AP Calc Review: Unit 3 Differentiation Application MC WS

Key

$2^3 - x(2)^2 = 4$, $8-4x=4$, $4=4x$, $\underline{x=1}$

*implicit
*product rule

1. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

(A) -2 (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ $\boxed{\text{(D)}}$ $\frac{1}{2}$ (E) 2

$3y^2\left(\frac{dy}{dx}\right) + \underbrace{-1}_{f'}\cdot\underbrace{y^2}_{g} + \underbrace{-x}_{f}\cdot\underbrace{2y\left(\frac{dy}{dx}\right)}_{g'} = 0$

$3y^2\left(\frac{dy}{dx}\right) - 2xy\left(\frac{dy}{dx}\right) = y^2$

$\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{2^2}{3(2)^2 - 2(1)(2)} = \boxed{\frac{1}{2}}$

2) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when → *vertical tangent when denominator of $\frac{dy}{dx} = 0$

(A) $y = 0$ (B) $y = \pm\sqrt{3}$ (C) $y = \frac{1}{2}$

$2y\left(\frac{dy}{dx}\right) + \underbrace{(-1)}_{f'}\cdot\underbrace{y}_{g} + \underbrace{(-x)}_{f}\underbrace{\left(\frac{dy}{dx}\right)}_{g'} = 0$

$\frac{dy}{dx}(2y - x) = y$

$\frac{dy}{dx} = \frac{y}{2y-x}$

$2y - x = 0$

$2y = x$

$y^2 - xy + 9 = 0$

$y^2 - (2y)(y) + 9 = 0$

$y^2 - 2y^2 + 9 = 0$

$-y^2 = -9$

$y^2 = 9$

$y = \pm 3$

$\boxed{\text{D}}$

3) The function $f(x) = x^4 - 4x^2$ has

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
$\boxed{\text{(E)}}$ two relative minima and one relative maximum

*1st Derivative Test

$f'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

$f'(x)$: $-$ $+$ $-$ $+$ (at -2, $-\sqrt{2}$, -1, 0, 1, $\sqrt{2}$, 2)

Rel. min: $x = -\sqrt{2}, \sqrt{2}$
Rel. max: $x = 0$

4) The number of inflection points of the curve in Question 3 is

(A) 0 (B) 1 $\boxed{\text{(C) 2}}$ (D) 3 (E) 4

* set $f''(x) = 0$
* create $f''(x)$ sign line

$f''(x) = 12x^2 - 8$

$0 = 4(3x^2 - 2)$

$3x^2 - 2 = 0$

$x = \pm\sqrt{\frac{2}{3}}$

$f''(x)$: $+$ $-$ $+$ (at -1, $-\sqrt{2/3}$, 0, $\sqrt{\frac{2}{3}}$, 1)

POI at $x = -\sqrt{\frac{2}{3}}$, $x = \sqrt{\frac{2}{3}}$

5) The maximum value of the function $y = -4\sqrt{2 - x}$ is

$\boxed{\text{(A) 0}}$ (B) -4 (C) 2 (D) -2 (E) none of these

*Max value is y-value
*Find critical pt, set $f'(x) = 0$

$y = -4(2-x)^{1/2}$

$y' = -2(2-x)^{-1/2}(-1)$

$y' = \frac{2}{(2-x)^{1/2}}$

critical pt: $2 - x = 0$

$\underline{x = 2}$

$y(0) = -4\sqrt{2-2} = 0$

$\boxed{\text{Max value is } y = 0}$

at $x = 0$

* Related Rates #6-8

Find $\frac{dS}{dt}$ ^ *Related Rates

6) A balloon is being filled with helium at the rate of 4 ft³/min. The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft³ is

(A) 4π (B) 2 $\boxed{\text{(C) } 4}$ (D) 1 (E) 2π

$* V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3} \cdot 3\pi r^2 \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4$

$4 = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$4 = 4\pi (2)^2 \left(\frac{dr}{dt}\right)$

$\frac{1}{4\pi} = \frac{dr}{dt}$

$V = \frac{32\pi}{3}$

$\frac{32\pi}{3} = \frac{4}{3}\pi r^3$

$32\pi = 4\pi r^3$

$8 = r^3$

$2 = r$

$\frac{dS}{dt} = 8\pi (2)\left(\frac{1}{4\pi}\right)$

$\boxed{\frac{dS}{dt} = 4 \text{ ft}^2/\text{min}}$

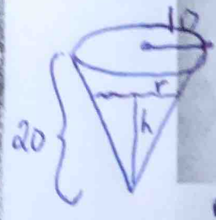
7) A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}$ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

