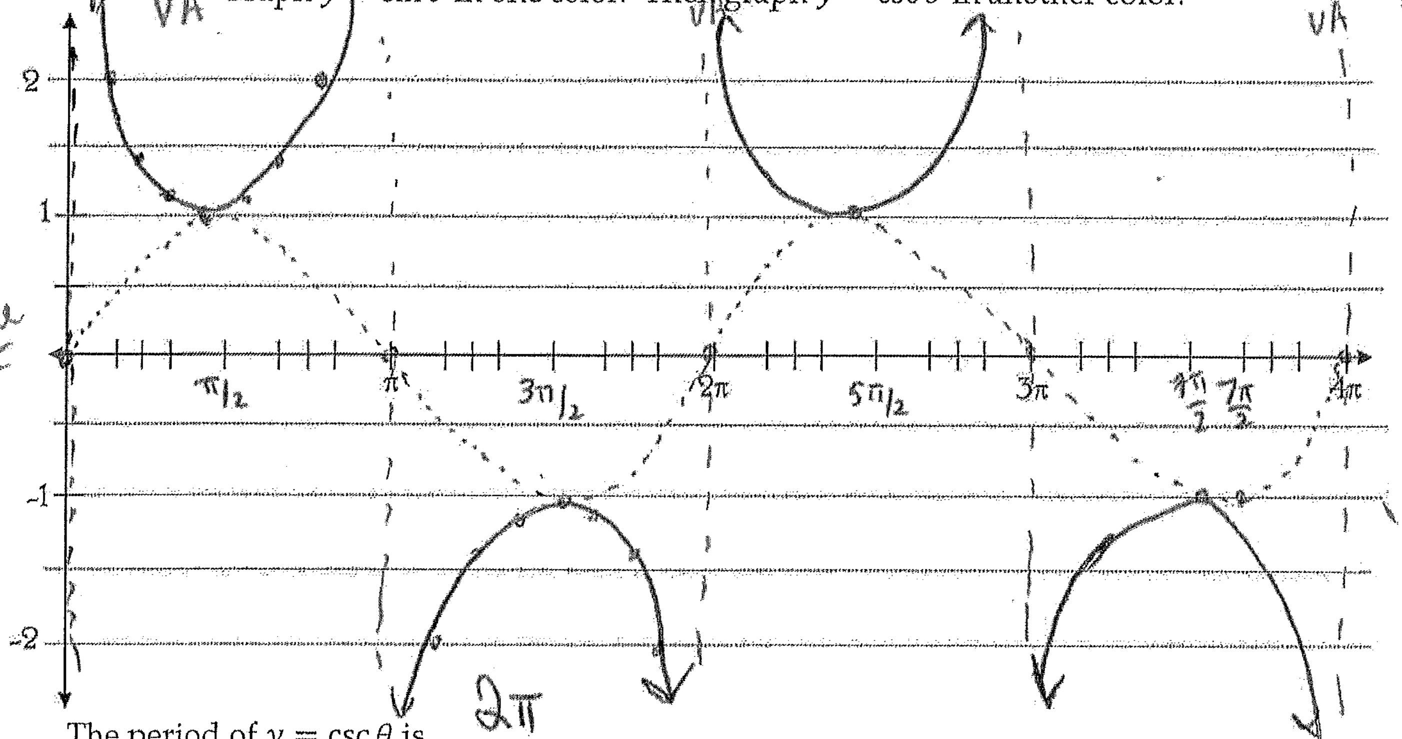
Accelerated Precalculus Name ____________________

2.09: Exploring the Cosecant & Secant Graphs Date __________ Period ______

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Sine (decimal)	0	.5	.707	.866	1	.866	.707	.5	0	-.5	-.707	-.866	-1	-.866	-.707	-.5	0
Cosecant (decimal)	und	2	1.41	1.15	1	1.15	1.41	2	und.	-2	-1.41	-1.15	-1	-1.15	-1.41	-2	und.

$\frac{1}{\sin\theta}$

Graph $y = \sin\theta$ in one color. Then graph $y = \csc\theta$ in another color.



The period of $y = \csc\theta$ is 2π

The domain of $y = \csc\theta$ is $\mathbb{R}$ except πn where $n \in \mathbb{Z}$

The range of $y = \csc\theta$ is $(-\infty, -1] \cup [1, \infty)$

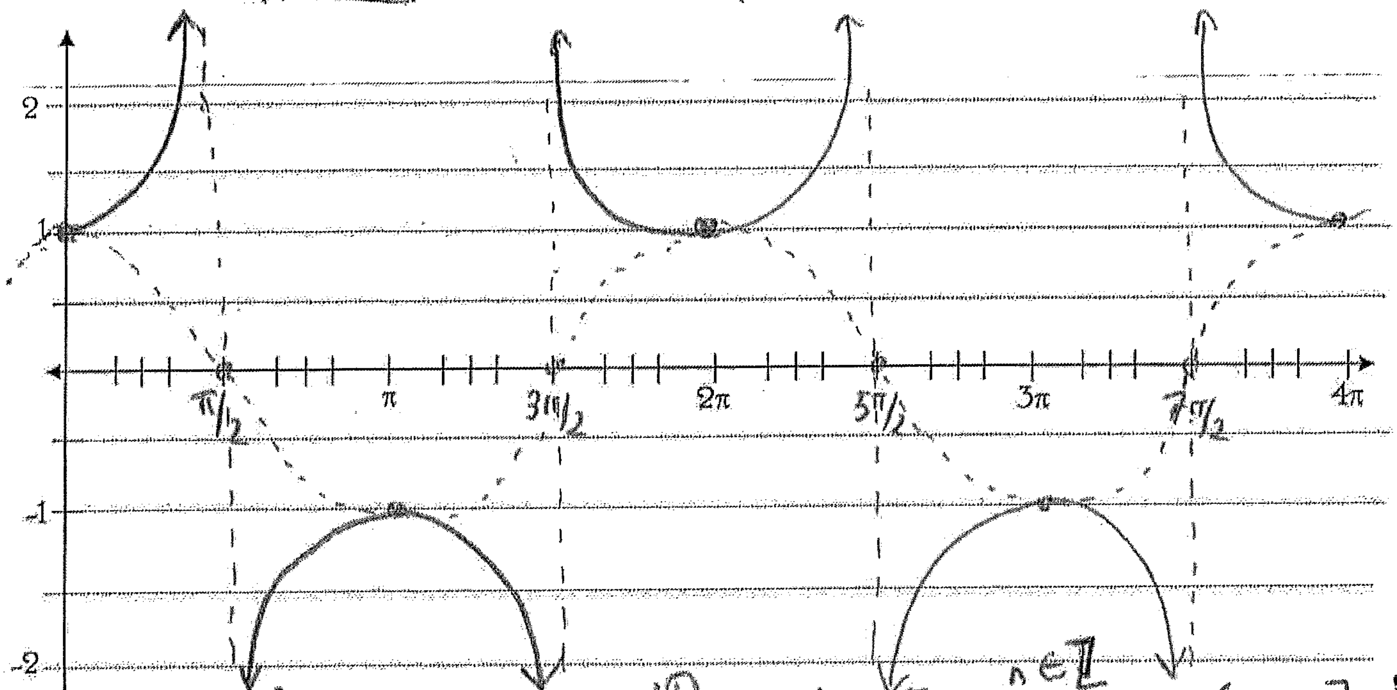
To graph cosecant functions:

1. Plot your points like you're graphing $\sin\theta$.

2. You'll have vertical asymptotes where you see points on the x-axis ("d-value")

3. Plot parabolas at max and min points. max points open up and min points open down

Now let's do the same for cosine and secant.
Graph $y = \cos\theta$ in one color. Then graph $y = \sec\theta$ in another color.



period 2π Domain $\mathbb{R}$ except $\frac{\pi}{2}+\pi n$ $n\in\mathbb{Z}$ R: $(-\infty,-1]\cup[1,\infty)$

Graph each function by first graphing their reciprocal trigonometric function.

1. $y = -3\,\cancel{\csc}^{\,\sin}\left(\frac{\theta}{4}+\frac{\pi}{8}\right) - 2$

$y=-3\csc\left[\frac{1}{4}\left(\theta+\frac{\pi}{2}\right)\right]-2$

$a=-3$
$b=1/4$
period $=\frac{2\pi}{1/4}=8\pi$
$I=\frac{1}{4}\cdot 8\pi=2\pi$

Vertical Stretch: 3

Period: 8π

Phase Shift: left $\pi/2$

Vertical Shift: down 2

	$\boxed{-\pi/2}$	$\boxed{3\pi/2}$	$\boxed{7\pi/2}$	$\boxed{11\pi/2}$	$\boxed{15\pi/2}$
θ	$0-\frac{\pi}{2}$	$2\pi-\frac{\pi}{2}$	$4\pi-\frac{\pi}{2}$	$6\pi-\frac{\pi}{2}$	$8\pi-\frac{\pi}{2}$
$\sin\theta$ $\csc\left(\frac{\theta}{4}\right)$	0	1	0	-1	0
$-3\csc\left(\frac{\theta}{4}\right)$	0	-3	0	3	0

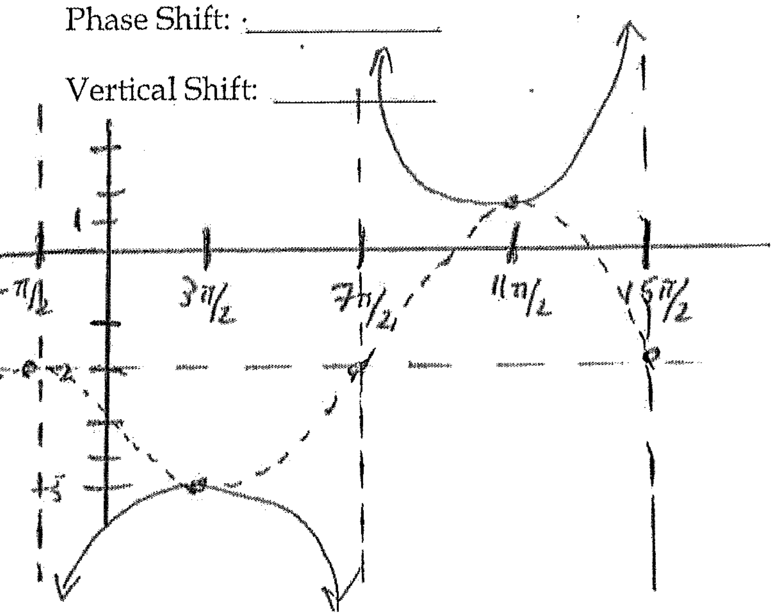
~~2.~~ $y = -0.5\,sec(2\theta - \pi) + 3$

Vertical Stretch: ________

Period: ________

Phase Shift: ________

Vertical Shift: ________



2.09 Practice: Graphing Secant and Cosecant Functions Date ____________

Graph each.

1. $y = \csc 3\theta$

Per: ______ PS: ______ VS: ______

2. $y = -\sec\left(\theta - \frac{\pi}{2}\right)$

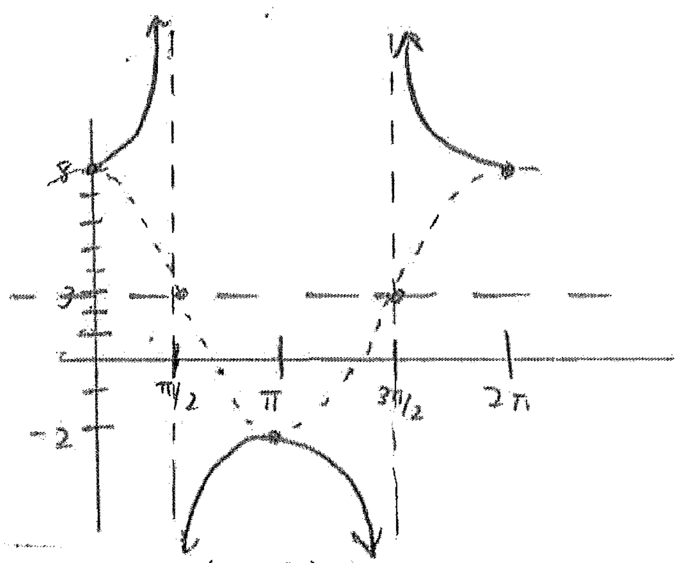
Per: ______ PS: ______ VS: ______

3. $y = 5\sec\theta + 3$ (*Graph $\cos\theta$ first)

Per: 2π PS: none VS: up 3

$a = 5$
$b = 1$
$c = 0$
$d = 3$

θ	0	$\pi/2$	π	$3\pi/2$	2π
$(\cos\theta)$ $\sec\theta$	1	0	-1	0	1
$5\sec\theta$	5	0	-5	0	5



Per: ______ PS: ______ VS: ______

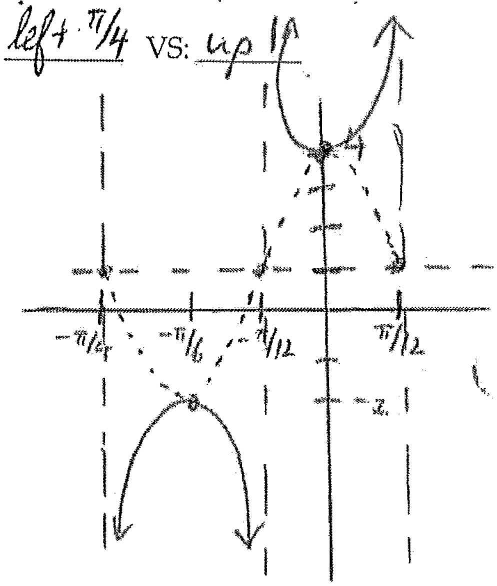
6. $y = -3\csc\left(6\theta + \frac{3\pi}{2}\right) + 1$ (*Graph $\sin\theta$ first)

Per: $\pi/3$ PS: left $\pi/4$ VS: up 1

6) $y = -3\csc\left[6\left(\theta + \frac{\pi}{4}\right)\right] + 1$

$a = -3$
$b = 6$
$\text{period} = \frac{2\pi}{6} = \frac{\pi}{3}$
$c = \pi/4$
$d = 1$

$I = \frac{1}{4}\cdot\frac{\pi}{3} = \frac{\pi}{12}$	$\boxed{-\pi/4}$	$-\pi/6$	$-\frac{\pi}{12}$	0	$\pi/12$
θ	$0 - \pi/4$	$\frac{\pi}{12} - \pi/4$	$\frac{2\pi}{12} - \pi/4$	$\frac{3\pi}{12} - \pi/4$	$\frac{4\pi}{12} - \pi/4$
*sin $\csc(6\theta)$	0	1	0	-1	0
$-3\csc\theta$	0	-3	0	3	0



$a=1 \quad b=\pi/5 \quad \text{period} = \frac{2\pi}{b} \to \frac{2\pi}{\pi/5} = 2\pi \cdot \frac{5}{\pi} = 10$

$I = \frac{1}{4} \cdot 10 = \frac{5}{2} = 2.5$

$y = \sec\left[\frac{\pi}{5}\left(\theta - \frac{5}{2}\right)\right] - 1.5$

$a=1$
$b=\pi/5$
$c=5/2$
$d=-1.5$

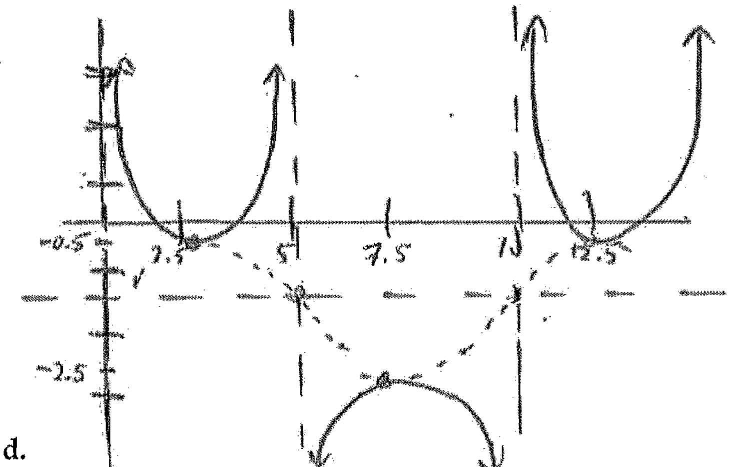
7. $y = \sec\left(\frac{\pi\theta}{5} - \frac{\pi}{2}\right) - 1.5$

8. $y = \csc(2\pi\theta + \pi)$

Per: 10 PS: 2.5 right VS: down 1.5

Per: ______ PS: ______ VS: ______

θ	$\boxed{2.5}$ 0 +2.5	$\boxed{5}$ 2.5 +2.5	$\boxed{7.5}$ 5 +2.5	$\boxed{10}$ 7.5 +2.5	$\boxed{12.5}$ 10 +2.5
cos θ / sec θ	1	0	−1	0	1
c θ	1	0	−1	0	1



Write an equation for each function described.

9. Secant function, vertically stretched to be 5 times taller than normal, horizontally compressed to have a period one-half the length as normal, shifted $\frac{\pi}{4}$ units to the right and 10 units down.

$a=5$
period $= \pi$
period $= \frac{2\pi}{b}$
$\pi = \frac{2\pi}{b}$
$b\pi = 2\pi \to b = 2$

$c = -\pi/4$
$d = -10$

$\boxed{y = 5\sec\left[2\left(\theta - \frac{\pi}{4}\right)\right] - 10}$

10. Cosecant function, reflected upside down, period = $\frac{\pi}{3}$, phase shift right $\frac{\pi}{2}$, and vertical shift up 4

$a=-1$
$\frac{\pi}{3} = \frac{2\pi}{b}$
$b\pi = 6\pi$
$b=6$

$c = -\frac{\pi}{2}$
$d = 4$

$\boxed{y = -\csc\left[6\left(\theta - \frac{\pi}{2}\right)\right] + 4}$

11. Secant function, period = 8, phase shift right 5, and vertical shift up 7.5

period $= \frac{2\pi}{b}$
$8 = \frac{2\pi}{b}$
$8b = 2\pi$
$b = \frac{2\pi}{8} = \frac{\pi}{4}$

$c = -5$
$d = 7.5$

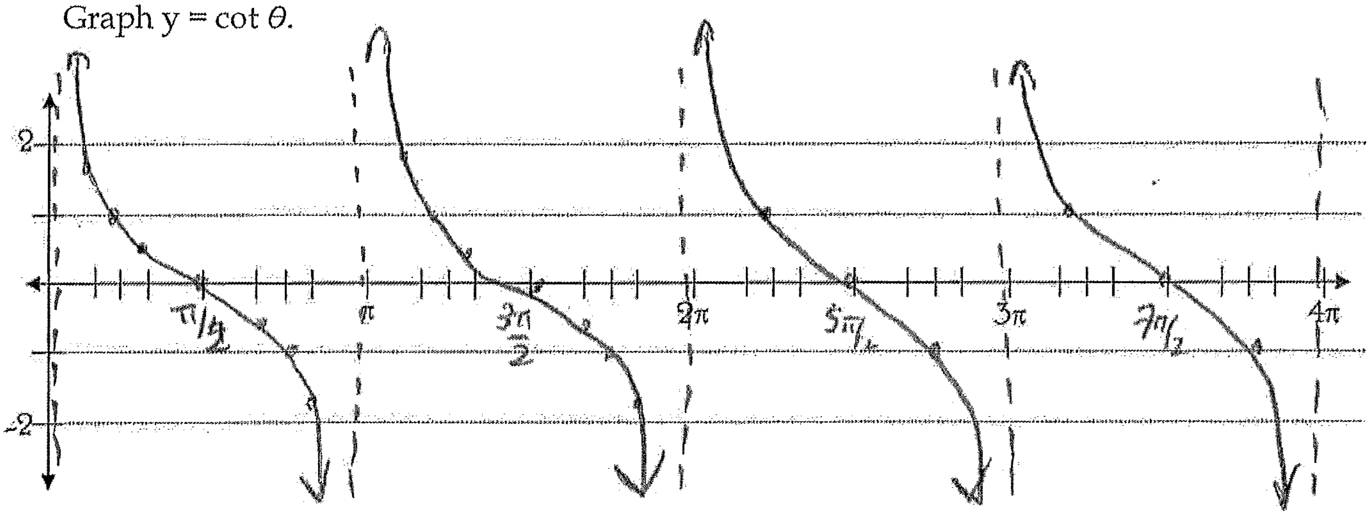
$\boxed{y = \sec\left[\frac{\pi}{4}(\theta - 5)\right] + 7.5}$

2.10: Exploring the Cotangent & Tangent Graphs

Date ____________________

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
cot θ (exact)	und.	$\sqrt{3}$	1	$\sqrt{3}/3$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	und.	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	und.
cot θ (decimal)	und.	1.73	1	≈0.577	0	-0.577	-1	-1.73	und.	1.73	1	0.577	0	-0.57	-1	1.73	und.

Graph y = cot θ.



The period of $y = \cot\theta$ is π

The domain of $y = \cot\theta$ is $\mathbb{R}$ except $x = n\pi$ where $n \in \mathbb{Z}$

The range of $y = \cot\theta$ is $(-\infty, \infty)$

$\boxed{* \text{ period} = \frac{\pi}{b}}$

1. Graph $y = 5\cot\theta - 1$

Vertical Stretch: 5

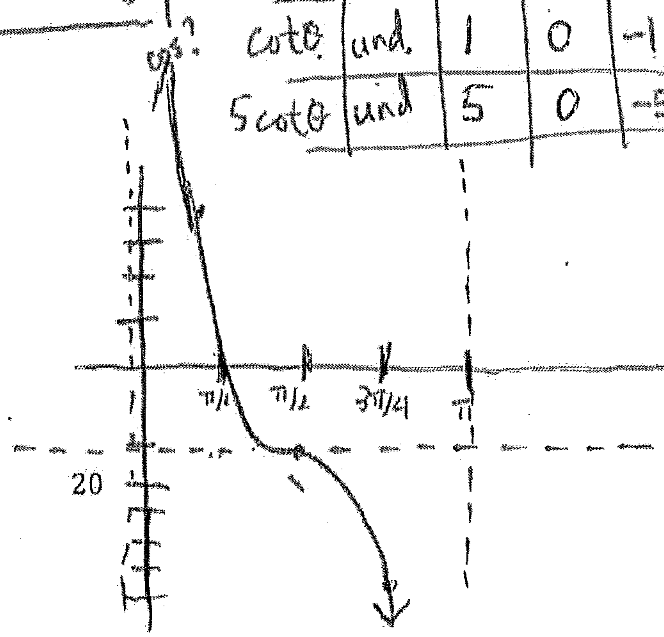
Period: π

Phase Shift: none

Vertical Shift: down 1

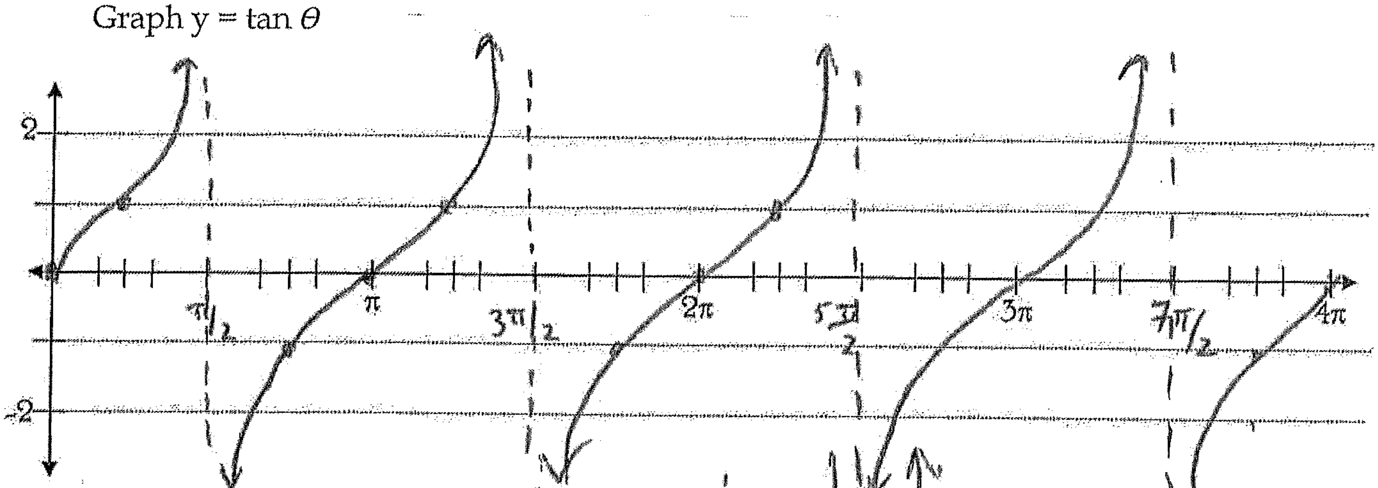
$I = \frac{1}{4}P = \frac{1}{4}\cdot\pi = \frac{\pi}{4}$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\cot\theta$	und.	1	0	-1	und.
$5\cot\theta$	und	5	0	-5	und.



Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
tan θ (exact)	0		1		und.		−1		0		1		und.		−1		0
tan θ (decimal)	0		1		und		−1		0		1		und		−1		0

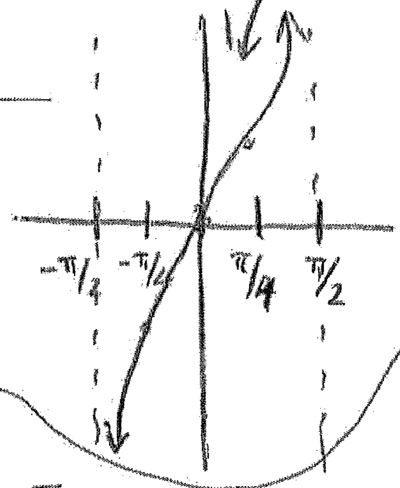
Graph y = tan θ



The period of $y = \tan\theta$ is π

Practice: Graph one period of tangent.

2. $y = \tan\left(\frac{\theta}{2} - \frac{3\pi}{4}\right)$



θ	−π/2	−π/4	0	π/4	π/2
	und.	−1	0	1	und.

Vertical Stretch: ________

Period: ________

Phase Shift: ________

Vertical Shift: ________

$a = -2$ $c = \pi/12$

$b = 3$ $d = -1$

$y = -2\tan\left[3\left(\theta - \frac{\pi}{12}\right)\right] - 1$ * period $= \frac{\pi}{b} \to \frac{\pi}{3}$

3. $y = -2\tan\left(3\theta - \frac{\pi}{4}\right) - 1$

$I = \frac{1}{4} \cdot \frac{\pi}{3} = \frac{\pi}{12}$

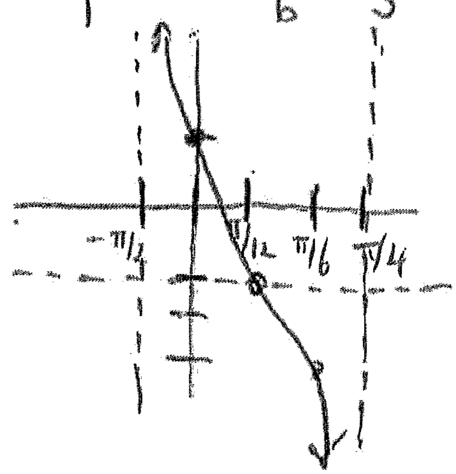
θ	$-\frac{\pi}{12}$ ($-\frac{2\pi}{12}$)	0 ($-\frac{\pi}{12}+\frac{\pi}{12}$)	$\pi/12$	$\pi/6$ ($\frac{\pi}{12}$)	$\pi/4$ ($\frac{2\pi}{12}+\frac{\pi}{12}$)
tan3θ	und.	−1	0	1	und.
−2tan3θ	und	2	0	−2	und.