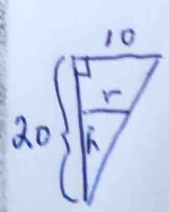
*Related Rates

$\frac{dh}{dt} = -\frac{1}{2}$

$h = 8$

(A) 4π $\boxed{\text{(B) } 8\pi}$ (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$





$\frac{r}{10} = \frac{h}{20}$

$10h = 20r$ $\boxed{r = \frac{h}{2}}$

$10\left(\frac{dh}{dt}\right) = 20\left(\frac{dr}{dt}\right)$

$\cancel{10\left(-\frac{1}{2}\right) = 20\left(\frac{dr}{dt}\right)}$

$\cancel{-5 = 20\left(\frac{dr}{dt}\right)}$

$\cancel{\frac{-5}{20} = -\frac{1}{4} = \frac{dr}{dt}}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{4} \cdot h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{3\pi}{12} (8)^2 \left(-\frac{1}{2}\right)$

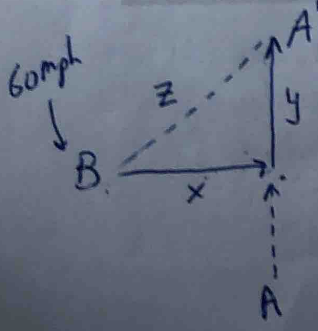
$\frac{dV}{dt} = -8\pi \text{ ft}^3/\text{hr}$

water is leaving tank at rate of 8π ft³/hr

8) *Related Rates

Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon, when car A reaches the intersection, car B is 90 mi away, and moving toward it. At 1 P.M. the rate, in miles per hour, at which the distance between the cars is changing is

(A) -40 (B) 68 (C) 4 $\boxed{\text{(D) } -4}$ (E) 40



$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 30$ $\frac{dx}{dt} = -60$

$y = 40$ $\frac{dy}{dt} = 40$

$z = 50$

$2(30)(-60) + 2(40)(40) = 2(50)\left(\frac{dz}{dt}\right)$

$-3600 + 3200 = 100\left(\frac{dz}{dt}\right)$

$-4 = \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = -4 \text{ mph}}$

9) If $f(x) = ax^4 + bx^2$ and $ab > 0$, then

$f'(x) = 4ax^3 + 2bx \rightarrow$ when $x=0$, $f'(x)=0$ horizontal tangent at $x=0$.

$f''(x) = 12ax^2 + 2b$

$0 = 12ax^2 + 2b$

$\frac{-2b}{12a} = x^2 \rightarrow$ since a, b have same signs, there is no inflection point/critical pt.

(A) the curve has no horizontal tangents
(B) the curve is concave up for all x
(C) the curve is concave down for all x
$\boxed{\text{(D)}}$ the curve has no inflection point
(E) none of the preceding is necessarily true

10) A function f is continuous and differentiable on the interval [0,4], where f' is positive but f'' is negative. Which table could represent points on f?

* need positive slope and slope values decreasing from left to right

(A)

x	0	1	2	3	4
y	10	12	14	16	18

2 2 2 2

(B)

x	0	1	2	3	4
y	10	12	15	19	24

2 3 4 5

$\boxed{\text{(C)}}$

x	0	1	2	3	4
y	10	18	24	28	30

8 6 4 2

pos. slope decreasing from left to right

(D)

x	0	1	2	3	4
y	30	28	24	18	10

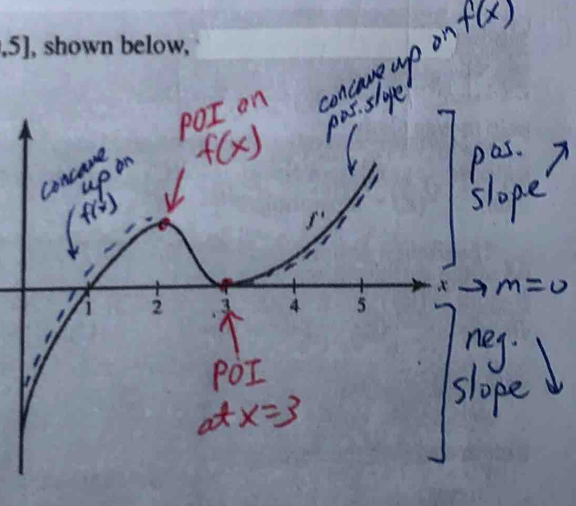
-2 -4

(E)

x	0	1	2	3	4
y	24	19	15	12	10

-5 -4 -3 -2

Use the graph of f' on [0,5], shown below,



$f'(x)$: $-$ on (0,1), $+$ on (1,3), $+$ on (3,5); marks at 0, 1, 3, 5

Rel. min at $x=1$

$f''(x)$: $+$ (∪) on (0,2), $-$ (∩) on (2,3), $+$ (∪) on (3,5); marks at 0, 2, 3, 5