Vertical Stretch: ________

Period: ________

Phase Shift: ________

Vertical Shift: ________



5. $y = \cot\left(4\theta - \frac{\pi}{2}\right) + 2$

Per: $\pi/4$ PS: right $\pi/8$ VS: up 2

$y = \cot\left[4\left(\theta - \frac{\pi}{8}\right)\right] + 2$

$a = 1$ $\quad c = \pi/8$

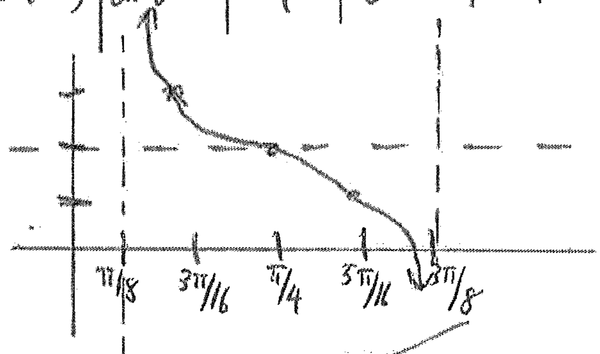
$b = 4$ $\quad d = 2$

$*\text{period} = \frac{\pi}{b}$

$\text{period} = \frac{\pi}{4}$

$I = \frac{1}{4}\cdot P \rightarrow \frac{1}{4}\cdot\frac{\pi}{4} = \frac{\pi}{16}$

	$\pi/8$	$3\pi/16$	$\pi/4$	$5\pi/16$	$\frac{6\pi}{16} \rightarrow \frac{3\pi}{8}$
θ	$0 + \pi/8$	$\pi/16 + \pi/8$	$2\pi/16 + \pi/8$	$3\pi/16 + \pi/8$	$4\pi/16 + \pi/8$
$\cot(4\theta)$	und	1	0	-1	und
$\cot(4\theta)$	und	1	0	-1	und



6. $y = -\cot\left(3\theta + \frac{\pi}{2}\right) + 1$

Per: ______ PS: ______ VS: ______

7. $y = 2\tan\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 3$

Per: ______ PS: right π VS: down 3

$y = 2\tan\left[\frac{1}{2}(\theta - \pi)\right] - 3$

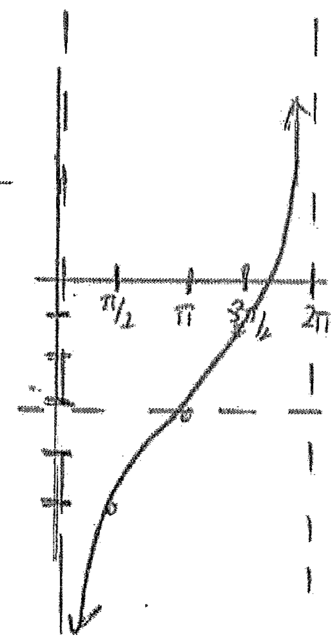
$a = 2$ $\quad b = 1/2$ $\quad c = -\pi$ $\quad d = -3$

$\text{period} = \frac{\pi}{b} = \frac{\pi}{1/2} = 2\pi$

$I = \frac{1}{4}\cdot P = \frac{1}{4}\cdot 2\pi = \frac{\pi}{2}$

θ	$\boxed{0}$ $-\pi + \pi$	$\boxed{\pi/2}$ $-\pi/2 + \pi$	$\boxed{\pi}$ $0 + \pi$	$\boxed{3\pi/2}$ $\pi/2 + \pi$	$\boxed{2\pi}$ $\pi + \pi$
$\tan\theta$	und	-1	0	1	und
$2\tan\theta$	und	-2	0	2	und

$\boxed{\text{period} = \frac{\pi}{b}}$



8. $y = -3\cot(2\theta + 3\pi) + 5.5$

Per: ______ PS: ______ VS: ______

Write an equation for each function described.

9. Tangent function, period = 3π, phase shift left $\frac{\pi}{4}$, and vertical shift down 6

$\frac{3\pi}{1} = \frac{\pi}{b}$ | $3\pi b = \pi$ | $b = \frac{\pi}{3\pi} = \frac{1}{3}$ | $a = 1$, $b = 1/3$ | $c = \pi/4$, $d = -6$ | $y = \tan\left[\frac{1}{3}\left(\theta + \frac{\pi}{4}\right)\right] - 6$

10. Cotangent function, period = $\frac{\pi}{4}$, phase shift right $\frac{\pi}{2}$, and vertical shift up 9

$\frac{\pi}{4} = \frac{\pi}{b}$, $b\pi = 4\pi$ | $b = \frac{4\pi}{\pi}$, $b = 4$ | $c = -\pi/2$, $d = 9$ | $y = \cot\left[4\left(\theta - \frac{\pi}{2}\right)\right] + 9$

2.11 Additional Practice: Graphing Sec, Csc, Tan, Cot   Date________

Find the Vertical Stretch, Period, Phase Shift, and Vertical Shift. Then graph the function.

1. $y = \csc\left(4\theta + \frac{\pi}{2}\right)$

Vertical Stretch: none

Period: $\pi/2$

Phase Shift: left $\pi/8$

Vertical Shift: none

$y = \csc\left[4\left(\theta + \frac{\pi}{8}\right)\right]$

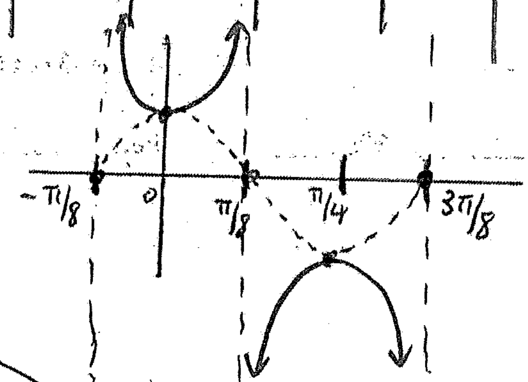
$a = 1 \quad b = 4$

$c = \pi/8 \quad d = 0$

period $= \frac{2\pi}{b} \to \frac{2\pi}{4} = \frac{\pi}{2}$

$I = \frac{1}{4} \cdot P \to \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

θ	$\boxed{-\pi/8}$ $0 - \pi/8$	$\boxed{0}$ $\pi/8 - \pi/8$	$\boxed{\pi/8}$ $\frac{2\pi}{8} - \pi/8$	$\boxed{\pi/4}$ $\frac{3\pi}{8} - \pi/8$	$\boxed{3\pi/8}$ $\frac{4\pi}{8} - \pi/8$
$\csc(4\theta)$	0	1	0	−1	0
$\csc(4\theta)$	0	1	0	−1	0



2. ~~$y = \sec\left(\frac{\theta}{2} - \pi\right)$~~

Vertical Stretch: ________

Period: ________

Phase Shift: ________

Vertical Shift: ________

3. $y = -3\sec(2\theta) + 1$

Vertical Stretch: 3

Period: π

Phase Shift: none

Vertical Shift: up 1

$y = -3\sec\left[2(\theta)\right] + 1$

$a = -3 \quad c = 0$

period $= \frac{2\pi}{b} \to \frac{2\pi}{2} = \pi$

$b = 2 \quad d = 1$

4. ~~$y = 2.5\csc\left(\theta - \frac{\pi}{4}\right) + 5$~~

$I = \frac{1}{4} \cdot \pi = \pi/4$

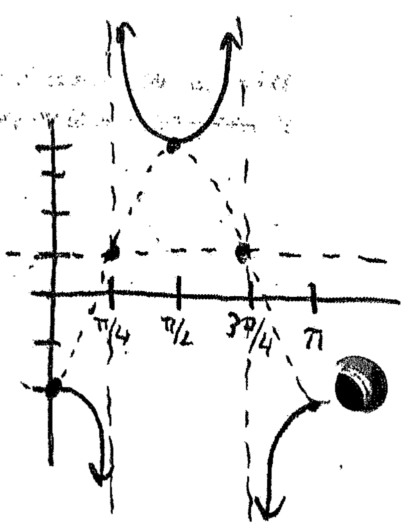
Vertical Stretch: ________

Period: ________

Phase Shift: ________

Vertical Shift: ________

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\sec\theta$	1	0	−1	0	1
$-3\sec\theta$	−3	0	3	0	−3



$a=2 \quad c=0 \qquad period = \frac{\pi}{b} \rightarrow \frac{\pi}{1} = \pi$

$b=1 \quad d=4 \qquad I = \frac{1}{4}\cdot P \rightarrow \frac{1}{4}\cdot\pi = \frac{\pi}{4}$

5. $y = 2\tan\theta + 4$

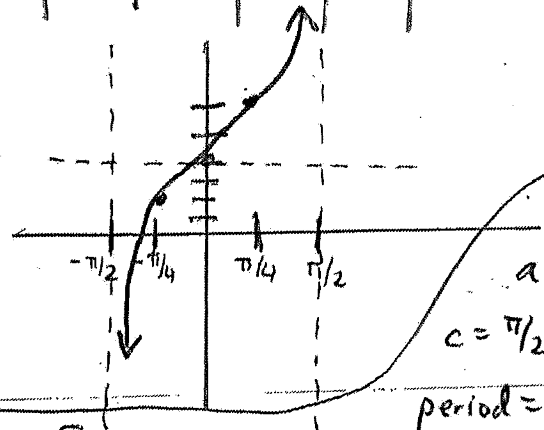
Vertical Stretch: 2

Period: π

Phase Shift: none

Vertical Shift: up 4

θ	$-\frac{\pi}{2}$	$-\pi/4$	0	$\pi/4$	$\pi/2$
$\tan\theta$	und	-1	0	1	und
$2\tan\theta$	und	-2	0	2	und



~~6. $y = \cot\left(2\theta - \frac{\pi}{2}\right)$~~

~~Vertical Stretch: _____~~

~~Period: _____~~

~~Phase Shift: _____~~

~~Vertical Shift: _____~~

$a=-3 \quad b=\frac{1}{3}$

$c=\pi/2 \quad d=1$

$period = \frac{\pi}{b} \rightarrow \frac{\pi}{1/3} \rightarrow \pi\cdot\frac{3}{1} = 3\pi$

7. $y = -3\cot\left(\frac{\theta}{3} + \frac{\pi}{6}\right) + 1 \qquad y = -3\left[\frac{1}{3}\left(\theta + \frac{\pi}{2}\right)\right] + 1$

$I = \frac{1}{4}\cdot P \rightarrow \frac{1}{4}\cdot 3\pi = \frac{3\pi}{4}$

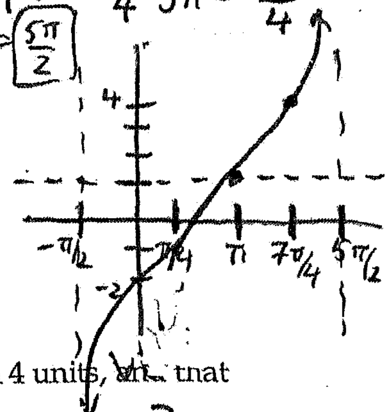
Vertical Stretch: 3

Period: 3π

Phase Shift: left $\pi/2$

Vertical Shift: up 1

θ	$0-\frac{\pi}{2}$ $\boxed{-\pi/2}$	$\frac{3\pi}{4}-\frac{\pi}{2}$ $\boxed{\pi/4}$	$\frac{6\pi}{4}-\frac{\pi}{2}$ $\boxed{\pi}$	$\frac{9\pi}{4}-\frac{\pi}{2}$ $\boxed{7\pi/4}$	$3\pi-\frac{\pi}{2}=\boxed{\frac{5\pi}{2}}$
$\cot\theta$	und	1	0	-1	und.
$-3\cot\theta$	und	-3	0	3	und.



Write the equation described.

8. A secant function that has been shifted to the left $\frac{\pi}{3}$ units and down 4 units, and that has a wavelength of π units.

$period = \pi \qquad period = \frac{2\pi}{b}$

$\frac{\pi}{1} = \frac{2\pi}{b} \qquad b\pi = 2\pi$

$b=2 \quad c=\pi/3 \quad d=-4$

$y = \sec\left[2\left(\theta + \frac{\pi}{3}\right)\right] - 4$

9. A cotangent function that has a period of 3π, has been shifted to $\frac{\pi}{2}$ units to the right, and has been shifted 4 units up.

$period = \frac{\pi}{b} \qquad \frac{3\pi}{1} = \frac{\pi}{b}$

$3\pi b = \pi \qquad b = \frac{\pi}{3\pi} = 1/3$

$a=1 \quad b=1/3 \quad c=-\pi/2 \quad d=4$

$y = \cot\left[\frac{1}{3}\left(\theta - \frac{\pi}{2}\right)\right] + 4$

$\boxed{2.11}$ p. 24–25

2) $y = \sec\left(\frac{\theta}{2} - \pi\right)$

$a = 1$ $\quad c = 2\pi$

$y = \sec\left[\frac{1}{2}(\theta - 2\pi)\right]$ $\quad b = \frac{1}{2}$ $\quad d = 0$

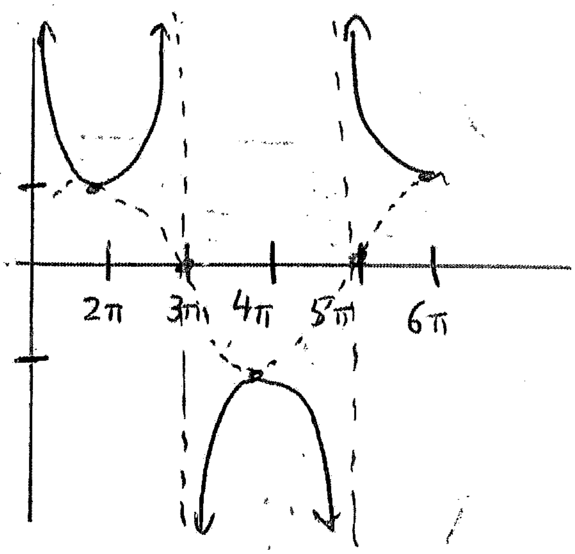
Vert. Stretch: none

Period: 4π $\quad$ period $= \frac{2\pi}{b} \rightarrow \frac{2\pi}{1/2} \rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$

PS: right 2π $\quad$ $I = \frac{1}{4} \cdot P \rightarrow \frac{1}{4} 4\pi = \pi$

VS: none

θ	$0 + 2\pi$ ($\boxed{2\pi}$)	$\pi + 2\pi$ ($\boxed{3\pi}$)	$2\pi + 2\pi$ ($\boxed{4\pi}$)	$3\pi + 2\pi$ ($\boxed{5\pi}$)	$4\pi + 2\pi$ ($\boxed{6\pi}$)
$\sec\theta$	1	0	−1	0	1
$\sec\theta$	1	0	−1	0	1



4) $y = 2.5\csc\left(\theta - \frac{\pi}{4}\right) - 5$

$a = 2.5 \quad c = \pi/4$

$b = 1 \quad d = -5$

vert. stretch 2.5

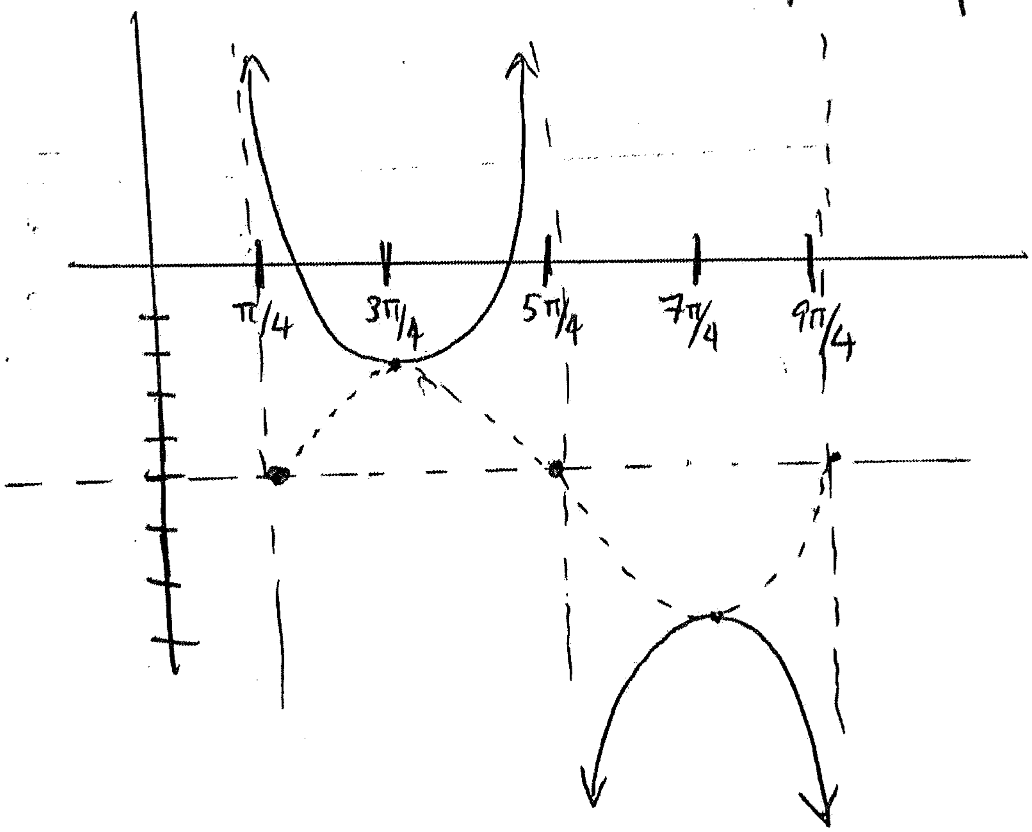
PS right $\pi/4$

VS down 5

Period 2π

$I = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$

θ	$0 + \pi/4$ $\boxed{\pi/4}$	$\pi/2 + \pi/4$ $\boxed{3\pi/4}$	$\pi + \pi/4$ $\boxed{5\pi/4}$	$3\pi/2 + \pi/4$ $\boxed{7\pi/4}$	$2\pi + \pi/4$ $\boxed{9\pi/4}$
$\csc\theta$	0	1	0	-1	0
$2.5\csc\theta$	0	2.5	0	-2.5	0



6) $y = \cot\left(2\theta - \frac{\pi}{2}\right)$

$y = \cot\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

Vert stretch: none

period: π

PS: right $\pi/4$

VS: none

$a = 1$ $c = {}^{-}\pi/4$

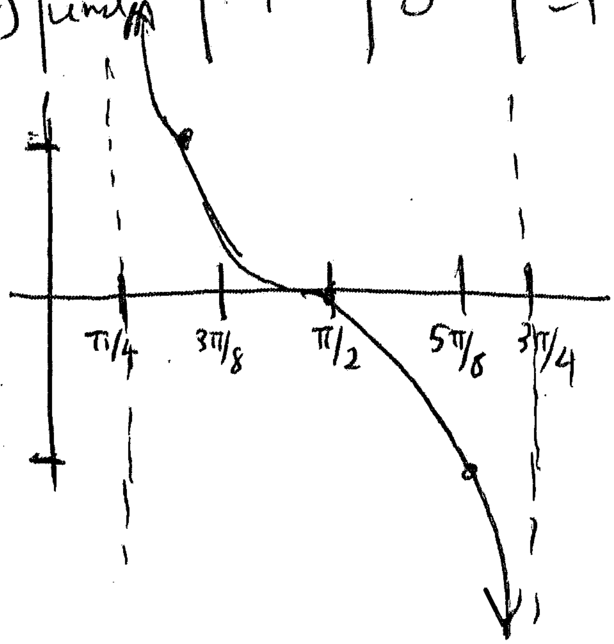
$b = 2$ $d = 0$

period $= \frac{\pi}{b} \rightarrow \frac{\pi}{2} = \frac{\pi}{2}$

$I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

θ	$\boxed{\pi/4}$ $0 + \frac{\pi}{4}$	$\boxed{3\pi/8}$ $\frac{\pi}{8} + \frac{2\pi}{8}$	$\boxed{\pi/2}$ $\frac{2\pi}{8} + \frac{2\pi}{8}$	$\boxed{5\pi/8}$ $\frac{3\pi}{8} + \frac{2\pi}{8}$	$\boxed{3\pi/4}$ $\frac{4\pi}{8} + \frac{2\pi}{8}$
$\cot 2\theta$	und.	1	0	-1	und.
$\cot(2\theta)$	und	1	0	-1	und



2.13 Unit 2 Review: Graphing Trig Functions & Sinusoidal Modeling Date: ________

State the requested characteristics of each function. Graph at least one period of the function.

period $= \frac{2\pi}{b}$ $= \frac{2\pi}{1/4} = 8\pi$

$I = 2\pi$

1. $y = \sin\left(\frac{\theta}{4} + \frac{3\pi}{8}\right) + 1$

$y = \sin\left[\frac{1}{4}\left(\theta + \frac{3\pi}{2}\right)\right] + 1$

$a = 1$ $c = 3\pi/2$
$b = 1/4$ $d = 1$

$\boxed{13\pi/2}$

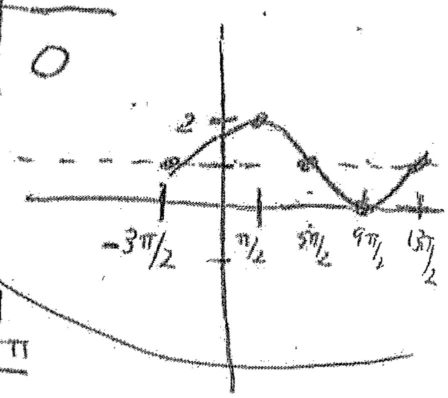
Amplitude 1

Period 8π

Phase Shift left $3\pi/2$

Vertical Shift up 1

θ	$0 - \frac{3\pi}{2}$ ($\boxed{-3\pi/2}$)	$2\pi - \frac{3\pi}{2}$ ($\boxed{\pi/2}$)	$4\pi - \frac{3\pi}{2}$ ($\boxed{5\pi/2}$)	$6\pi - \frac{3\pi}{2}$ ($\boxed{9\pi/2}$)	$8\pi - 3\pi/2$
$\sin\theta$	0	1	0	−1	0
$\sin\theta$	0	1	0	−1	0



*think $\cos\theta$

2. $y = -4\sec(\theta - \pi) - 2$

$a = -4$ $c = \pi$
$b = 1$ $d = -2$

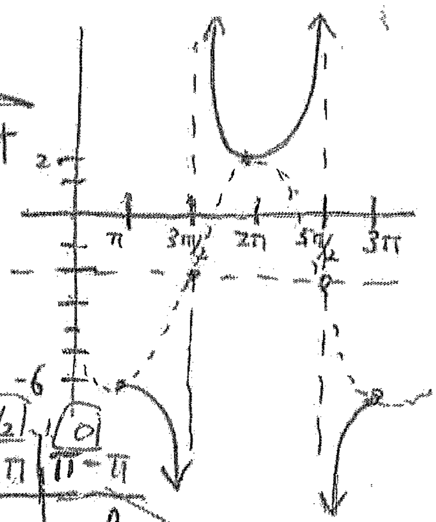
Vertical Stretch 4

Period 2π

Phase Shift right π

Vertical Shift down 2

θ	$0 + \pi$ ($\boxed{\pi}$)	$\frac{\pi}{2} + \pi$ ($\boxed{3\pi/2}$)	$\pi + \pi$ ($\boxed{2\pi}$)	$\frac{3\pi}{2} + \pi$ ($\boxed{5\pi/2}$)	$2\pi + \pi$ ($\boxed{3\pi}$)
$\sec\theta$	1	0	−1	0	1
$-4\sec\theta$	−4	0	4	0	−4



$y = 5\tan\left[\frac{1}{2}(\theta + \pi)\right] + 3$

$a = 5$ $c = \pi$
$b = 1/2$ $d = 3$

3. $y = 5\tan\left(\frac{\theta}{2} + \frac{\pi}{2}\right) + 3$

Vertical Stretch 5

Period 2π

Phase Shift left π

Vertical Shift up 3

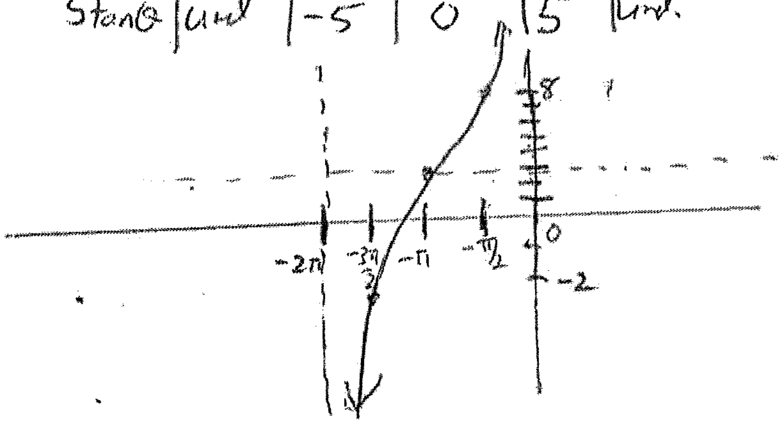
θ	$-\pi - \pi$ ($\boxed{-2\pi}$)	$-\pi/2 - \pi$ ($\boxed{-3\pi/2}$)	$0 - \pi$ ($\boxed{-\pi}$)	$\pi/2 - \pi$ ($\boxed{-\pi/2}$)	$\pi - \pi$ ($\boxed{0}$)
$\tan\theta$	und	−1	0	1	und
$5\tan\theta$	und	−5	0	5	und

*period $= \frac{\pi}{b}$

period $= \frac{\pi}{1/2}$

period $= 2\pi$

$I = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$



$y=\frac{1}{2}\cot\left[2\left(\theta+\frac{\pi}{2}\right)\right]-1$

4. $y = 0.5\cot(2\theta + \pi) - 1$

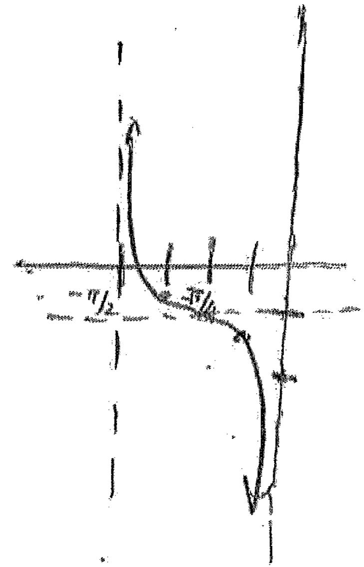
Vertical Stretch compress by $\frac{1}{2}$

Period $\pi/2$

Phase Shift left $\pi/2$

Vertical Shift down 1

θ	$0-\frac{\pi}{2}$ $\left(-\frac{\pi}{2}\right)$	$\frac{1\pi}{8}-\frac{4\pi}{8}$ $\left(-\frac{3\pi}{8}\right)$	$\frac{2\pi}{8}-\frac{4\pi}{8}$ $\left(-\frac{\pi}{4}\right)$	$\frac{3\pi}{8}-\frac{4\pi}{8}$ $\left(-\frac{\pi}{8}\right)$	$\frac{4\pi}{8}-\frac{4\pi}{8}$ (0)
$\cot\theta$	und	1	0	-1	und
$\frac{1}{2}\cot\theta$	und	$\frac{1}{2}$	0	$-\frac{1}{2}$	und.