POI at $x=2,3$

11) f has a local minimum at $x =$

(A) 0 $\boxed{\text{(B)}}$ 1 (C) 2 (D) 3 (E) 5

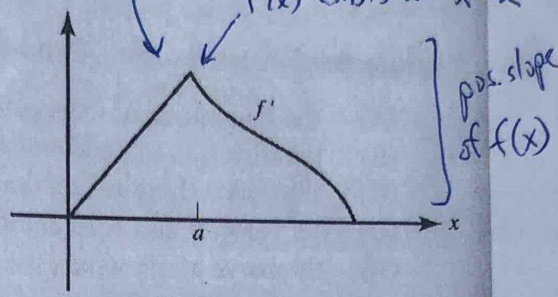
12) The graph of f has a point of inflection at $x =$

(A) 1 only (B) 2 only (C) 3 only
$\boxed{\text{(D)}}$ 2 and 3 only (E) none of these

peaks/valleys of $f'(x)$ graph is POI of $f(x)$.

13) It follows from the graph of f', shown at the right, that

(A) f is not continuous at $x = a$
(B) f is continuous but not differentiable at $x = a$
(C) f has a relative maximum at $x = a$
$\boxed{\text{(D)}}$ The graph of f has a point of inflection at $x = a$
(E) none of these

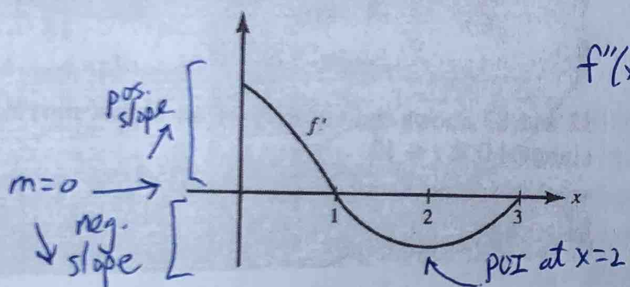


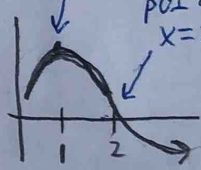
* Even though $f'(x)$ is not differentiable at $x=a$, $f(x)$ is differentiable everywhere. Since $f'(x)$ exists at $x=a$, that means slope exists for $f(x)$, so $f(x)$ is differentiable.

14) Given f' as graphed, which could be the graph of f?

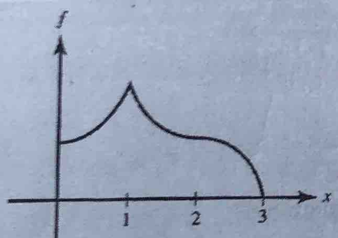
$f'(x)$: 0 — + — 1 — − — 3

$f''(x)$: 0 — − — 2 — + — 3

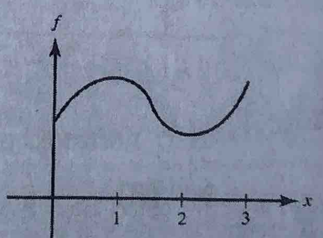




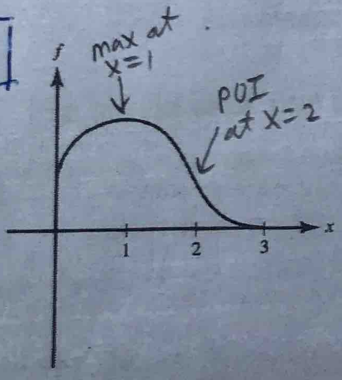
(A)



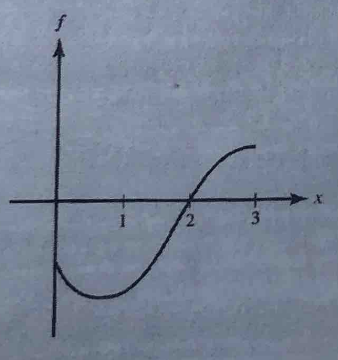
(B)



$\boxed{\text{(C)}}$



(D)



(E)