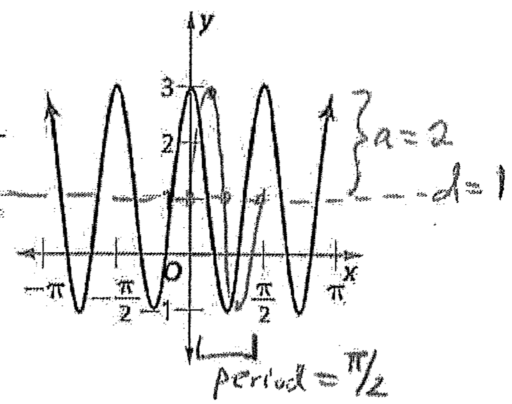
*period $= \frac{\pi}{b} = \frac{\pi}{2}$

$I = \frac{1}{4}\cdot P \rightarrow \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}$

Write two functions for each graph using the specified functions.

5. Using sine: $y=2\sin\left[4\left(\theta+\frac{\pi}{8}\right)\right]+1$

Using cosine: $y=2\cos[4(\theta)]+1$



*start w/ cosine graph

$a=2$ | period $= \frac{2\pi}{b}$ | $I = \frac{1}{4}\cdot P$

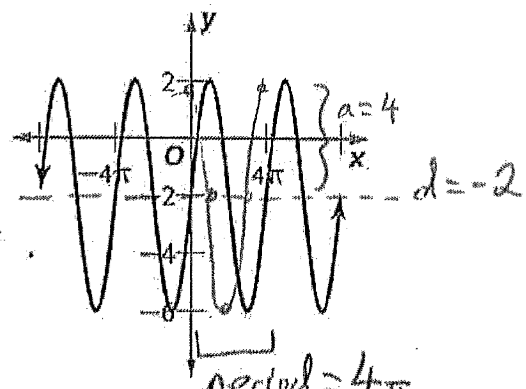
$b=$ ___ | $\frac{\pi}{2} = \frac{2\pi}{b}$ | $I = \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}$

$c=0$ |

$d=1$ | $b\pi = 4\pi \quad b=4$

6. Using sine: $y=4\sin\left[\frac{1}{2}(\theta)\right]-2$

Using cosine: $y=4\cos\left[\frac{1}{2}(\theta-\pi)\right]-2$



*start with sine equation:

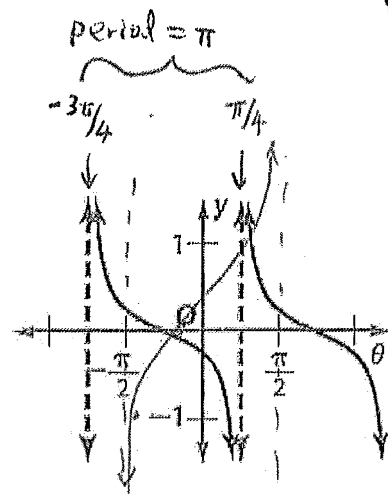
$a=4$ | period $= \frac{2\pi}{b}$ | $I = \frac{1}{4}\cdot P$

$b=\frac{1}{2}$ | $4\pi = \frac{2\pi}{b}$ | $I = \frac{1}{4}\cdot 4\pi = \pi$

$c=0$ |

$d=-2$ | $4\pi b = 2\pi$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$



7. Using tangent: $y = -\tan\left[\theta + \pi/4\right]$

Using cotangent: $y = \cot\left[\theta - \frac{\pi}{4}\right] + 0$

*start with $\cot\theta$

$a = 1$
$b = 1$
$c =$
$d = 0$

8. True or False, and explain why: Every sine function of the form $y = a\sin b(x - c) + d$ can also be written as a cosine function of the form $y = a\cos b(x - c) + d$.

True, since every cosine and sine functions are shifted versions of each other

$* \cos(\theta) = \sin(\theta + \pi/2)$

9. True or False, and explain why: The period of $f(x) = \cos 8\theta$ is equal to four times the period of $g(x) = \cos 2\theta$.

$f(x) = \cos(8\theta)$
$b = 8$
$\text{period} = \frac{2\pi}{b} \rightarrow \frac{2\pi}{8} = \frac{\pi}{4}$

$g(x) = \cos(2\theta)$
$b = 2$
$\text{period} = \frac{2\pi}{b} \rightarrow \frac{2\pi}{2} = \pi$

False
Since $\cos 2\theta$ has period π, $\cos(2\theta)$ has period 4 times that of $\cos(8\pi) \rightarrow (\pi/4)$

10. True or False, and explain why: If $x = \theta$ is an asymptote of $y = \csc x$, then $x = \theta$ is also an asymptote of $y = \cot x$.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\csc x$	0		1		0
$\cot x$	0 und				0 und

* from sin x

$\csc x = \frac{1}{\sin x}$

$\cot x = \frac{\cos x}{\sin x}$

whenever $\sin x = 0$, both csc x and cot x are undefined, therefore they share same asymptotes.

11. How many zeros does $y = \cos 1500\theta$ have on the interval from $0 \leq \theta \leq 2\pi$?

$b = 1500$

* For context, $\cos(\theta)$ has 2 zeros
→ $\cos(2\theta)$ means graph will complete 2 full cycle in 2π → means $2 \times 2 = 4$ zeros

$\cos(1500\theta)$ means 1500 cycle within (2π)

$1500 \times 2 = \boxed{3000 \text{ zeros}}$

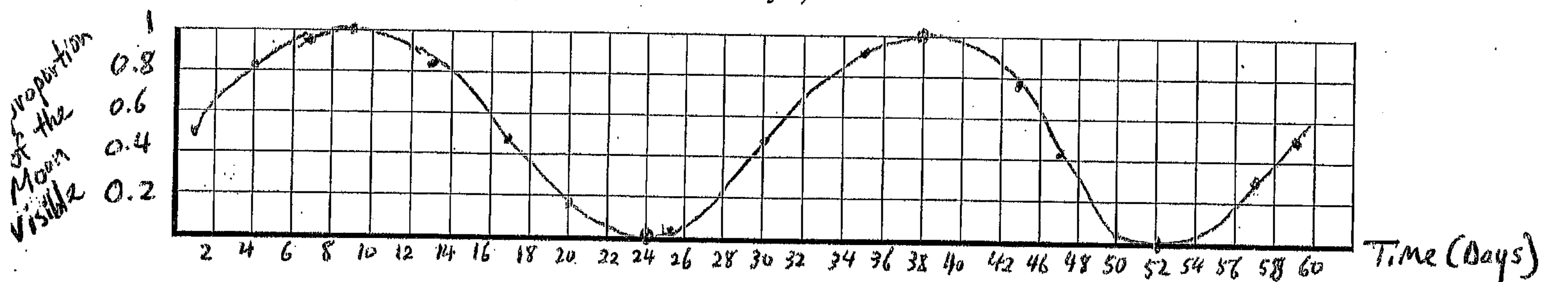
2.15 Applications of Sinusoidal Functions Day 1 **Date:** ______________

Problem 1: Moon Watch

As an avid sky-watcher, you know that the moon is always half illuminated by the sun and that how much of the moon we can see illuminated depends on where it is in its orbit around Earth. For the first months of the year, you watched the moon and kept data for the amount of the moon that was visible each day.

Day of Year	1	4	7	9	13	17	20	24	27	30	35	38	43	47	50	52	57	59
Proportion of Moon Visible	0.5	0.81	0.98	1	0.82	0.46	0.17	0	0.16	0.5	0.92	1	0.79	0.41	0.12	0	0.33	0.5

Part 1. Graph this relationship. (Hint: scale the x-axis by 2)



Part 2. a) Explain the practical meaning of each of the following characteristics.
b) Find the value of each characteristic for this application.

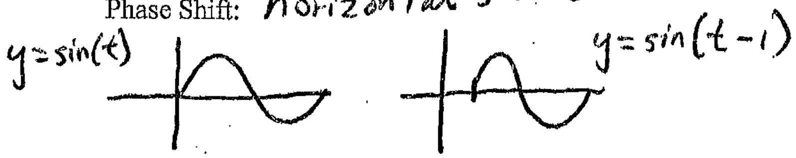
Sinusoidal Function: smooth curve with a repeating oscillating pattern around the midline. (comes from the word "sine")

Period: time period needed to complete a full cycle.
$P = 30 - 1 = 29$ days

Vertical Shift: midline shift (midline = average value). midline is $y = 0.5$
$d = 0.5$

Amplitude: distance from midline to the peak (or to minimum)
$a = 0.5$

Phase Shift: horizontal shift



Part 3. Write the equation of the sinusoidal function that models the visibility of the moon.

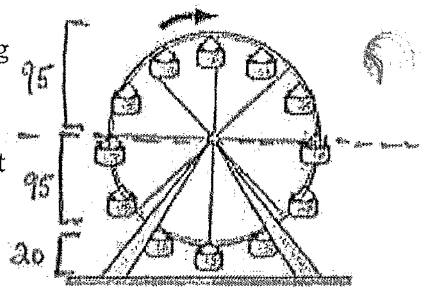
$a = 0.5$
period $= 29$
period $= \frac{2\pi}{b}$

$29 = \frac{2\pi}{b}$
$29b = 2\pi$
$b = \frac{2\pi}{29}$

$y = a\sin[b(x-c)] + d$
$y = 0.5\sin\left[\frac{2\pi}{29}(x-1)\right] + 0.5$

Problem 2: A Day at the Amusement Park

There are many rides at the amusement park whose movement can be modeled using trigonometric functions. The Ferris wheel is a good example of periodic movement.



You want to ride the Ferris wheel. It has a radius of 95 feet and is suspended 20 feet above the ground. The wheel rotates at a rate of 2 revolutions every 20 minutes.

Part 1.

a) What is the period of the function that models the movement of the Ferris wheel?

1 revolution = 10 mins. $p = 10$ mins. period $= \frac{2\pi}{b}$ | $10 = \frac{2\pi}{b}$ | $10b = 2\pi$ | $b = \frac{2\pi}{10}$ | $b = \pi/5$

b) What is midline of the function? What is the practical meaning of midline?

$d = 20 + 95 = 115$
$d = 115$ $y = 115$ | midline is the average height during the ride.

c) What is amplitude of the function? What is the practical meaning of amplitude?

Amplitude is 95 ft. This is the height above/below the midline (avg. height of ferris wheel)

d) Write the equation of a sinusoidal function that models your height above the ground over time.

You enter the ride at the lowest point (20 ft) and exit at the same location.

$a = -95$ $c = 0$
$b = \pi/5$ $d = 115$

$$\boxed{f(t) = -95\cos\left[\pi/5(t)\right] + 115}$$

or

$$f(x) = -95\cos\left(\frac{\pi}{5}x\right) + 115$$

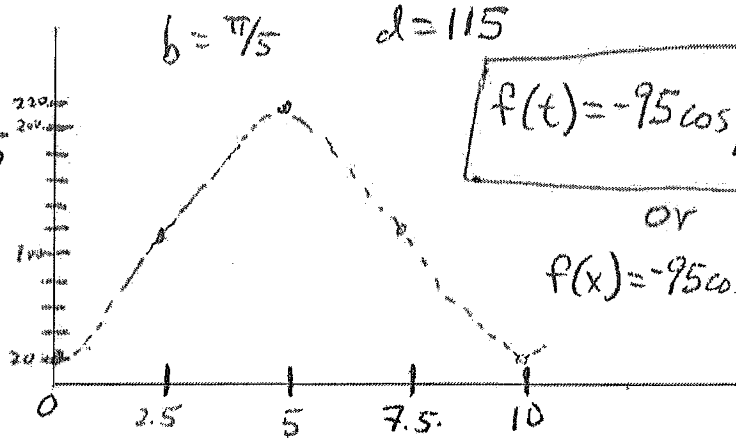
e) Graph the function:

period = 10

$I = \frac{1}{4} \cdot P = \frac{1}{4}(10) = \frac{5}{2} = 2.5$

(time, height)

(0, 20) | (5, 210)
(2.5, 115) | (7.5, 115)
| (10, 20)



Part 2.

a) Determine your height above the ground at 6 minutes.

$f(6) = -95\cos\left(\frac{\pi}{5}(6)\right) + 115 \approx 191.85 \approx \boxed{192 \text{ ft.}}$

b) Suppose the Ferris wheel moved faster and completed a revolution in 5 minutes. How would the function change? (Rewrite the equation reflecting this change).

$p = 5$ | $5 = \frac{2\pi}{b}$ | $5b = 2\pi$ | $b = \frac{2\pi}{5}$ | $y = -95\cos\left(\frac{2\pi}{5}t\right) + 115$

c) If the radius of the Ferris wheel remained the same, but the height of the wheel was raised 15 feet higher, how would the function change? (Rewrite the equation reflecting this change).

midline would change

$d = 115 + 15 = 130$

$y = -95\cos\left(\frac{\pi}{5}t\right) + 130$

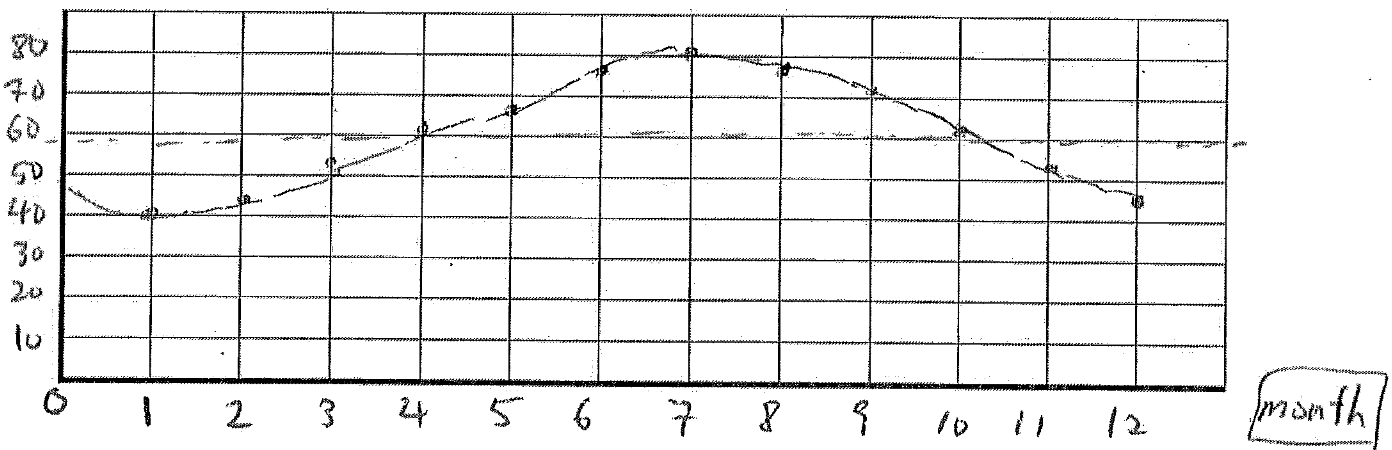
Homework: Global Warming?

Scientists are continually monitoring the average temperatures across the globe to determine if Earth is experiencing climate change. One statistic scientists use to describe the climate of an area is average monthly temperature. The average monthly temperature of a region is the mean of its average high and low temperatures.

The table below shows the average monthly temperature (°F) in Atlanta from January to December.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
40°	44°	53°	61°	68°	76°	80°	78°	72°	62°	53°	45°

Part 1. Graph this relationship. Use January as month 1 of the year, meaning that for January, $t = 1$.



Part 2.

a) What is the period of the function that models the monthly temperatures in Atlanta? Find b.

period $= 12$ $\quad 12 = \frac{2\pi}{b}$ | $12b = 2\pi$ $\quad b = \frac{2\pi}{12} = \frac{\pi}{6}$

b) What is the vertical shift of this model? Find ~~k~~ (d)

$d = 60$

c) What is the amplitude of this model? Find a.

$a = 20$

d) What is the phase shift of this model? Find c.

$c = 1$

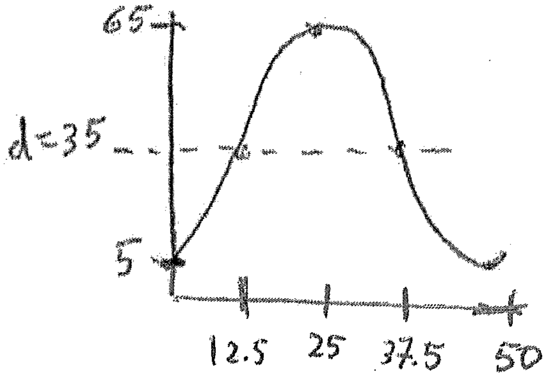
e) Write the equation of a sinusoidal function that models the average monthly temperatures.

$$y = -20\cos\left[\frac{\pi}{6}(x-1)\right] + 60$$

f) Use the equation to predict the temperature in October.

$$y = -20\cos\left[\frac{\pi}{6}(10-1)\right] + 60$$ $\boxed{y \approx 60}$

7. A Ferris wheel 60 ft in diameter makes one revolution every 50 seconds. If the center of the wheel is 35 feet above the ground, how long after reaching the low point is a rider 50 ft. above the ground? Write the function and the time. Show all your work.



period = 50

$50 = \frac{2\pi}{b}$

$50b = 2\pi$

$b = \frac{2\pi}{50} = \frac{\pi}{25}$

$I = \frac{1}{4} \cdot P$

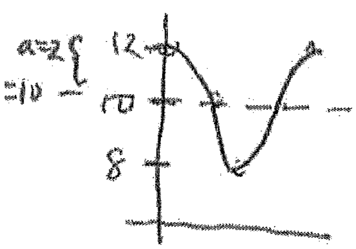
$I = \frac{1}{4} \cdot 50 = 12.5$

7. Function: $h(t) = -30\cos\left(\frac{\pi}{25}t\right) + 35$

Time: ≈ 17 secs.

8. Ebb and Flow: on a particular Labor Day, the high tide in South California occurs at 7:15 am. At that time, you measure the water at the end of the Santa Monica Pier to be 12 feet deep. At 1:36 pm, it is low tide and you measure the water to be only 8 feet deep. What is the depth at noon? Write the function and the depth.

$a = 2$



period = 381 + 381 = 762

$762 = \frac{2\pi}{b}$

$b = \frac{2\pi}{762} = \frac{\pi}{381}$

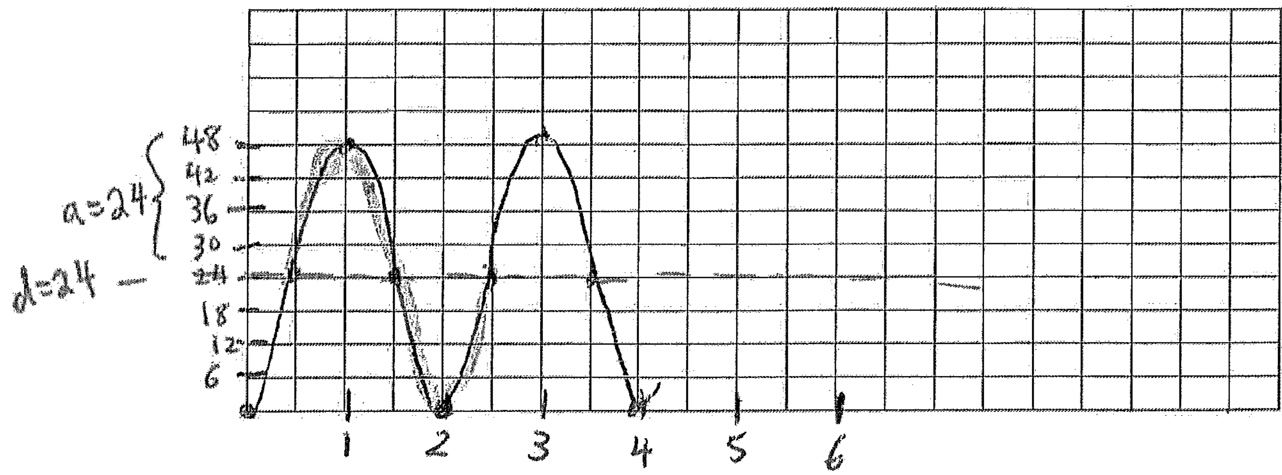
8. Function: $h(t) = 2\cos\left(\frac{\pi}{381}t\right) + 10$

Depth: $h(285) \approx 8.595$ ft.

noon

5. Sam is riding his bike one day and picks up a nail in his tire. The nail hits the ground every 2 seconds and reaches a maximum height of 48 cm (assume the tire does not deflate).

a) Use the information above to sketch a diagram of this sinusoidal movement.



period = 2

$I = \frac{1}{4} \cdot P$

$I = \frac{2}{4} = \frac{1}{2}$

$2 = \frac{2\pi}{b}$

$2b = 2\pi$

$b = \pi$

b) Write a cosine function that describes the situation in part a.

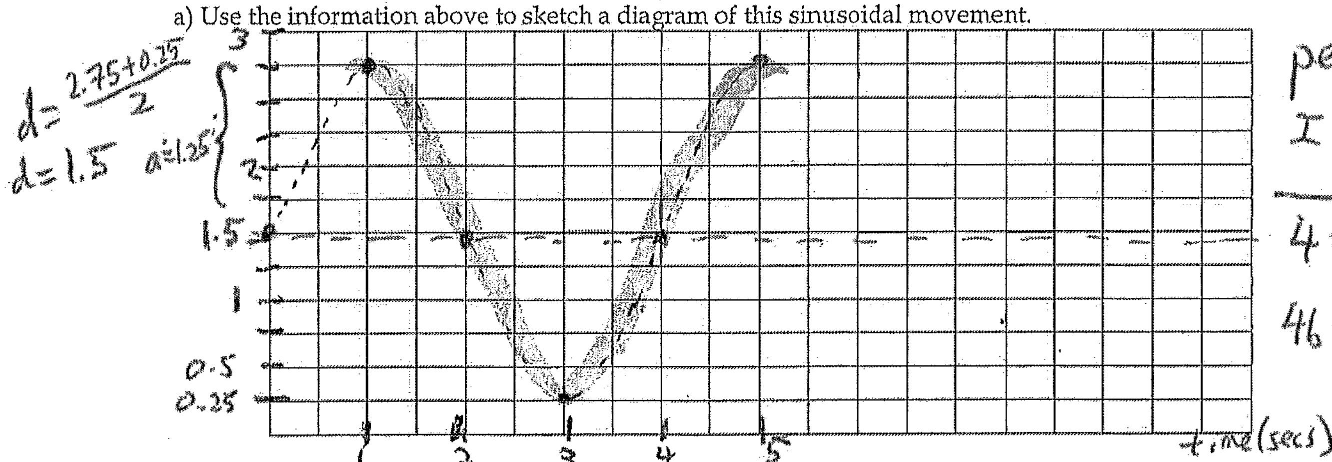
b) $h(t) = -24\cos(\pi t) + 24$

c) Write a sine function that describes the situation in part a.

c) $h(t) = 24\sin\left[\pi\left(t - \frac{1}{2}\right)\right] + 24$

6. Jackie, Nicolle and Maegan are playing skip rope. As the rope rotates it is observed that its maximum height is 2.75m after 1 second. The first minimum height of 0.25m occurs 2 seconds after the maximum height.

a) Use the information above to sketch a diagram of this sinusoidal movement.



period = 4

$I = \frac{1}{4} \cdot 4 = 1$

$4 = \frac{2\pi}{b}$

$4b = 2\pi$

$b = \frac{\pi}{2}$

b) Write a cosine function that describes the situation in part a.

b) $h(t) = 1.25\cos\left[\frac{\pi}{2}(t-1)\right] + 1.5$

c) Write a sine function that describes the situation in part a.

c) $h(t) = 1.25\sin\left(\frac{\pi}{2}t\right) + 1.5$

d) What is the height of the rope at 2.75 seconds?

d) $h(2.75) = 0.345$ m

$1 = \frac{2\pi}{b} \quad b = 2\pi$

2.16 Sinusoidal Modeling Day 2

Date Key

1. A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, write a sine function that models the person's blood pressure.

period $= 1$ $a = 30$ 140   80 $d = 110$ $a = 30$

period $= \frac{2\pi}{b}$

$a = 30$, $b = 2\pi$, $c = 0$, $d = 110$

$y = 30\sin(2\pi x) + 110$

2. A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by $h = -3\cos\left(\frac{5\pi}{3}t\right) + 3.5$, where t is the time measured in seconds. $a = 3$ $d = 3.5$

   a. What is the highest point reached by the knot? 6.5 feet

   b. What is the lowest point reached by the knot? 0.5 feet

   $a = 3$, $a = 3$, 6.5, 0.5, $d = 3.5$

   c. What is the period of the model?

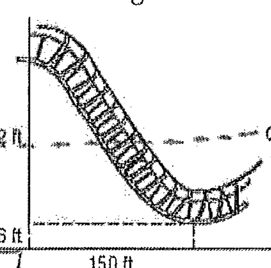
   $\text{period} = \frac{2\pi}{5\pi/3} \rightarrow 2\pi \cdot \frac{3}{5\pi} = 6/5 = 1.2 \text{ seconds}$

   d. According to the model, find the height of the knot after 25 seconds.

   $h(25) = -3\cos\left(\frac{5\pi}{3} \cdot 25\right) + 3.5 = \boxed{2 \text{ feet}}$

3. Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically as shown. The low point is 6 feet above the ground.

   a. Write a sinusoidal function that models the distance the roller coaster track is above the ground at a given horizontal distance x.



$a = 41$, 41, $d = 47$

150 feet is half the period (half the cycle)

period $= 300$ period $= \frac{2\pi}{b}$ $300 = \frac{2\pi}{b}$ $300b = 2\pi$

$b = \frac{2\pi}{300} = \frac{\pi}{150}$ $a = 41$ $c = 0$ $d = 47$

$\boxed{y = 41\cos\left(\frac{\pi}{150}x\right) + 47}$

   b. Point A is 40 feet to the right of the *y-axis*. How far above the ground is the track at point A?

   $y(40) = 41\cos\left(\frac{\pi}{150} \cdot 40\right) + 47 \approx \boxed{74.434 \text{ feet}}$

4. A buoy, bobbing up and down in the water as waves pass it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

   $a = 1.5$ 3 feet

   a. Determine the amplitude and period of a sinusoidal function that models the bobbing buoy.

   $a = 1.5$ | period $= 10$ | period $= \frac{2\pi}{b}$ | $10 = \frac{2\pi}{b}$ | $10b = 2\pi$, $b = \frac{2\pi}{10} \rightarrow b = \frac{\pi}{5}$

   b. Write an equation of a sinusoidal function that models the bobbing buoy, using $x = 0$ as its highest point.

   *use cosine $\boxed{y = 1.5\cos\left(\frac{\pi}{5}x\right)}$

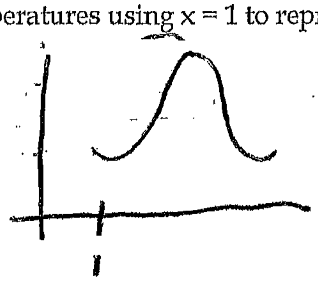
   $a = 1.5$, $b = \pi/5$, $c = 0$, $d = 0$

5. The average monthly temperatures for Baltimore, Maryland, are shown below.

Month	Temperature (°F)	Month	Temperature (°F)
Jan	32	July	77
Feb	35	Aug	76
Mar	44	Sept	69
Apr	53	Oct	57
May	63	Nov	47
June	73	Dec	37

a. Determine the amplitude, period, phase shift, and vertical shift of a sinusoidal function that models the monthly temperatures using x = 1 to represent January.

$a = 22.5$ 77, 32, $d = 54.5$



phase shift: $c = 1$

$p = 12$

$p = \frac{2\pi}{b}$

$12 = \frac{2\pi}{b}$

$12b = 2\pi$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

b. Write an equation of a sinusoidal function that models the monthly temperatures.

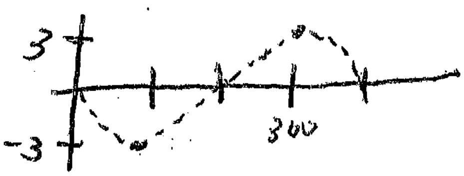
$$y = -22.5\cos\left[\frac{\pi}{6}(x-1)\right] + 54.5$$

c. According to your model, what is Baltimore's average temperature in July? In December?

$y(7) \approx 77°$ $y(12) \approx 35°$

6. One day all 322 million people in the United States climb up on tables. We all jump off and land on the floor/ground at $t = 0$. The resulting shock as we hit Earth's surface will start the entire earth vibrating in such a way that its surface first moves down from its normal position. At $t = 300$ milliseconds, it has moved up an equal distance above its normal position. The total difference in Earth's vertical position is 6 mm. For an extremely brief amount of time, the vertical position of the surface closely resembles a sinusoidal function.

a. Sketch a graph of Earth's vertical movement over time for this scenario.



b. At what time will the first minimum occur? 100 millisecond

c. What is the period of this function? 400 millisecond

d. What is the amplitude of this function? $a = 3$

e. Write an equation of a sinusoidal function that models Earth's vertical position.

$$d(t) = -3\sin\left[\frac{\pi}{200}(t)\right]$$

period $= \frac{2\pi}{b}$

$400 = \frac{2\pi}{b}$

$400b = 2\pi$

$b = \frac{2\pi}{400}$

$b = \frac{\pi}{200}$

f. If the wave were to continue, predict the vertical position of Earth 3.25 seconds after our jump.

$t = 3250$ milliseconds

$d(3250) \approx -2.121$

$= 2.121$ mm below normal

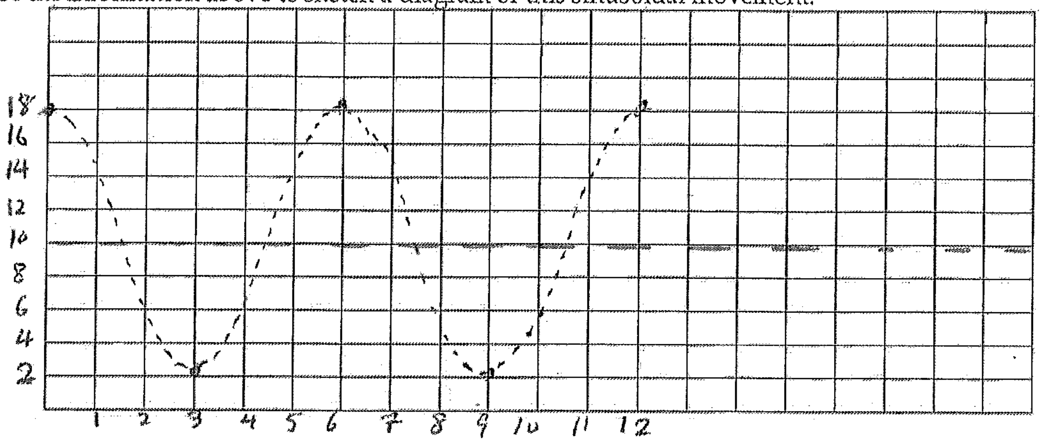
2.17 Sinusoidal Modeling Day 3 Date: ____________

1. In Canada's wonderland there is a roller coaster that is a continuous series of identical hills that are 18m high from the ground. The platform to get on the ride is on top of the first hill. It takes 3 seconds for the coaster to reach the bottom of the hill 2m off the ground.

   a) Use the information above to sketch a diagram of this sinusoidal movement.

$a=8$

$d=10$



period $= 6$

period $= \frac{2\pi}{b}$

$6 = \frac{2\pi}{b}$

$6b = 2\pi$

$b = \frac{2\pi}{6} = \frac{\pi}{3}$

   b) Write a cosine function, h(t), that describes the situation in part a.

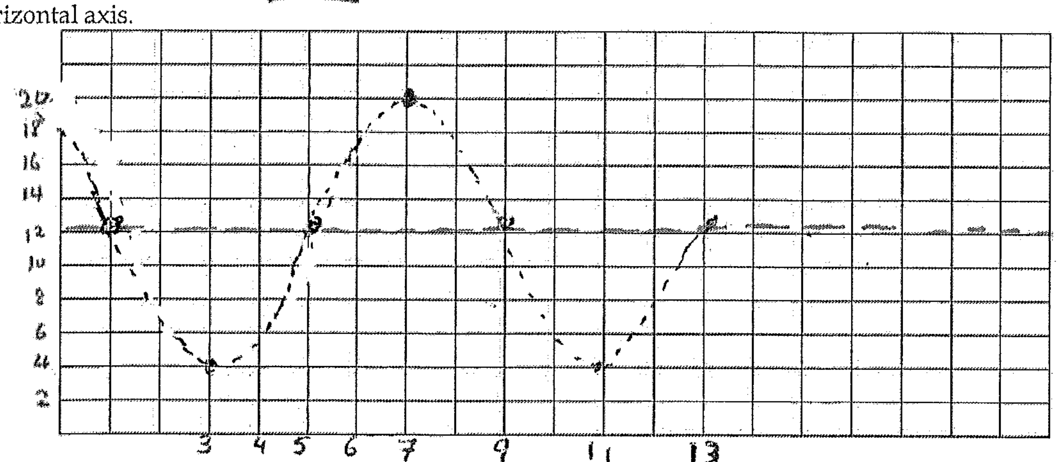
b) $h(t) = 8\cos\left(\frac{\pi}{3}t\right) + 10$

   c) Determine the height of the rider at 11 seconds. Answer in function notation.

c) $h(11) = 14$ ft.

2. Mr. Jones, disguised as Mathman, a costumed crime fighter, is swinging back and forth in front of the window for the Front Office. At t = 3 sec, he is at one end of his swing and 4m from the window. At t = 7 sec, he is at the other end of his swing and 20m from the window.

   a) Sketch the curve. Use the distance from the window on the vertical axis and the time in seconds along the horizontal axis.



period $= 8$

$8 = \frac{2\pi}{b}$

$8b = 2\pi$

$b = \frac{2\pi}{8} = \pi/4$

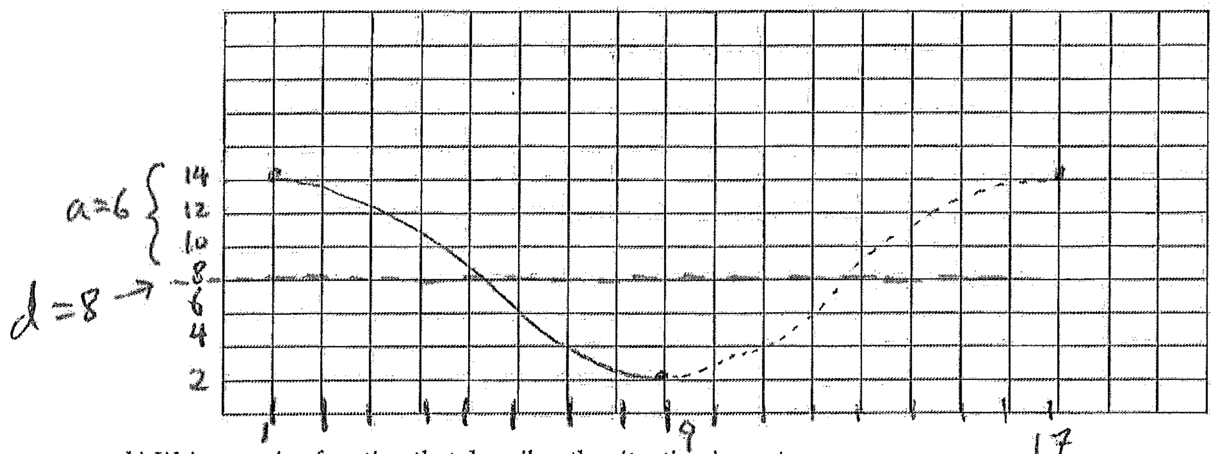
   b) Write a sine function that describes the situation in part a.

b) $d(t) = 8\sin\left[\pi/4(t-5)\right] + 12$

   c) When is the first time Mathman reaches 16m?

c) $t \approx 1/3$ second.

3. John is floating on a tube in a wave tank. At t = 1 second, John reaches a maximum height of 14m above the bottom of the pool. At t = 9 seconds, John reaches a minimum height of 2m above the bottom of the pool
a) Use the information above to sketch a diagram of this sinusoidal movement.



period = 8+8 = 16

$16 = \frac{2\pi}{b}$

$16b = 2\pi$

$b = \frac{2\pi}{16} = \frac{\pi}{8}$

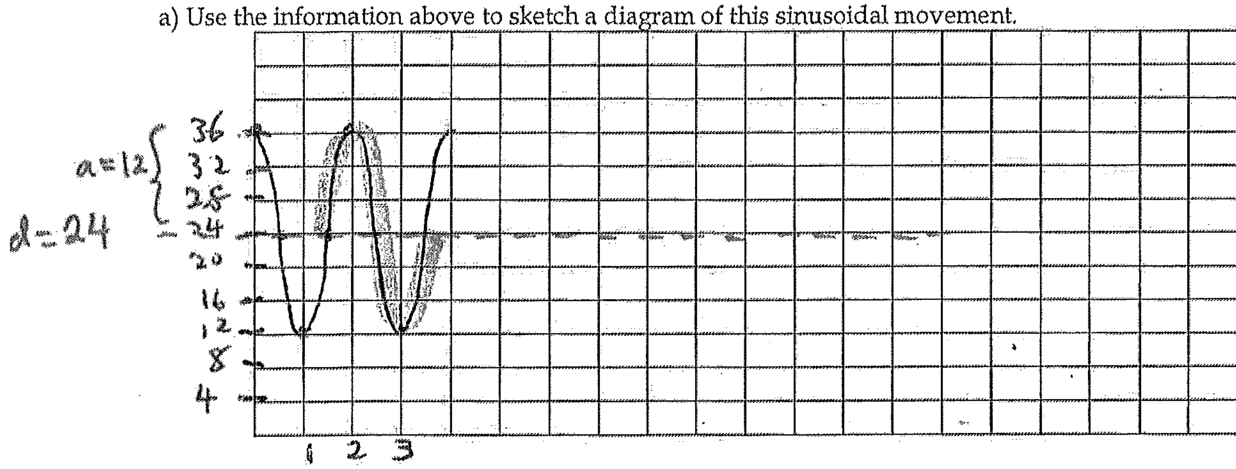
b) Write a cosine function that describes the situation in part a.

b) $h(t) = 6\cos\left[\frac{\pi}{8}(t-1)\right] + 8$

c) What is John's height from the bottom of the pool at 21 seconds? c) 8m

4. A pendulum on a grandfather clock is swinging back and forth as it keeps time. A device is measuring the distance the pendulum is above the floor as it swings back and forth. At the beginning of the measurements the pendulum is at its highest point, 36 cm high exactly one second later it was at its lowest point of 12 cm. One second later it was back to its highest position.

a) Use the information above to sketch a diagram of this sinusoidal movement.



period = 2

$I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot 2 = \frac{1}{2}$

$2 = \frac{2\pi}{b}$

$2b = 2\pi$

$b = \pi$

b) Write a cosine function that describes the situation in part a.

b) $h(t) = 12\cos(\pi t) + 24$

c) Write a sine function that describes the situation in part a.

$h(t) = -12\sin[\pi(t-0.5)] + 24$

c) $h(t) = 12\sin[\pi(t-1.5)] + 24$

d) When will the pendulum be at 15 cm if 2 < t < 3?

d) ≈ 2.7 secs.

Key

Unit 2 Graphing Trig Morning Review

1. $y = -2\tan\left(2\theta - \frac{\pi}{4}\right) - 1$ $\qquad y = -2\tan\left[2\left(\theta - \frac{\pi}{8}\right)\right] - 1$

vertical stretch: 2

period: $\pi/2$

phase shift: right $\pi/8$

vertical shift: down 1

$a = -2$, $b = 2$, $c = \pi/8$, $d = -1$

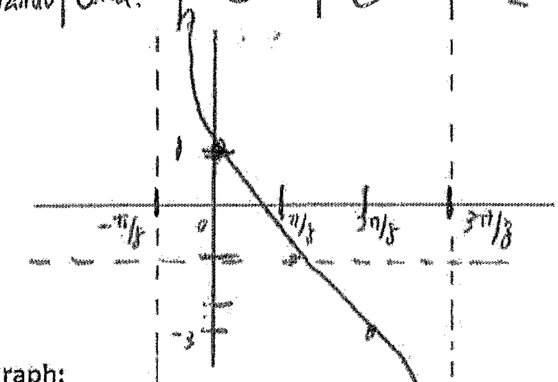
$\text{period} = \frac{\pi}{b}$

$\text{period} = \frac{\pi}{2}$

$I = \frac{1}{4}\cdot P \rightarrow \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}$

Graph:

	$-\pi/8$	0	$\pi/8$	$2\pi/8$	$3\pi/8$
θ	$-\frac{2\pi}{8}+\frac{\pi}{8}$	$-\frac{\pi}{8}+\frac{\pi}{8}$	$0+\pi/8$	$\frac{\pi}{8}+\pi/8$	$\frac{2\pi}{8}+\pi/8$
$\tan 2\theta$	und.	-1	0	1	und.
$-2\tan 2\theta$	und.	2	0	-2	und.



2. $y = -2\sec\left(\frac{\theta}{4} + \frac{3\pi}{2}\right) + 1$

vertical stretch: 2

period: 8π

phase shift: left 6π

vertical shift: up 1

$y = -2\sec\left[\frac{1}{4}(\theta + 6\pi)\right] + 1$

$a = -2$, $b = 1/4$, $c = 6\pi$, $d = 1$

$\text{period} = \frac{2\pi}{b}$

$\text{period} = \frac{2\pi}{1/4} = 2\pi\cdot\frac{4}{1} = 8\pi$

$I = \frac{1}{4}\cdot P$

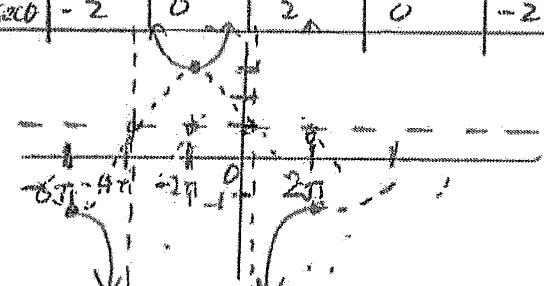
$I = \frac{1}{4}\cdot 8\pi$

$I = 2\pi$

Graph:

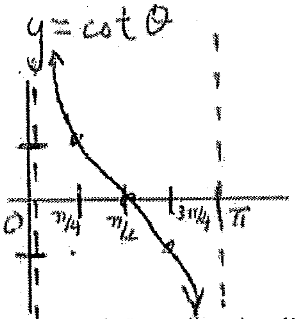
* $\cos\theta$

	-6π	-4π	-2π	0	2π
θ	$0-6\pi$	$2\pi-6\pi$	$4\pi-6\pi$	$6\pi-6\pi$	$8\pi-6\pi$
$\sec\theta$	1	0	-1	0	1
$-2\sec\theta$	-2	0	2	0	-2

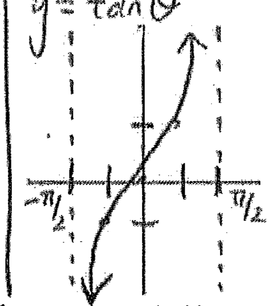


3. Given: y = tan θ. Write a function of cotangent that would produce the same graph.

$y = \cot\theta$



$y = \tan\theta$



$y = -\cot\left(\theta + \pi/2\right)$

$a = 1$, $b = 1$, $c = \_\_$, $d = 0$

4. Describe the difference in period between y = 2cos(2θ) *with that of* y = 2cos(7θ).

$y = 2\cos(2\theta)$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{2}$

$\text{period} = \pi$

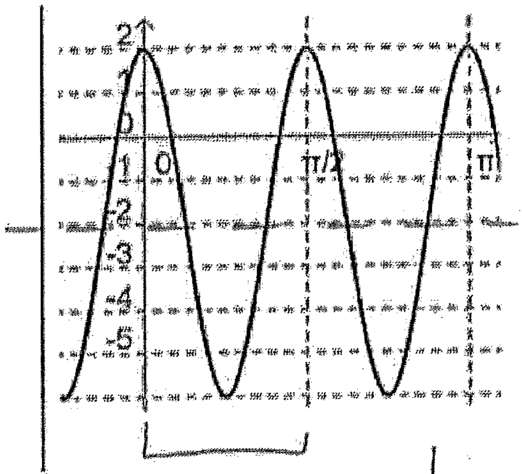
$y = 2\cos(7\theta)$

$\text{period} = \frac{2\pi}{b} \rightarrow \frac{2\pi}{7}$

$\text{period} = \frac{2}{7}\pi$

$y = 2\cos(7\theta)$ has a period that is $\frac{2}{7}$ that of $y = 2\cos(2\theta)$

5. Write two functions for the graph below; one for cosine and one for sine.



$a = 4$

$d = -2$

a) $y = 4\cos(4\theta) - 2$

b) $y = 4\sin[4(\theta + \pi/8)] - 2$

period $= \pi/2$

$\frac{\pi}{2} = \frac{2\pi}{b}$

$b\pi = 4\pi$

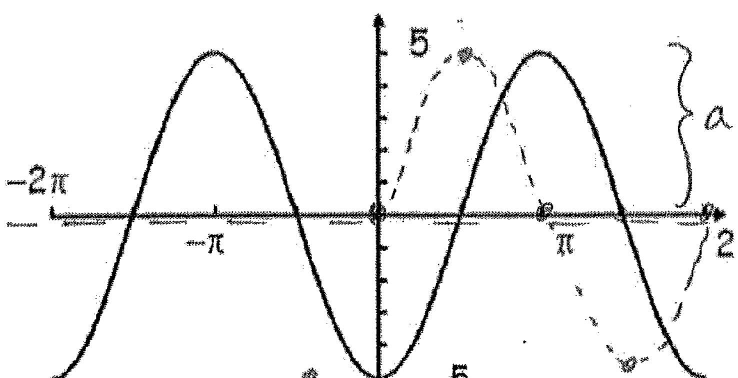
$b = 4$

$I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot \pi/2$

$I = \pi/8$

6. Write two functions for the graph below; one for cosine and one for sine.



$a = 5$

$d = 0$

period $= 2\pi$

$a = -5$

$b = 1$

$c = 0$

$d = 0$

period $= \frac{2\pi}{b} \rightarrow \frac{2\pi}{1} = 2\pi$

$\boxed{y = -5\cos(\theta)}$

$I = \frac{1}{4} \cdot P \rightarrow \frac{1}{4} \cdot 2\pi = \pi/2$

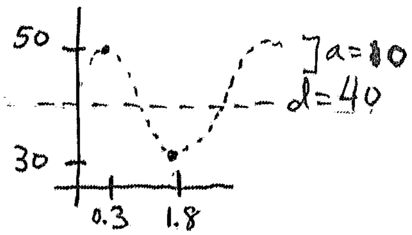
$\boxed{y = 5\sin(\theta - \pi/2)}$

GSE Accel. Precalculus   **Sinusoidal Modeling Quiz Review WS #2**

Key

1) A weight is attached to the end of a spring. As it bounces, its distance from the floor varies sinusoidally with time. When the stopwatch reads 0.3 seconds, the weight reaches a high point at 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 seconds.

Write an equation that models the height of the weight as a function of time.



1.5 is half the period

period = 3

$\text{period} = \frac{2\pi}{b}$

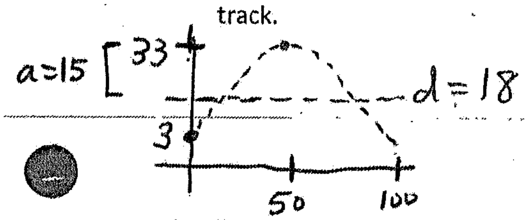
$3 = \frac{2\pi}{b}$

$b = \frac{2\pi}{3}$

$$\boxed{y = 10\cos\left[\frac{2\pi}{3}(x - 0.3)\right] + 40}$$

2) A portion of a roller coaster track is to be built in the shape of one full period of a sinusoid (from one minimum point to the next minimum point). You have been hired to calculate the lengths of the horizontal and vertical timer supports. The high and low points on the track are separated by 50 meters horizontally and 30 meters vertically. The low point is 3 meters above the ground.

a. Let the first minimum point be found at (0,3). Write an equation that models the path of the track.



period = 100

$100 = \frac{2\pi}{b}$

$b = \frac{2\pi}{100} = \frac{\pi}{50}$

$$y = -15\cos\left(\frac{\pi}{50}x\right) + 18$$

b. A vertical support is needed at $x = 10$ meters. How tall should this support be?

$$y = -15\cos\left(\frac{\pi}{50}\cdot 10\right) + 18 = \boxed{5.86\text{m}}$$

3) As a wave passes by an offshore piling, the height of the water is modeled by the function

$$h(t) = 3\cos\left(\frac{\pi}{10}t\right)$$

where $h(t)$ is the height in feet above mean sea level at time t seconds.

a. Find the period of the wave. Using correct units, explain what this value represents.

$\text{period} = \frac{2\pi}{b}$

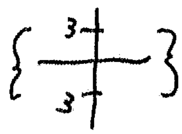
$= \frac{2\pi}{\pi/10}$

$= 2\pi \cdot \frac{10}{\pi} = 20$

period = 20 seconds. for the time period between 2 waves.

b. Find the wave height (the vertical distance between the trough and the crest of the wave)

amplitude = 3



$3 + 3 = \boxed{6 \text{ feet}}$

4) A tuning fork is struck, producing a pure tone as its tines vibrate. The vibrations are modeled by the function

$$v(t) = 0.7\sin(880\pi t)$$

where $v(t)$ is the displacement of the tines in millimeters at time t seconds.

Find the period of the vibration.

$\text{period} = \frac{2\pi}{880\pi} = \boxed{\frac{1}{440} \text{ second}}$

5) At a particular beach on August 2, you find that at 2:00 pm (high tide), the depth of the water at the end of a pier is 1.5 meters. At 7:30 pm (low tide), the depth of the water is 1.1 meters. Assume that the depth varies sinusoidally with time.

a. Find an equation expressing depth as a function of the time that has elapsed since 12:00 am midnight at the beginning of August 2.

2pm

max $(14, 1.5)$

min $(19.5, 1.1)$

7:30pm

1.5, 1.1, $d = 1.3$, $a = 0.2$

$19.5 - 14 = 5.5$ is half the period

period $= 11$

$11 = \frac{2\pi}{b}$

$b = \frac{2\pi}{11}$

$\boxed{y = 0.2\cos\left[\frac{2\pi}{11}(x-14)\right] + 1.3}$

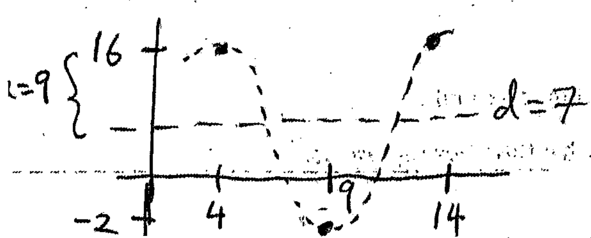
b. Use this model to predict the depth of the water at 5:00 pm on August 3.

$t = 17 + 24 = 41$

$y(41) = 0.2\cos\left[\frac{2\pi}{11}(41-14)\right] + 1.3 = \boxed{1.108\text{m}}$

6) Mark Twain sat on the deck of a river steamboat. As the paddle wheel turned, a point on the paddle blade moved so that its distance, d, from the water's surface was a sinusoidal function of time. When Twain's stopwatch read 4 s, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft, and it completed a revolution every 10 s.

a. Sketch the graph of the sinusoid.



period $= 10$

$10 = \frac{2\pi}{b}$

$10b = 2\pi$

$b = \frac{2\pi}{10}$

$b = \frac{\pi}{5}$

b. Find a sinusoidal function that models the height d as a function of time t.

$d = 9\cos\left[\frac{\pi}{5}(t-4)\right] + 7$

c. How far above the surface was the point when the stopwatch read 17 s?

$d(17) = 9\cos\left[\pi/5(17-4)\right] + 7 = \boxed{4.2 \text{ feet}}$

Domain / Range:

$\sin\theta$ D $(-\infty, \infty)$ R $[-1, 1]$

$\cos\theta$ D $(-\infty, \infty)$ R $[-1, 1]$

$\csc\theta$ D: $\mathbb{R}$ except $x = \pi n$, $n \in \mathbb{Z}$
R: $(-\infty, -1] \cup [1, \infty)$

$\sec\theta$ D: $\mathbb{R}$ except $x = \frac{\pi}{2} + \pi n$ $n \in \mathbb{Z}$
R: $(-\infty, -1] \cup [1, \infty)$

$\tan\theta$ D: $\mathbb{R}$ except $x = \frac{\pi}{2} + \pi n$ $n \in \mathbb{Z}$
R: $(-\infty, \infty)$

$\cot\theta$ D: $\mathbb{R}$ except $x = \pi n$ $n \in \mathbb{Z}$
R: $(-\infty, \infty)$